No. Fisika Statistik

Date: 25-02-2025

Aproksimasi Sterling

$\ln N! = ?$

$N! = 1 \cdot 2 \cdot 3 \cdot 4 \ldots N$

$\ln N! = \ln_1 + \ln_2 + \ln_3 \ldots \ln N$

$= \sum_{x=1}^{N} \ln x$

$$\ln N! = \sum_{x=1}^{N} \ln x$$

$$\cong \int_1^N \ln x \, dx \rightarrow \text{gunakan integral parsial}$$

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

Misal: $u = \ln x$ $\quad dv = dx$

$du = \frac{1}{x} dx$ $\quad v = x$

Maka

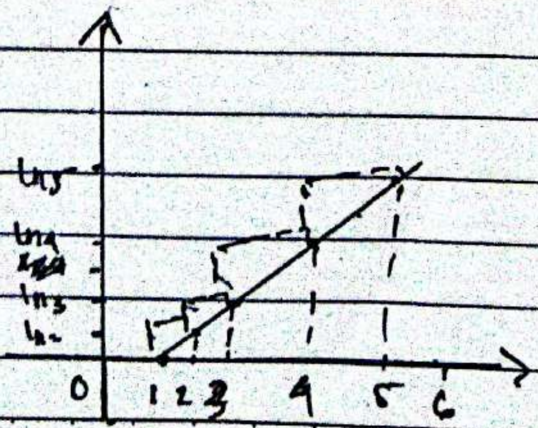
$$= x \ln x \Big|_1^N - \int_1^N x \frac{1}{x} dx$$

$$= (N \ln N - 0) - (N - 1)$$

$$\ln N! = N \ln N - N + 1$$

untuk $N \gg 1$ ($N \cong 1000$).

No	x	$\ln x$
1.	0	1
2.	1	2
3.	2	3
4.	3	4
5.	4	5
6.	5	6



No.

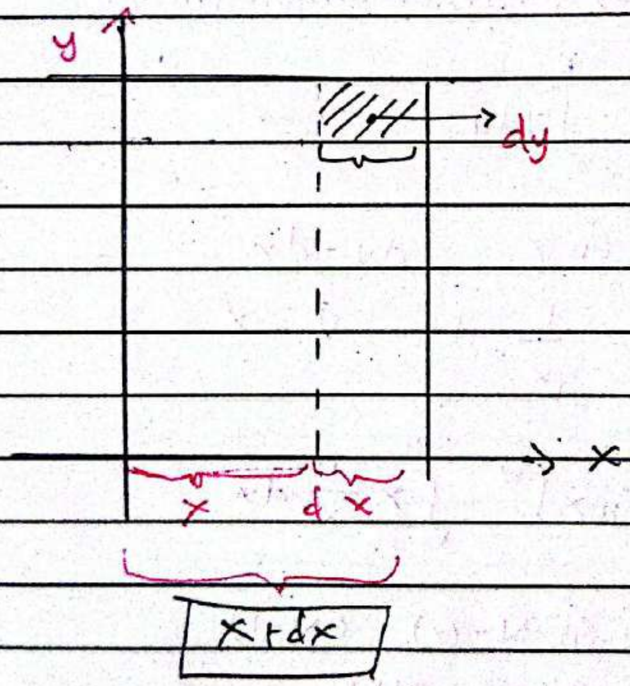
Date :

$$\int_{-\infty}^{\infty} e^{-ax^2}\,dx = ?$$

$$\int_{-\infty}^{\infty} e^{-ax^2}\,dx = I = \sqrt{\frac{\pi}{a}}$$

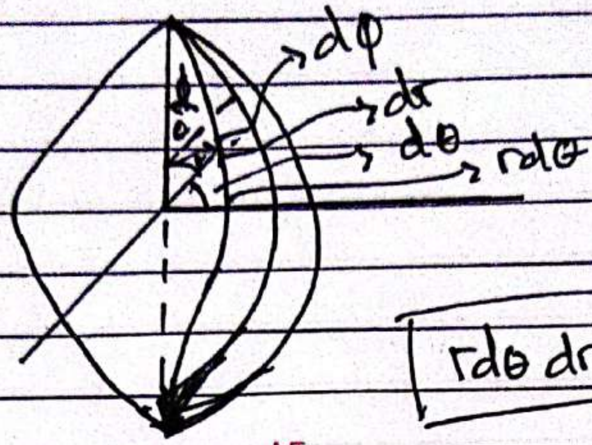
$$\int_{-\infty}^{\infty} e^{-ay^2}\,dy = I = \sqrt{\frac{\pi}{a}} \rightarrow \frac{\pi}{a}$$

$$\boxed{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-a(x^2+y^2)}\,dx\,dy = I^2}$$



No.

Date :



$$\boxed{r d\theta\, dr}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\, dx\, dy \qquad = \int_0^{2\pi} d\theta$$

$$\int_{r=0}^{\infty}\int_{\theta=0}^{2\pi} e^{-ar^2}\, r\, dr\, d\theta \qquad = \theta\Big|_0^{2\pi}$$

$$\boxed{\int_0^{\infty} e^{-x} dx =}$$

$$= 2\pi \int_0^{\infty} e^{-ar^2}\, r\, dr$$

$$= \frac{2\pi}{2a}\int_0^{\infty} e^{-ar^2}\, d(ar^2) = \frac{2\pi}{2a}\left[-e^{-ar^2}\Big|_0^{\infty}\right] = \frac{\pi}{a}$$

$$I^2 = \frac{\pi}{a}$$

$$I = \sqrt{\frac{\pi}{a}}$$

$$\frac{d}{da}\int_{-\infty}^{\infty} e^{-ax^2}\, dx = \frac{d}{da}\left[\sqrt{\frac{\pi}{a}}\right]$$

$$\int_{-\infty}^{\infty} \frac{d}{da}\left(e^{-ax^2}\right) dx = \frac{d}{da}\sqrt{\frac{\pi}{a}}$$

$$-\int_{-\infty}^{\infty} x^2 e^{-ax^2}\, dx = \frac{d}{da}\sqrt{\frac{\pi}{a}}$$

$$= \sqrt{\pi}\,\frac{d}{da}\left(a^{-\frac{1}{2}}\right)$$

$$-\int_{-\infty}^{\infty} x^2 e^{-ax^2}\, dx = \sqrt{\pi}\left(-\frac{1}{2}\, a^{-3/2}\right)$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{a^3}}$$