

25/2025
2

No
Date

Aproksimasi sterling

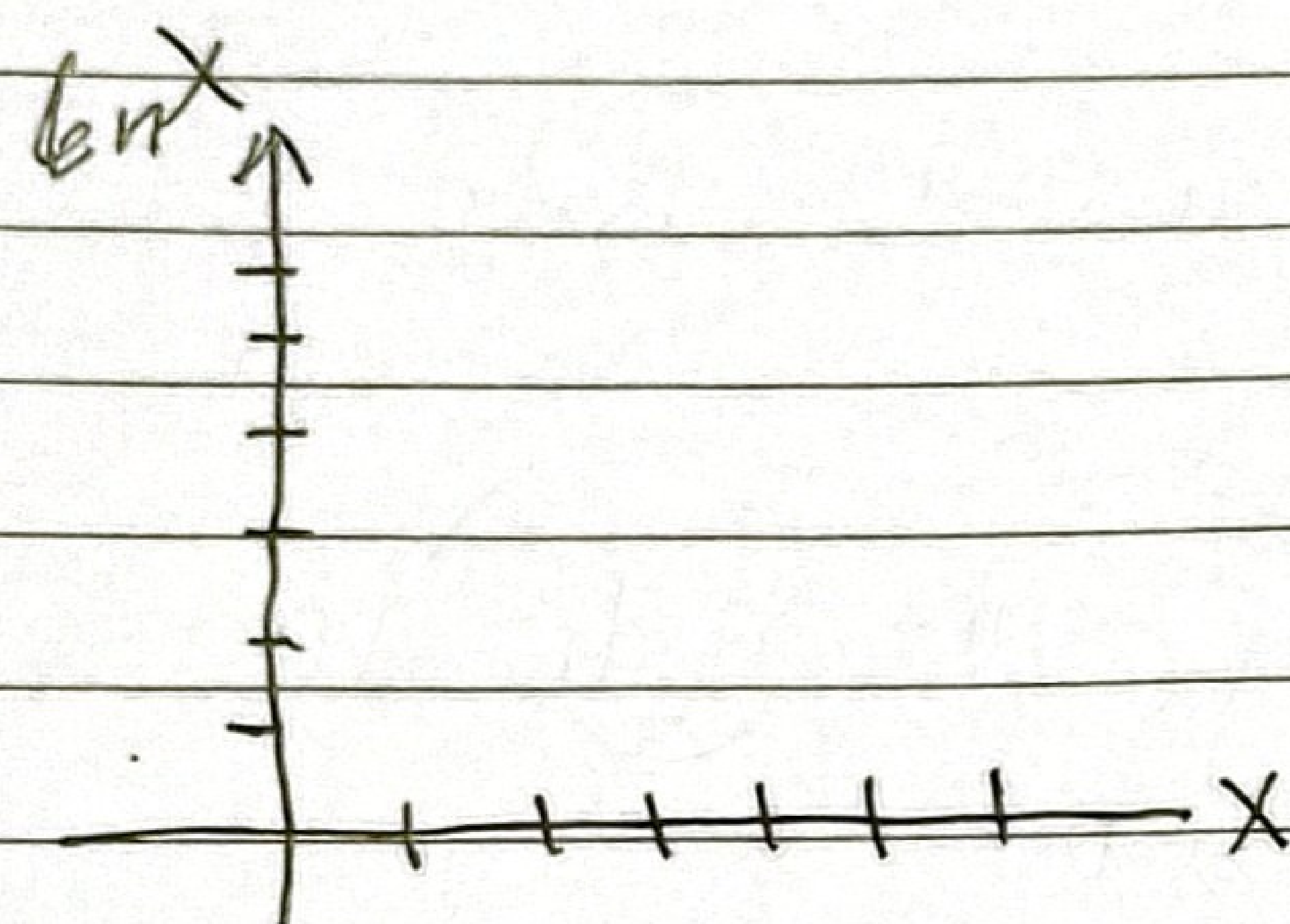
$$\ln N! = ?$$

$$N! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \dots \cdot N$$

$$\begin{aligned} \ln N! &= \ln 1 + \ln 2 + \ln 3 + \dots + \ln N \\ &= \sum_{x=1}^N \ln x \end{aligned}$$

untuk menjawab hal tersebut diatas, sterling membuat grafik

Persamaan sterling



$\ln 0$

x	$\ln x$
1	
2	
3	
4	
5	
6	

Jika $\ln 1 = 0$

$\ln 2 = 0.6$

$\ln 3 = 1$

$\ln 4 = 2$

$\ln 5 = 3$

angka $\ln 5! =$

$$\ln N! = \sum_{x=1}^N \ln x$$

$$= \int_1^N \ln x dx$$

$$\int_a^b v dv = uv \Big|_a^b - \int_a^b v dv$$

Misal : $v = \ln x$

$$dv = dx$$

$$dv = \frac{1}{x} dx$$

$$v = x$$

$$\text{Maka : } x \ln x \Big|_1^N - \int_1^N x \frac{1}{x} dx$$

$$= (N \ln N - 0) - (N - 1)$$

$$\ln N! = N \ln N - N + 1$$

Untuk $N \gg 1$ ($N = 1000$), maka

$$\int_1^N \ln x dx = \ln N! = N \ln N - N$$

$\int$ = Integral

$!$ = Faktorial

∞ = Tak terhingga

$\sin$ = Sin

hal yg harus diingat !

$$\frac{1}{e+v} + \frac{1}{e-v}$$

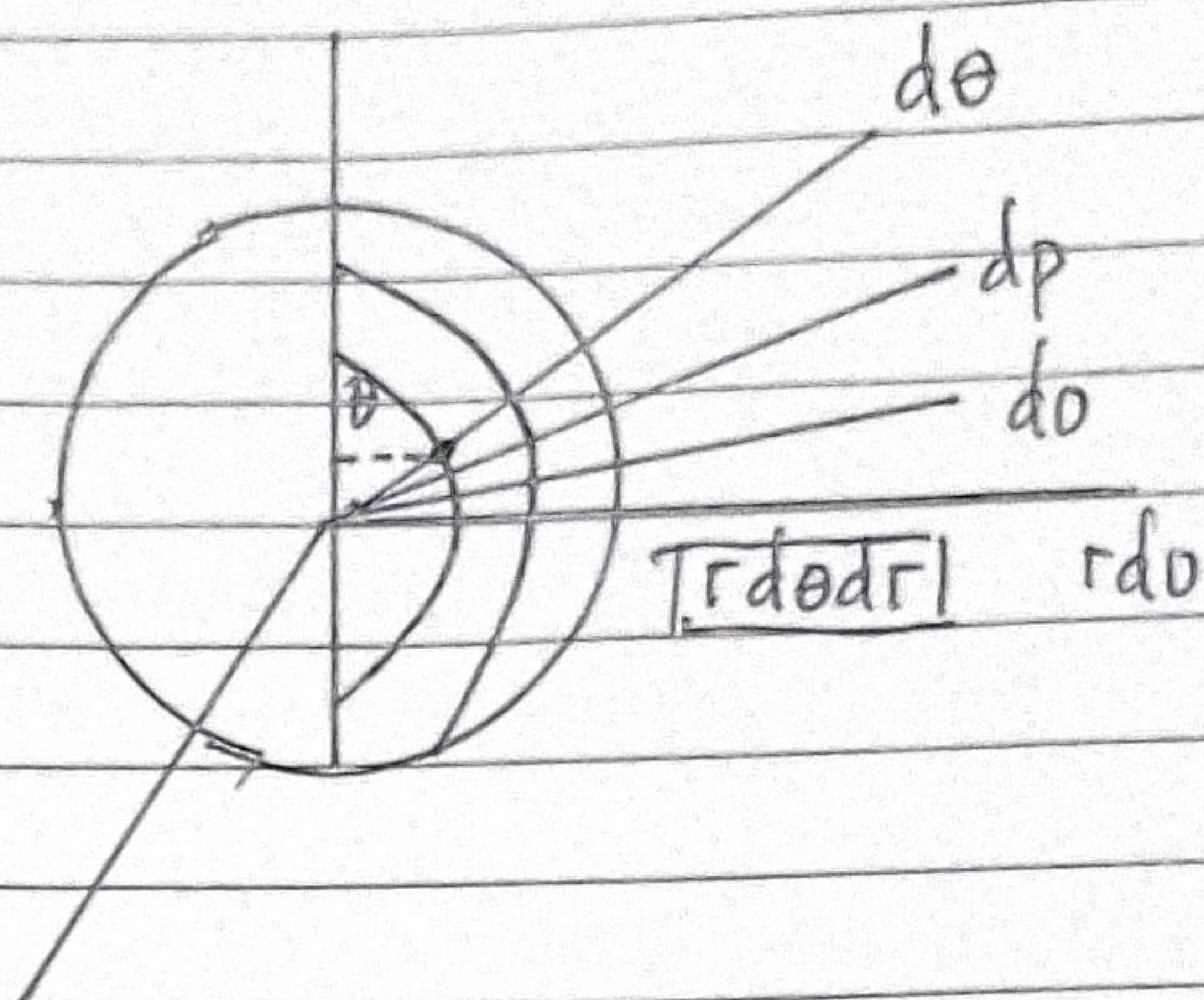
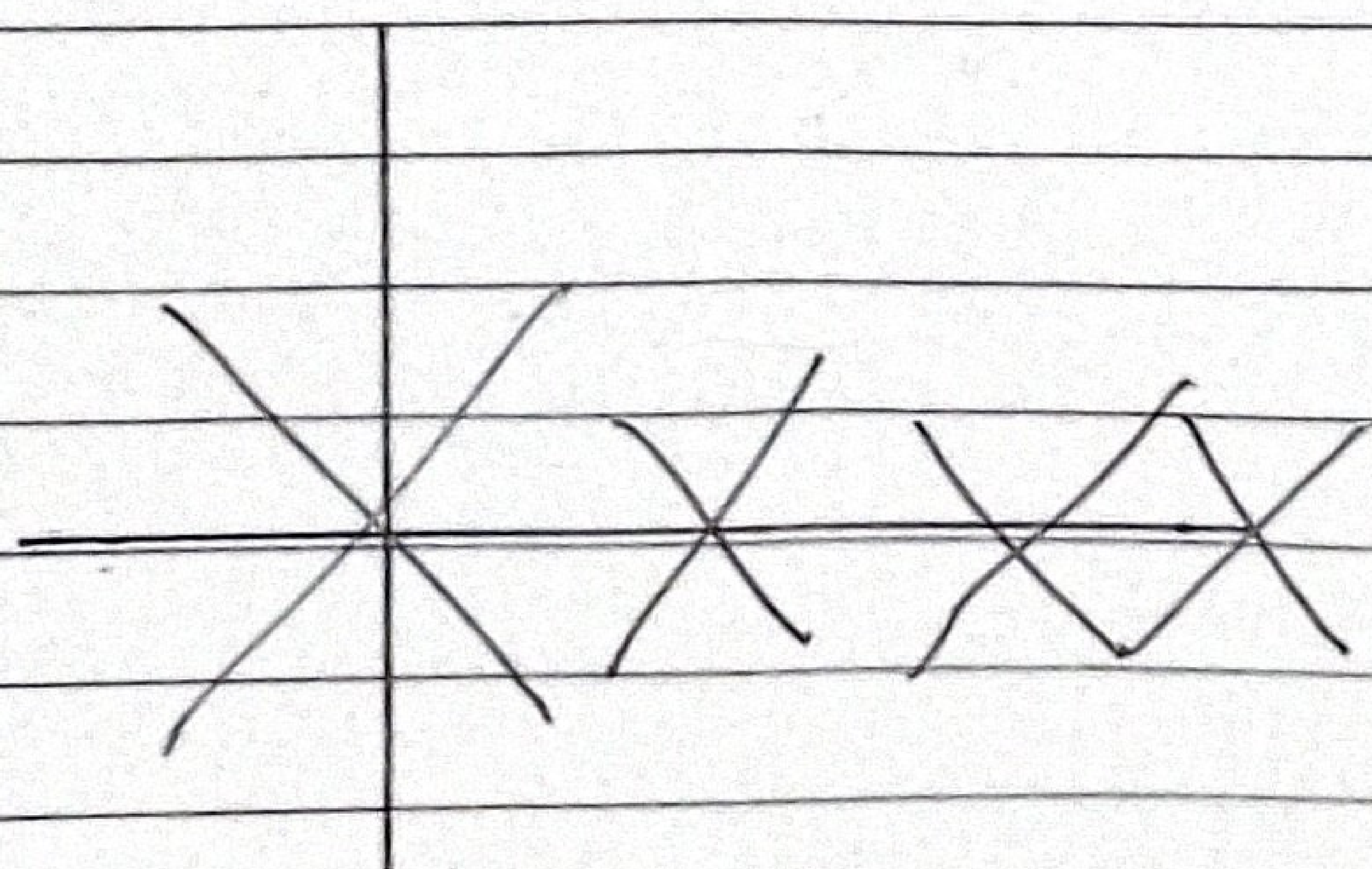
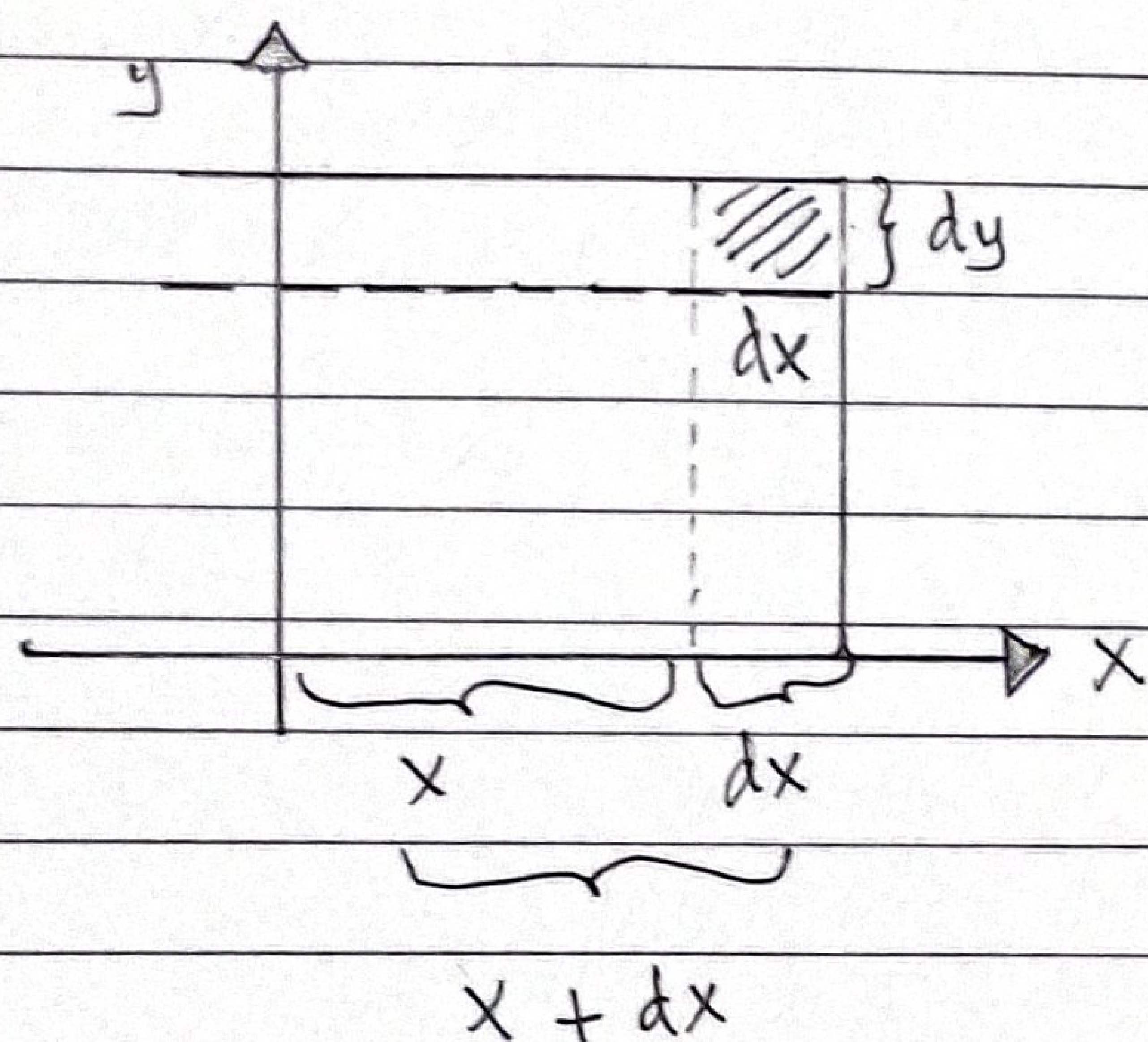
$$= \frac{(c+v) + (c-v)}{c^2 - v^2} = \frac{(c+v) + (c-v)}{c^2 - v^2} = \frac{2c}{c^2 - v^2}$$

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = ?$$

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = 1$$

$$\int_{-\infty}^{\infty} e^{-ay^2} dy = 1$$

$$\iint_{-\infty}^{\infty} a(x^2 + y^2) dx dy = 1^2$$



$$\iint_{-\infty}^{\infty} a(x^2 + y^2) dx dy = \int_0^{2\pi} d\theta$$

$$\int_{r=0}^{\infty} \int_{\theta=0}^{2\pi} e^{-ar^2} r dr d\theta$$

$$r=0 \quad \theta=0$$

$$2\pi \int_0^{\infty} e^{-ar^2} r dr$$

$$= \frac{2\pi}{2a} \int_0^{\infty} e^{-ar^2} d(ar^2) = \frac{2\pi}{2a}$$

$$= \left[-e^{-ar^2} \right]_0^{\infty}$$

$$\int_0^{\infty} e^{-x} dx = -e^{-x} \Big|_0^{\infty}$$

$$\pi^2 = \frac{\pi}{a}$$

$$1 = \sqrt{\frac{\pi}{a}}$$

$$\frac{d}{da} \int_{-\infty}^{\infty} e^{-ax^2} dx = \frac{d}{da} \left[\sqrt{\frac{\pi}{a}} \right]$$

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \frac{d}{da} \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{d}{da} \sqrt{\frac{\pi}{a}}$$

$$= \sqrt{\pi} \frac{d}{da} \left(a^{-\frac{1}{2}} \right)$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \sqrt{\pi} \left(-\frac{1}{2} a^{-\frac{3}{2}} \right)$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a^3}}$$