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UTS Kelistrikan dan Kemagnetan

2. a. Koordinat dengan kawat dari $x=0$, $x=L$ dan titik P, berada di atas titik $x=0$, sejauh z

$$r = \sqrt{x^2+z^2}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda\,dx}{x^2+z^2}\cdot\frac{z}{\sqrt{x^2+z^2}} = \frac{1}{4\pi\varepsilon_0}\cdot\lambda z\cdot\frac{dx}{(x^2+z^2)^{3/2}}$$

$$E = \frac{\lambda z}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(x^2+z^2)^{3/2}}$$

$$= \int_0^L \frac{dx}{(x^2+z^2)^{3/2}} = \left[\frac{x}{z^2\sqrt{x^2+z^2}}\right]_0^L = \frac{L}{z^2\sqrt{L^2+z^2}}$$

$$E = \frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{L}{z\sqrt{L^2+z^2}}$$

b. Kawat dari $x_1 = -\frac{1}{2}\cdot$ hingga $x = \frac{L}{2}$ titik P di atas titik tengannya.

$$dE = \frac{1}{4\pi\varepsilon_0}\cdot\lambda z\cdot\frac{dx}{(x^2+z^2)^{3/2}}$$

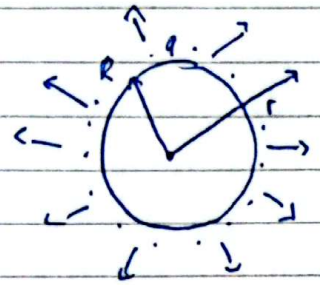
$$E = \frac{\lambda z}{4\pi\varepsilon_0}\int_{-L/2}^{L/2}\frac{dx}{(x^2+z^2)^{3/2}}$$

$$E = \frac{\lambda z}{2\pi\varepsilon_0}\int_0^{-L/2}\frac{dx}{(x^2+z^2)^{3/2}}$$

$$= \int_0^{L/2}\frac{dx}{(x^2+z^2)^{3/2}} = \left[\frac{x}{z^2\sqrt{x^2+z^2}}\right]_0^{L/2} = \frac{L}{2z^2\sqrt{(L/2)^2+z^2}}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0}\cdot\frac{L}{2z\sqrt{(L/2)^2+z^2}} = \frac{\lambda L}{4\pi\varepsilon_0\, z\sqrt{(L^2/4)+z^2}}$$

3. ★ Besar Medan listrik di luar bola ($r > R$)



$$F = k\frac{q_1 \cdot q_2}{r^2}$$

$$E = \frac{F}{q_1}$$

$$E = k\frac{q}{r^2}$$

$$k = \frac{1}{4\pi\varepsilon_0}$$

$$\sigma = \frac{Q}{4\pi R^2}$$

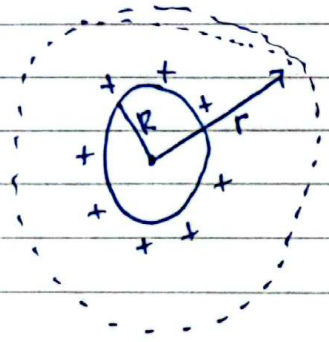
$$Q = \sigma\, 4\pi R^2$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4\pi R^2\sigma}{r^2}$$

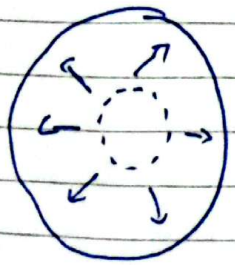
$$E = \frac{\sigma R^2}{\varepsilon_0 r^2}$$

★ Besar Medan Listrik di Permukaan bola ($r = R$)



$$E = \frac{\sigma}{\varepsilon_0}$$

★ Besar Muatan listrik di dalam Permukaan bola ($r > R$)



$E = 0$, karena muatan hanya berada di permukaan.

④ a) Rapat muatan (ρ)

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla E = \frac{1}{r^2} \frac{d}{dr} (r^2 E_{(r)})$$

$$\nabla \vec{E} = \frac{1}{r^2} \frac{d}{dr} (kr^5)$$

$$\nabla \cdot \vec{E} = \frac{1}{r^2} 5kr^4$$

$$\nabla \cdot \vec{E} = 5kr^2$$

$$\rho = \varepsilon_0 (\nabla \cdot \vec{E})$$

$$\rho_{(r)} = 5k\varepsilon_0 r^2$$

b) Hitung muatan total dalam bola jari-jari (R)

$$Q = \int_v \rho(r)\, dv, \quad dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$Q = \int_0^{2\pi} \int_0^{\pi} \int_0^{R} 5k\varepsilon_0 r^2 \cdot r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$Q = 5k\varepsilon_0 \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta \int_0^{R} r^4\, dr$$

$$Q = 5k\varepsilon_0 \cdot 2\pi \cdot 2 \cdot \frac{R^5}{5}$$

$$Q = 4\pi\varepsilon_0 kR^5$$

⑤ $E_z = \frac{2k\lambda R \sin\theta}{z}$

$$E = 4 \cdot \frac{2k\lambda \sin\theta}{z}$$

$$E = \frac{8k\lambda \sin\theta}{z}; \quad \theta = \tan^{-1}\left(\frac{s}{2z}\right); \quad \sin(\tan^{-1}(x)) = \frac{x}{\sqrt{1+x^2}}$$

$$E = \frac{8k\lambda}{z} \cdot \frac{s}{2z\sqrt{1+(s/2z)^2}} = \frac{4k\lambda s}{z^2\sqrt{1+s^2/4z^2}}$$