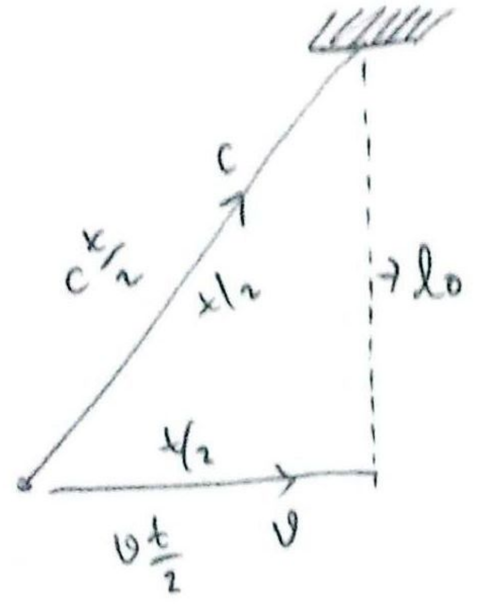
Pemuaian Waktu

Modern Physics
P. Tipler



$$\left(c\frac{t}{2}\right)^2 = l_0^2 + \left(v\frac{t}{2}\right)^2$$

$$c^2\frac{t^2}{4} = l_0^2 + v^2\frac{t^2}{4}$$

$$l_0^2 = \frac{t^2}{4}(c^2 - v^2)$$

$$4l_0^2 = t^2(c^2 - v^2)$$

$$t^2 = \frac{4l_0^2}{c^2 - v^2}$$

$$t^2 = \frac{4l_0^2/c^2}{\left(1 - \frac{v^2}{c^2}\right)}$$

$$t = \frac{2l_0/c}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\boxed{t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

Contoh

* Berapa kelajuan roket yang loncengnya berjalan 1 detik terlambat dalam 1 jam relatif terhadap lonceng di bumi

Dik : lonceng dalam keadaan bergerak, berarti $t = 3600$ detik, maka

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow \frac{t}{t_0} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\frac{t_0}{t} = \sqrt{1 - \frac{v^2}{c^2}} \rightarrow \frac{v^2}{c^2} = 1 - \frac{t_0^2}{t^2}$$

$$v^2 = c^2 \left(1 - \frac{t_0^2}{t^2}\right)$$

$$v = c\sqrt{1 - \frac{t_0^2}{t^2}}$$

$$= 3 \cdot 10^8 \sqrt{1 - \left(\frac{3600}{3601}\right)^2}$$

Sekarang mari kita bahas efek doppler

$$t = t_0 \left(\frac{1 + v/c}{1 - v/c}\right)$$

v = kecepatan pengamat
v' = kecepatan sumber

Contoh

* Berapa kelajuan roket yang loncengnya berjalan 1 detik terlambat dalam 1 jam relatif terhadap lonceng di bumi

Dik : lonceng dalam keadaan bergerak, berarti

$t = 3600$ detik, maka

$$t = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}} \rightarrow \frac{t}{t_o} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{t_o}{t} = \sqrt{1-\frac{v^2}{c^2}} \rightarrow \frac{v^2}{c^2} = 1 - \frac{t_o^2}{t^2}$$

$$v^2 = c^2\left(1-\frac{t_o^2}{t^2}\right)$$

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$$= 3 \cdot 10^8 \sqrt{1-\left(\frac{3600}{3601}\right)^2}$$

Sekarang mari kita bahas efek doppler

$$t = t_o\left(\frac{1+v/c}{1-v/c}\right)$$

v = kecepatan pengamat

v' = kecepatan sumber

$$x_1' = \frac{x_1 - v.t}{\sqrt{1-\frac{v^2}{c^2}}}, \quad x_2' = \frac{x_2 - v.t}{\sqrt{1-v^2/c^2}}$$

$$x_1 = v.t + x_1'\sqrt{1-v^2/c^2}$$
$$x_2 = v.t + x_2'\sqrt{1-v^2/c^2}$$

$$x_2 = vt + x_2'\sqrt{1-v^2/c^2}$$
$$x_1 = vt + x_1'\sqrt{1-v^2/c^2}$$

$$x_2 - x_1 = (x_2' - x_1')\sqrt{1-v^2/c^2}$$
$$L = L_0\sqrt{1-v^2/c^2}$$

Jadi akan nampak lebih pendek dari panjang mula-mula kalau benda tersebut bergerak dengan kecepatan v

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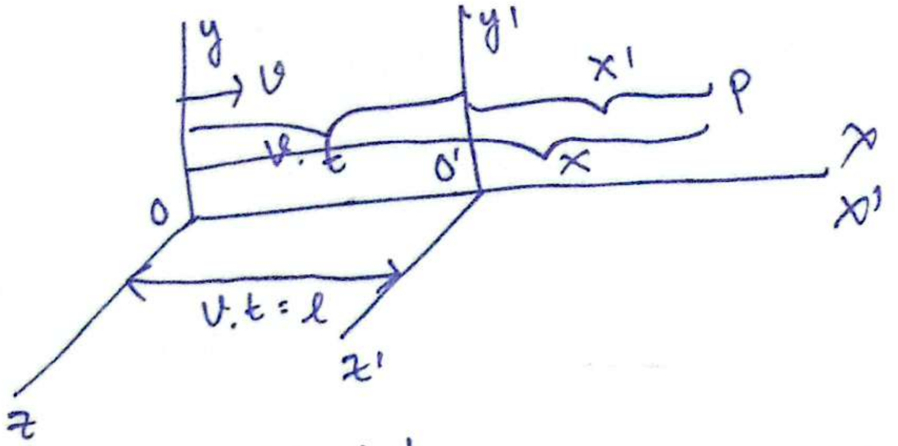
Kontraksi lorentz



$$x = x' + v.t$$

secara relativistik $x = \dfrac{x' + v.t}{\sqrt{1-\frac{v^2}{c^2}}}$

selanjutnya $t = \dfrac{t' + \frac{v.x'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$

Bagaimana prosesnya?

Posisi P dilihat dari X

$$x = x' + v.t \quad \dots\dots \text{(1)}$$

$$= x' + l$$

↓

atau pembuktian dengan kontraksi panjang

- Dalam relativitas klasik kita memperoleh

$$x = x' + v.t \longrightarrow x = \frac{x' + v.t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = t_0 \longrightarrow t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = t' + \frac{x'}{c^2} \cdot v \longrightarrow t = \frac{t' + \frac{x'}{c^2} v}{\sqrt{1 - v^2/c^2}}$$

$$v = v + v' \longrightarrow v \frac{v_1 + v_2}{\left(1 - \frac{v_1 \cdot v_2}{c}\right)}$$

Problem

1) $L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$

$0{,}98 L_0 = L_0 \sqrt{1 - \frac{v^2}{c^2}}$

$0{,}98^2 = 1 - \frac{v^2}{c^2}$

$\frac{v^2}{c^2} = 1 - 0{,}98$

$v = 0{,}199\, c$

2) $L_0 = 12{,}740\ \text{km} = 1{,}274 \times 10^7\ \text{m}$

$v = 3 \times 10^9\ \text{m/s}$

$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$

$= (1{,}274 \times 10^7) \times \sqrt{1 - \left(\frac{3 \times 10^9}{3 \times 10^8}\right)^2}$

$L = 1{,}274 \times 10^7 \times \sqrt{1 . 10^{-8}}$

$= 1{,}274 \times 10^7 - 637\ \text{cm}$

3) $Lx' = Lx \sqrt{1 - \frac{v^2}{c^2}}$

$= 0{,}5 \times \sqrt{1 - 0{,}81}$

$= 0{,}5 \times 0{,}435$

$L' = \sqrt{Lx'^2 + Ly^2}$

$= \sqrt{0{,}097 + 0{,}75}$

$= 0{,}892\ \text{m}$

$\tan \theta' = \frac{Ly}{Lx'}$

$= 3{,}99$

$\theta = 75{,}9°$

4) a. $v = v' + v$

$$= 2 m/s + 0{,}1 c$$

b. $$v = \frac{2 m/s + 0{,}1 c}{1 + \frac{2 . m/s + 0{,}1 c}{c^2}}$$

$$v = 0{,}1 c + 2 m/s$$

5) $$v' = \frac{0{,}8c + 0{,}6 c}{1 + \frac{0{,}8 + (0{,}6)}{c^2}} = 0{,}996 c$$