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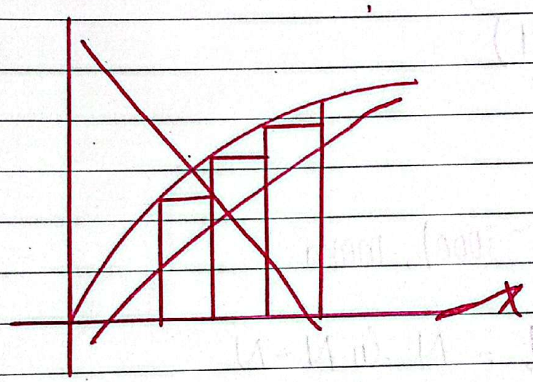
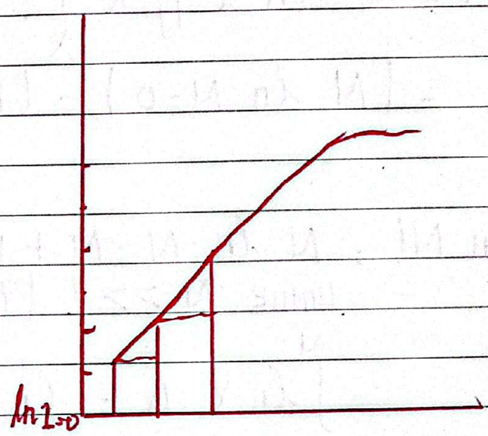
Aproksimasi Sterling

$\ln N! = ?$
$N! = 1 . 2 . 3 . 4 \ldots N$

$\ln N! = \ln_1 + \ln_2 + \ln_3 + \ldots + \ln N$
$= \sum_{x=1}^{N} \ln x$

Grafik Sterling





~~ln N!~~ ≈ 0

Persamaan Sterling

$$\boxed{\begin{aligned} \text{Jika } \ln 1 &= 0 \\ \ln 2 &= 0{,}6 \\ \ln 3 &= 1 \\ \ln 4 &= 2 \\ \ln 5 &= 3 \end{aligned}}$$

$\Sigma \rightarrow$ Penjumlahan partikel terbatas

$\int \rightarrow$ Penjumlahan partikel terbatas

Contoh $\int$ misal atom dikramik bila $\int$ berarti hanya ada 1 tapi panjang

$u = \ln$ $v = x$

$$\ln N! = \sum_{x=1}^{N} \ln x$$

$$\cong \int_1^N \ln x \, dx \rightarrow \text{Integral Persial}$$

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

$u = \ln x$	$dv = dx$
$du = \frac{1}{x}\,dx$	$v = x$

Misal

$$u = x \ln x \Big|_1^N x \frac{1}{x}\,dx$$

$$= (N \ln N - 0) - (N - 1)$$

$$\ln N! = N \ln N - N + 1$$

untuk $N >> 1$ ($N \cong 1000$), maka

$$\int_1^N \ln x \, dx = \ln N! = N \ln N - N$$

$$\int_{-\infty}^{\infty} e^{-ax^2}\,dx = ?$$

Misal $= \int_{-\infty}^{\infty} e^{-ax^2}\,dx = 1$

$$= \int_{-\infty}^{\infty} e^{-ay^2}\,dy = 1$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-ax^2} \cdot e^{-ay^2}\,dx\,dy = 1^2$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-a(x+y)}\,dx\,dy = I^2$$

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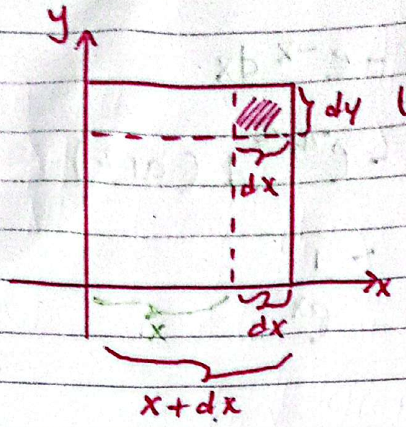
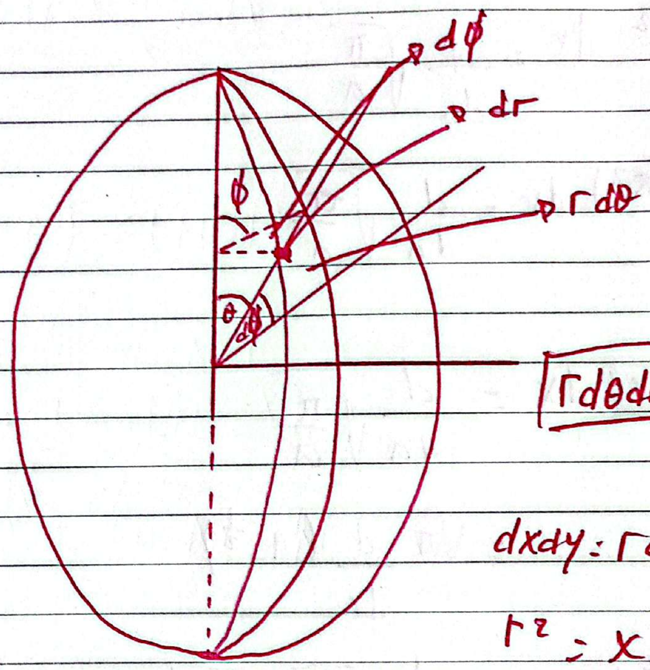
Transformasi koordinat



luas Permukaan

Ditransformasikan kedalam koordinat x & y



$\boxed{r\,d\theta\,dr}$

$dxdy = r\,d\theta\,dr$

$r^2 = x^2 + y^2$

Contoh:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-a(x^2+y^2)}\,dxdy$$

$$\int_{r=0}^{\infty}\int_{\theta=0}^{2\pi} e^{-ar^2}\,r\,dr\,d\theta \qquad = \int_0^{2\pi} d\theta$$

$$2\pi\int_0^{\infty} e^{-ar^2}\,r\,dr \qquad = \theta\Big|_0^{2\pi}$$

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$$\frac{2\pi}{2a}\int_0^\infty e^{-ar^2}\,d(ar^2)$$

$$\frac{2\pi}{2a}\left[-e^{-ar^2}\Big|_0^\infty\right] = \frac{\pi}{a}$$

$$-e^{-x}\,dx$$

$$-e^{-ar^2}\,d(ar^2)$$

$$-\frac{1}{e^x}$$

$$I^2 = \frac{\pi}{a}$$

$$I = \sqrt{\frac{\pi}{a}}$$

$$\frac{d}{da}\int_{-\infty}^{\infty} e^{-ax^2}\,dx = \frac{d}{da}\sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty}\frac{d}{da}\left(e^{-ax^2}\right)dx = \frac{d}{da}\sqrt{\frac{\pi}{a}}$$

$$-\int_{-\infty} x^2 e^{-ax^2}\,dx = \frac{d}{da}\sqrt{\frac{\pi}{a}}$$

$$= \sqrt{\pi}\,\frac{d}{da}\left(a^{-\frac{1}{2}}\right)$$

$$-\int_{-\infty}^{\infty} x^2 e^{-ax^2}\,dx = \cancel{\frac{d}{da}\sqrt{\frac{\pi}{a}}}\;\sqrt{\pi}\left(-\frac{1}{2}a^{-3/2}\right)$$

$$= \frac{1}{2}\sqrt{\frac{\pi}{a^3}}$$