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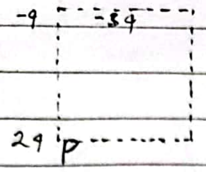
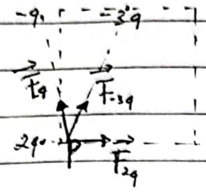
NPM : 2313022022
Kelas : 23 A

UTS

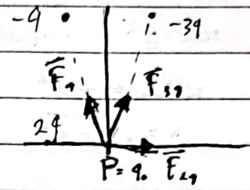
1.)



a.



misal muatan p = $q_0$



b. $F_p = kq_0\left[\frac{q}{r^2}\left[\frac{-\hat{i}+\hat{j}}{r}\right]+\frac{2q}{r^2}[\hat{i}]+\frac{3q}{r^2}\left[\frac{\hat{i}-\hat{j}}{r}\right]\right]$

$= kq_0\left[\left[\frac{-q}{r^3}+\frac{2q}{r^2}+\frac{3q}{r^3}\right]\hat{i}+\left[\frac{q}{r^3}-\frac{3q}{r^3}\right]\hat{j}\right]$

c. $E_p = k\left[\frac{q}{r^2}\left[\frac{-\hat{i}+\hat{j}}{r}\right]+\frac{2q}{r^2}[\hat{i}]+\frac{3q}{r^2}\left[\frac{\hat{i}-\hat{j}}{r}\right]\right]$

$= k\left[\left[\frac{-q}{r^3}+\frac{2q}{r^2}+\frac{3q}{r^3}\right]\hat{i}+\left[\frac{q}{r^3}+\frac{3q}{r^3}\right]\hat{j}\right]$

d.

e. $V_p = k\left[\frac{-q}{r}+\frac{2q}{r}-\frac{3q}{r}\right]$

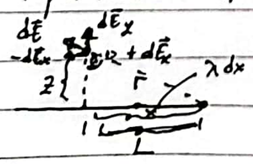
$= \frac{k}{r}\left[-q+2q-3q\right]$

$= -\frac{2kq}{r}$

2.) Diket :
panjang = L
rapat muatan = $\lambda$

Ditanya : a. kuat medan ujung kawat ?
b. kuat medan tengah kawat ?

a.



$d\vec{E} = d\vec{E}_x - d\vec{E}_x + d\vec{E}_\chi$

$= d\vec{E}_\chi$

$= d\vec{E}\cos\theta$

$\cos\theta = \frac{z}{r}$

$r = \frac{z}{\cos\theta}$

$r = \sqrt{z^2+x^2}$

$E = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{2\lambda\,dx}{r^2}\cos\theta$

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$$= \frac{1}{4\pi\varepsilon_0} \int_0^L \frac{2\lambda z}{(x^2+z^2)^{3/2}}\, dx$$

$$= \frac{2\lambda z}{4\pi\varepsilon_0} \int_0^L (x^2+z^2)^{3/2}\, dx$$

misal:

$u = x^2 + z^2$ batas 0 → L

$\frac{du}{dx} = 2x$ $u = z^2$ $u = L^2 + z^2$

$dx = \frac{du}{2x}$ $x^2 = u - z^2$ $z = \sqrt{u - x^2}$

$x = \sqrt{u - z^2}$

$$= \frac{2\lambda z}{4\pi\varepsilon_0} \int_{z^2}^{L^2+z^2} u^{\frac{3}{2}} \frac{du}{2x}$$

$$= \frac{2\lambda}{4\pi\varepsilon_0} \int_{z^2}^{L^2+z^2} u^{3/2}\, du \cdot \int_{z^2}^{L^2+z^2} \frac{\sqrt{u-x^2}}{2\sqrt{u-z^2}}\, du$$

$$= \frac{2\lambda}{4\pi\varepsilon_0} \left[ \frac{-2}{u^{\frac{1}{2}}} \Big|_{z^2}^{L^2+z^2} \right] \cdot \frac{1}{2} \Big[$$

$$= \frac{2\lambda}{4\pi\varepsilon_0} \left[ \frac{-2}{L+z} + \frac{2}{z} \right] \cdot \frac{1}{2} \left[ \frac{2}{3} (u-x^2)^{3/2} \cdot 2(u-x^2)^{\frac{1}{2}} \Big|_{z^2}^{L^2+z^2} \right]$$

$$= \frac{2\lambda}{4\pi\varepsilon_0} \left[ \frac{-2z + 2L + 2z}{Lz + z^2} \right] \cdot \frac{1}{2} \left[ \frac{4}{3} (L^2 + z^2 - x^2)^{\frac{3}{2}} \cdot (L^2 + z^2 - x^2)^{\frac{1}{2}} \right] - \frac{4}{3} \left[ z^2 - x^2 \right]^{\frac{3}{2}} \cdot (z^2 - x^2)^{\frac{1}{2}} \Big]$$

$$= \frac{2\lambda}{4\pi\varepsilon_0} \left[ \frac{2L}{Lz + z^2} \right] \frac{1}{2} \left[ \frac{4}{3} \left[ L^2 + z^2 - x^2 \right]^2 - \frac{4}{3} (z^2 - x^2)^2 \right]$$

$$= \frac{2\lambda L}{4\pi\varepsilon_0\, 2\sqrt{L^2 + z^2}}$$

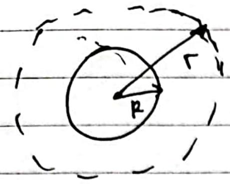
b. $$E = \frac{\lambda z}{4\pi\varepsilon_0} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dx}{(x^2+z^2)^{3/2}}$$

$$= \frac{2\lambda z}{4\pi\varepsilon_0} \int_0^{\frac{L}{2}} \frac{dx}{(x^2+z^2)^{3/2}}$$

$$= \frac{2\lambda z}{4\pi\varepsilon_0} \left[ \frac{x}{z^2\sqrt{x^2+z^2}} \Big|_0^{\frac{L}{2}} \right]$$

$$= \frac{2\lambda z}{4\pi\varepsilon_0} \left[ \frac{L/2}{z^2\sqrt{\frac{L^2}{4} + z^2}} - \frac{0}{z^2\sqrt{0+z^2}} \right]$$

$$= \frac{\lambda L}{4\pi\varepsilon_0 z\sqrt{\frac{L^2}{4}+z^2}}$$

3.) 

$r > R$

$$\oint \vec{E}\, dA = \frac{Q_{in}}{\varepsilon_0}$$

$$\vec{E}(4\pi r^2) = \frac{1}{\varepsilon_0}\int_0^{\pi}\int_0^{2\pi}\int_0^{R} \frac{\rho\, d\tau}{r}$$

$$= \frac{\rho}{\varepsilon_0}\int_0^{\pi}\int_0^{2\pi}\int_0^{R} r\sin\theta\, d\theta\, d\varphi\, dr$$

$$= \frac{\rho}{\varepsilon_0}\int_0^{\pi}\sin\theta\, d\theta \int_0^{2\pi} d\varphi \int_0^{R} r\, dr$$

$$= \frac{\rho}{\varepsilon_0}\left[-\cos\theta\Big|_0^{\pi}\right](2\pi)\ [R]$$

$$= \frac{\rho}{\varepsilon_0} 4\pi R$$

$$E = \frac{\rho 4\pi R}{4\pi\varepsilon_0 r^2} = \frac{\rho R}{\varepsilon_0 r^2}$$

$r < R$

$$\vec{E}(4\pi r^2) = \frac{1}{\varepsilon_0}\int_0^{\pi}\int_0^{2\pi}\int_0^{r} \rho\, r\sin\theta\, d\theta\, d\varphi\, dr$$

$$= \frac{\rho}{\varepsilon_0}\int_0^{\pi}\sin\theta\, d\theta\int_0^{2\pi} d\varphi\int_0^{r} r\, dr$$

$$= \frac{\rho}{\varepsilon_0}\left[-\cos\theta\Big|_0^{\pi}\right](2\pi)\ (r)$$

$$= \frac{\rho 4\pi r}{\varepsilon_0}$$

$$E = \frac{\rho 4\pi r}{4\pi r^2\varepsilon_0} = \frac{\rho}{\varepsilon_0 r}$$

4. Diket :

$\vec{E} = k r^3 \hat{r}$ , $k$ = konstan

a. $\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$

$$\nabla\cdot\vec{E} = \frac{1}{r^2}\cdot\frac{d}{dr}(r^2 E_r) = \frac{1}{r^2}\frac{d}{dr}(r^2 k r^3) = \frac{1}{r^2}\frac{d}{dr}(r^5 k) = \frac{1}{r^2}5kr^4$$

$$= 5kr^2$$

$$5kr^2 = \frac{\rho}{\varepsilon_0}$$

$$\rho = \varepsilon_0 . 5kr^2$$

b. $Q_{total} = \int_V \rho \, dv = \int_0^R \varepsilon_0 . 5kr^2 . 4\pi r^2 dr$

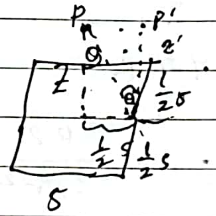
$$= 4\pi\varepsilon_0 \cdot 5k \int_0^R r^4 dr$$

$$= 4\pi\varepsilon_0 \cdot 5k \left[ \frac{r^5}{5} \Big|_0^R \right]$$

$$= 4\pi\varepsilon_0 \, 5k \, \frac{R^5}{5}$$

$$= 4\pi\varepsilon_0 k R^5$$

5.)


$$E = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda L}{z^2}$$

$$P = P' \cos\theta$$

$$P = 4P' \cos\theta$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda L}{z'^2} = 4 \frac{1}{4\pi\varepsilon_0} \frac{2\lambda L}{z'^2} \cos\theta$$

$L = \frac{1}{2} s \qquad z' = r = \sqrt{z^2 + (\frac{1}{2}s)^2} \qquad \cos = \frac{z}{\sqrt{z^2+(\frac{1}{2}s)^2}}$

$$\vec{E} = 4 \frac{1}{4\pi\varepsilon_0} \frac{2\lambda(\frac{1}{2}s) \cdot z}{(z^2+(\frac{1}{2}s)^2)\sqrt{z^2+(\frac{1}{2}s)^2}}$$

$$= \frac{1}{\pi\varepsilon_0} \frac{\lambda s z}{(z^2+\frac{s^2}{4})\sqrt{z^2+\frac{s^2}{4}}}$$

$$= \frac{\lambda s z}{\pi\varepsilon_0} \frac{1}{(z^2+\frac{s^2}{4})\sqrt{z^2+\frac{s^2}{4}}}$$

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