25/02/2025

Persamaan Strerling

• Aproksimasi Sterling

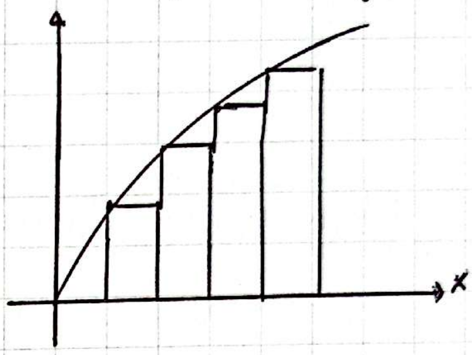
$\ln N! = ?$

$N! = 1.2.3.4 \cdots N$

$\ln N! = \ln_1 + \ln_2 + \ln_3 + \ln N$

$= \sum_{x=1}^{N} \ln x$

Grafik Sterling



Σ = penjumlahan partikel terbatas

$\int$ = penjumlahan partikel tak terbatas

• Misal:

Jika $\ln 1 = 0$ angka $\ln 5 = ?$

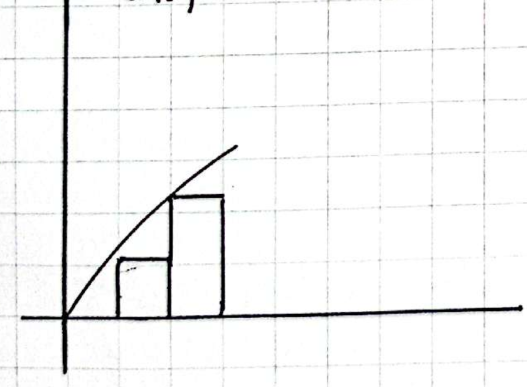
$\ln 2 = 0.6$

$\ln 3 = 1$

$\ln 4 = 2$

$\ln 5 = 3$

Grafik



$\ln N! = \sum_{x=1}^{N} \ln x$

$\cong \int_1^N \ln x\, dx$ → gunakan integral parsial

$\int_a^b u\, dv = uv|_a^b - \int_a^b v\, du$

• Misal

$u = \ln x$ $\quad dv = dx$

$du = \frac{1}{x} dx$ $\quad v = x$

Maka:

$= x \ln x |_1^N - \int_1^N x \frac{1}{x} dx$

$= N \ln N - 0) - (N-1)$

$\ln N! = N \ln N - N + 1$

untuk $N \gg 1$ ($N \cong 1000$) maka

$\int_1^N \ln x . dx$

Contoh :

$$\frac{1}{c+v} + \frac{1}{c-v} = \frac{(c+v)+(c-v)}{c^2 - cv^2 + cv - v^2}$$

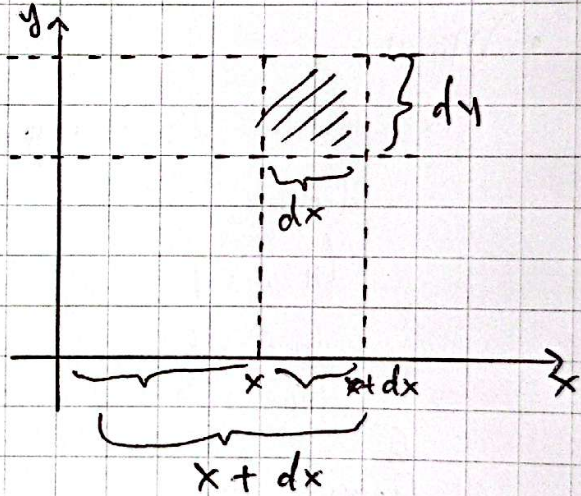
$$= \frac{(c+v)\ (c-v)}{c^2 - v^2}$$

$$= \frac{2c}{c^2 - v^2}$$

Hal yang harus diingat?

$$\int_{-\infty}^{\infty} e^{-ax^2}\, dx = ?$$

koordinat x dan y

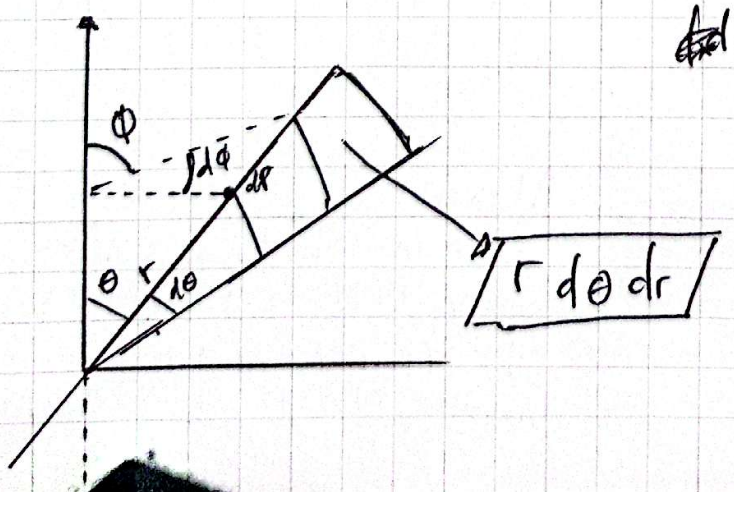


$$\int_{-\infty}^{\infty} e^{-ay^2}\, dy = I$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-ax^2} e^{-ay^2}\, dx\,dy = I^2$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-a(x+y)}\, dx\,dy = I^2$$

ditransformasi kedlm bentuk bola
koordinat x dan y



$dx\,dy = r\,d\theta\,dr$

$r^2 = x^2 + y^2$

$$\frac{\pi}{a} = I^2 \Rightarrow I = \sqrt{\frac{\pi}{a}}$$

$$\frac{d}{da}\int_{-\infty}^{\infty} e^{-ax^2}\,dx = \frac{d}{da}\left[\sqrt{\frac{\pi}{a}}\right]$$

$$\int_{-\infty}^{\infty} \frac{d}{da}\left(e^{-ax^2}\right) dx = \frac{d}{da}\sqrt{\frac{\pi}{a}}$$

$$-\int_{-\infty}^{\infty} x^2 e^{-ax^2}\,dx = \frac{d}{da}\sqrt{\frac{\pi}{a}}$$

$$= \sqrt{\pi}\,\frac{d}{da}\left(a^{-\frac{1}{2}}\right)$$

$$-\int_{-\infty}^{\infty} x^2 e^{-ax^2}\,dx = \sqrt{\pi}\left(-\frac{1}{2}a^{-3/2}\right)$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a^3}}$$