Selasa, 25/02 2025

"Fisika Statistik"

Persamaan ~~Sterling Sterling~~ Sterling

dan

Tumbukan.

$\ln N! = ?$

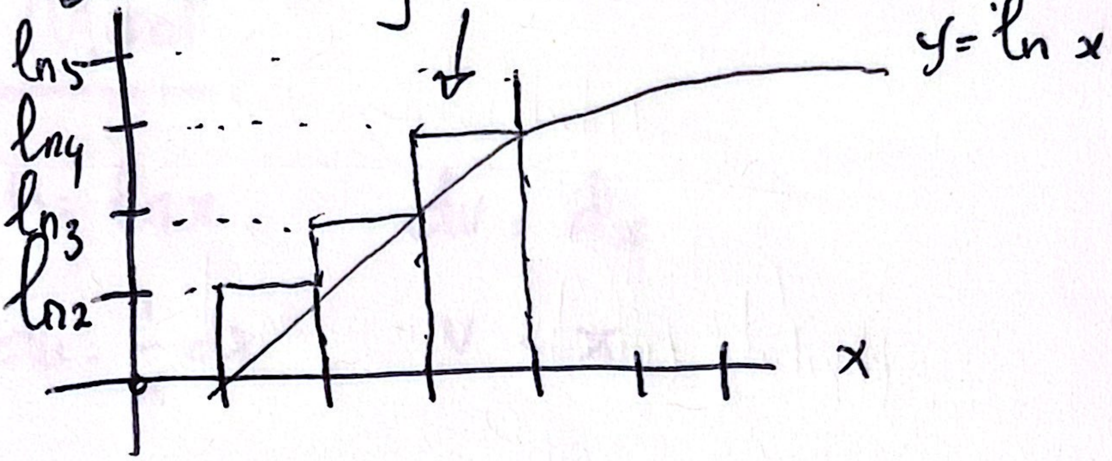
$N! = 1, 2, 3, 4, \ldots N$ (jenis partikel)

Partikel Terbatas.

$$\boxed{\begin{aligned} \ln N! &= \ln 1 + \ln 2 + \ln 3 + \ldots + \ln N \\ &= \sum_{x=1}^{N} \ln x . \quad \text{(partikel terbatas)} \end{aligned}}$$

Grafik hubungan x dengan $\ln x$

$\ln x = y = \ln x$



$\ln 1 = 0$

$\ln 2 = 1$, dst ...

$$\ln(N!) = \int_0^x \ln(x)\, dx = x \ln x - x + 1$$

Partikel tak terbatas

$$\ln N! = \sum_{x=1}^{N} \ln x$$

u = persamaan yang berbeda.

$\approx \ln x \, dx \rightarrow$ gunakan integral parsial

$$\int_a^b u\, dv = uv \Big|_b^a - \int_a^b v\, du$$

Misal

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}\,dx \qquad v = x$

Maka:

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

$$= x\ln x\Big|_1^N - \int_1^N x\,\frac{1}{x}\,dx$$

$$= (N\ln N - 0) - (N-1)$$

$$\ln N! = N\ln N - N + 1$$

Kasus tak terbatas

Hal yang harus di ingat !!

$$\boxed{\int_{-\infty}^{\infty} e^{-ax^2}\,dx = ?}$$

Contoh: $\dfrac{1}{e+v} + \dfrac{1}{e-v} = \dfrac{(c+v)+(c-v)}{c^2-v^2} = \dfrac{2c}{c^2}.$

$$\frac{l_A}{\sqrt{e^2+v^2}} + \frac{l_A}{\sqrt{e^2-v^2}} =$$

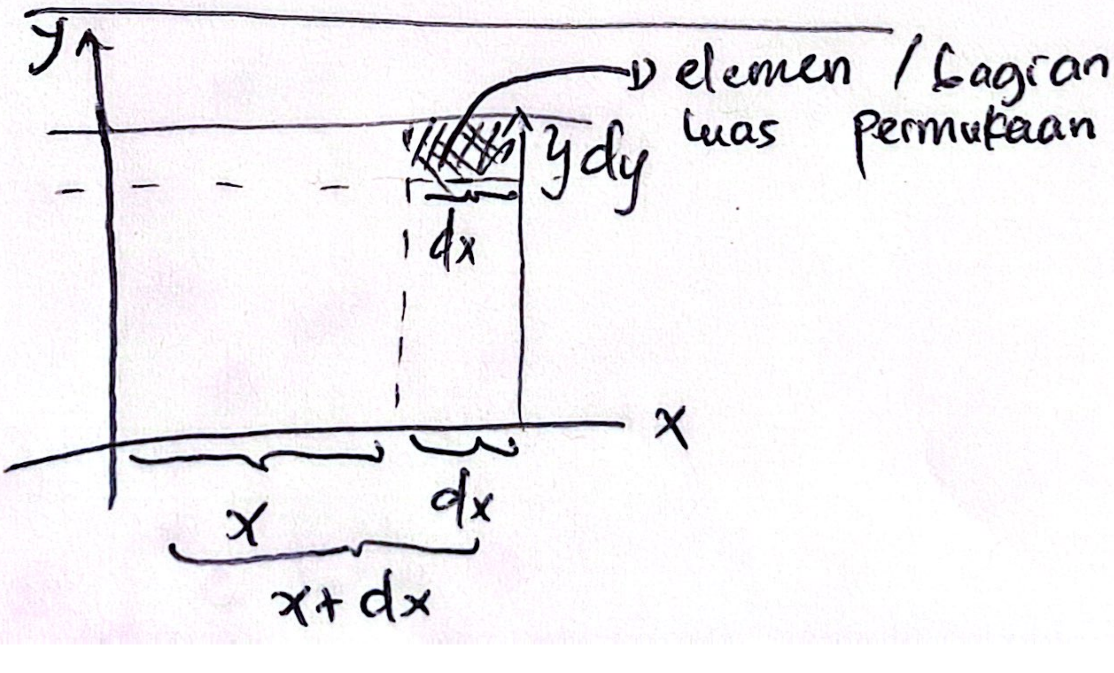
$$\int_{\infty}^{\infty} e^{-ax^2}\,dx = ?$$

$$\int_{-\infty}^{\infty} e^{-ax^2}\,dx = 1$$

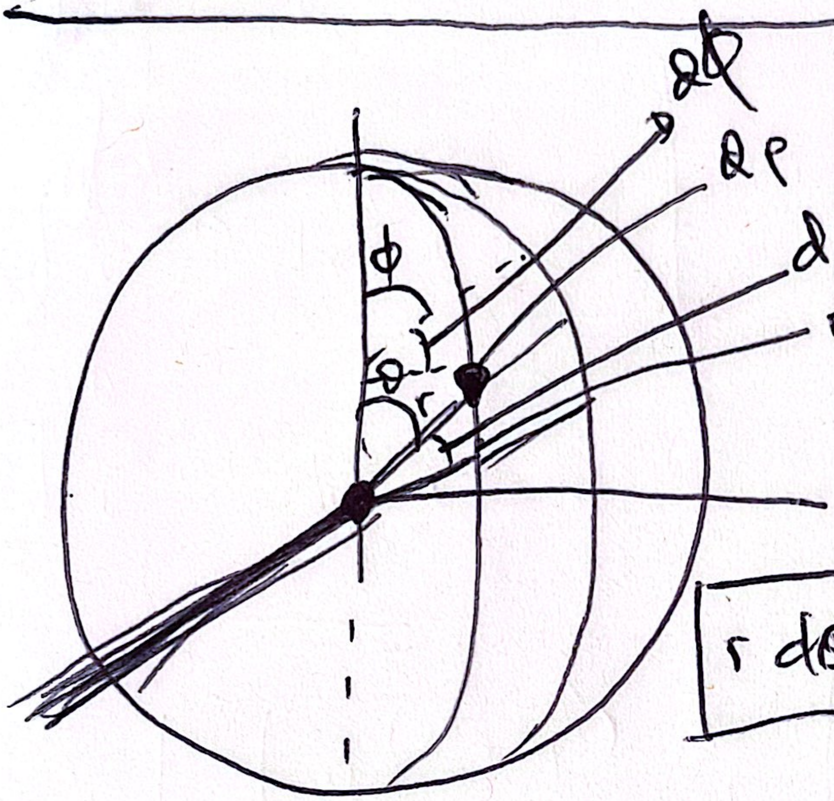
$$\int_{-\infty}^{\infty} e^{-ax^2}\,dy = 1$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a^{(x^2+y^2)}\,dxdy = I^2$$

Transformasi Koordinat



Transformasi koordinat bola x dan y



$$\boxed{r\,d\theta\,dr}$$

$$dydy = r\,d\theta\,dr$$

$$r^2 = x^2 + y^2$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\,dx\,dy$$

$$\int_{\rho=0}^{\infty}\int_{\theta=0}^{2\pi} e^{-ar^2}\,r\,dr\,d\theta$$

$$2\pi\int_0^{\infty} e^{-ar^2}\,r\,dr$$

→ harus di sama kan.

$$= \frac{2\pi}{2a}\int_0^{\infty} e^{-ar^2}\,d(ar^2)$$

$$= \frac{2\pi}{2\pi}\left[-e^{-ar^2}\Big|_0^{\infty}\right]$$

$$= -\frac{2\pi}{2\pi}(0-1) = \frac{\pi}{2}$$

$$I^2 = \frac{\pi}{a}$$

$$I = \sqrt{\frac{\pi}{a}}$$

$$\frac{d}{da}\int_{-\infty}^{\infty} e^{-ax} dx = \frac{d}{da}\left[\sqrt{\frac{\pi}{a}}\right]$$

$$\int_{-\infty}^{\infty} \frac{d}{da}\left(e^{-ax^2}\right) dx = \frac{d}{da}\sqrt{\frac{\pi}{a}}$$

$$-\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{d}{da}\sqrt{\frac{\pi}{a}}$$

$$= \sqrt{\pi}\,\frac{d}{da}\left(a^{-\frac{1}{2}}\right)$$

Sehingga:

$$-\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \sqrt{\pi}\left(-\frac{1}{2}\, a^{-3/2}\right)$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{a^3}}$$