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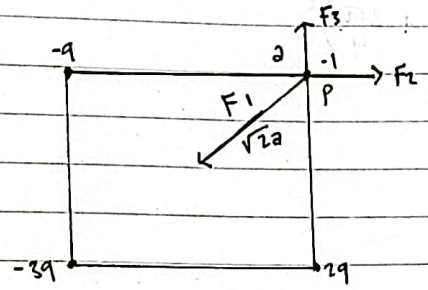
Mata kuliah : Kelistrikan dan kemagnetan (UTS

1. a)



b) * $F_1 = K \cdot \frac{q^2}{a^2}$

* $F_2 = K \cdot \frac{3q^2}{2a^2}$ (sudut 45°) terhadap sumbu x dan y

$F_{2x} = -\frac{3q^2}{2a^2} \cdot \cos 45^\circ = -\frac{3q^2}{2a^2} \cdot \frac{\sqrt{2}}{2} = -\frac{3q^2\sqrt{2}}{4a^2}$

$F_{2y} = -\frac{3q^2}{2a^2} \cdot \sin 45^\circ = -\frac{3q^2\sqrt{2}}{4a^2}$

* $F_3 = K \cdot \frac{2q^2}{a^2}$

Total komponen gaya

$F_x = -\frac{q^2}{a^2} - \frac{3q^2\sqrt{2}}{4a^2} = -\frac{q^2}{a^2}\left(1 + \frac{3\sqrt{2}}{4}\right)$

$F_y = \frac{2q^2}{a^2} - \frac{3q^2\sqrt{2}}{4a^2} = \frac{q^2}{a^2}\left(2 + \frac{3\sqrt{2}}{4}\right)$

$F_{total} = \frac{q^2}{a^2} \cdot K \cdot \sqrt{\left(1 + \frac{3\sqrt{2}}{4}\right)^2 + \left(2 + \frac{3\sqrt{2}}{4}\right)^2}$

c) $F_x = -\frac{q^2}{a^2}\left(1 + \frac{3\sqrt{2}}{4}\right)$

$F_y = \frac{q^2}{a^2}\left(2 + \frac{3\sqrt{2}}{4}\right)$

$\Rightarrow \tan\theta = \frac{F_y}{|F_x|} = \frac{2 - \frac{3\sqrt{2}}{4}}{1 + \frac{3\sqrt{2}}{4}}$

Gunakan $\sqrt{2} \approx 1{,}414$

$\tan\theta = \frac{2 - 1{,}061}{1 + 1{,}061} = \frac{0{,}939}{2{,}061} \approx 0{,}456.$

$\theta = \tan^{-1}(0{,}456) \approx 24{,}5^\circ$

$F_x < 0$ (ke kiri)
$F_y > 0$ (ke atas)
maka, gaya berada di kuadran II.
Sudutnya :
θ dari sumbu x positif $= 180^\circ - 24{,}5^\circ$
$= 155{,}5^\circ$
(ke atas)

d) dari I 6, besar gaya listrik total :

$$F_{tot} = \frac{q^2}{a^2} \cdot k \cdot \sqrt{\left(1+\frac{3\sqrt{2}}{4}\right)^2 + \left(2+\frac{3\sqrt{2}}{4}\right)^2}$$

maka, medan listrik di titik P :

$$E_{tot} = \frac{F_{tot}}{q} = \frac{q}{a^2} \cdot k \cdot \sqrt{\left(1+\frac{3\sqrt{2}}{4}\right)^2 + \left(2+\frac{3\sqrt{2}}{4}\right)^2}$$

arah medan listrik :

$$\tan\theta = \frac{F_y}{|F_x|} = \frac{2-\frac{3\sqrt{2}}{4}}{1+\frac{3\sqrt{2}}{4}}$$

$$\theta = \tan^{-1}\left(\frac{2-\frac{3\sqrt{2}}{4}}{1+\frac{3\sqrt{2}}{4}}\right)$$

arah medan listrik = $180^\circ - \theta$, kuadran II (karena $F_x < 0$, $F_y > 0$)

e) Tidak terjadi keseimbangan gaya, karena $F_{tot} \neq 0$

2. Diketahui : Panjang batang = L

Rapat muatan linear = λ (satuan C/m)

Ditanya : a. kuat medan listrik di titik P yang berjarak z diatas ujung kawat?

b. kuat medan listrik di titik P yang berjarak z ditengah-tengah kawat?

Penyelesaian :

Gunakan pendekatan $dq = \lambda \cdot dx$ pada titik x sepanjang batang.

Rumus medan listrik oleh elemen muatan di titik P (jarak $r = \sqrt{x^2+z^2}$)

$$dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r^2} \cdot \cos\theta$$

dengan : $dq = \lambda\, dx$, $\cos\theta = \frac{z}{\sqrt{x^2+z^2}}$, $r = \sqrt{x^2+z^2}$

Jadi, $dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda dx}{x^2+z^2} \cdot \frac{z}{\sqrt{x^2+z^2}} = \frac{\lambda z}{4\pi\epsilon_0} \cdot \frac{dx}{(x^2+z^2)^{3/2}}$

a) Batang dari $x = 0$ sampai $x = L$

$$E = \frac{\lambda z}{4\pi\epsilon_0} \int_0^L \frac{dx}{(x^2+z^2)^{3/2}}$$

$$\int \frac{dx}{(x^2+z^2)^{3/2}} = \frac{x}{z^2\sqrt{x^2+z^2}} + c$$

$$E = \frac{\lambda z}{4\pi\epsilon_0} \left[\frac{L}{z^2\sqrt{L^2+z^2}} - 0\right]$$

$$E = \frac{\lambda L}{4\pi\epsilon_0 z\sqrt{L^2+z^2}}$$

b) Batang dari $x = -\frac{L}{2}$ sampai $x = \frac{L}{2}$

$$E = \frac{\lambda z}{4\pi\epsilon_0} \int_{-L/2}^{L/2} \frac{dx}{(x^2+z^2)^{3/2}}$$

karena fungsi simetris, ambil dua kali integral dari 0 ke L/2 :

$$E = \frac{\lambda z}{4\pi\epsilon_0} \int_0^{L/2} \frac{dx}{(x^2+z^2)^{3/2}}$$

$$= \frac{\lambda z}{2\pi\epsilon_0} \cdot \left[\frac{x}{z^2\sqrt{x^2+z^2}}\right]_0^{L/2}$$

$$= \frac{\lambda z}{2\pi\epsilon_0} \cdot \frac{(L/2)}{x^2(L^2/4)+z^2} = \frac{\lambda L}{4\pi\epsilon_0 z\sqrt{(L^2/4)+z^2}}$$

3. Diketahui : Rapat muatan permukaan = σ
Jari-jari bola konduktor $r = R$

Ditanya : Medan listrik E di luar dan didalam bola konduktor?

Penyelesaian :

* Diluar bola : $(r > R)$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$$

Karena $Q = 4\pi R^2 \sigma$, maka:

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4\pi R^2 \sigma}{r^2}$$

$$= \frac{R^2 \sigma}{\varepsilon_0 r}$$

* Dipermukaan bola :

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{(hukum Gauss)}$$

* Didalam bola $(r < R)$

$$E = 0$$

4. a) $\nabla \cdot E = \frac{\rho}{\varepsilon_0}$

$$\nabla \cdot E = \frac{1}{r^2} \cdot \frac{d}{dr}\left(r^2 E_r\right)$$

dengan $E_r = Kr^3$, maka :

$$\nabla \cdot E = \frac{1}{r^2} \cdot \frac{d}{dr}\left(r^2 \cdot Kr^3\right)$$

$$= \frac{1}{r^2} \cdot \frac{d}{dr}\left(Kr^5\right)$$

$$= \frac{1}{r^2} 5Kr^4 = 5Kr^2$$

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

$$\frac{\rho}{\varepsilon_0} = 5Kr^2$$

$$\rho(r) = 5K\varepsilon_0 r^2$$

b) $Q = \int_V \rho(r)\, dv$

$dv = 4\pi r^2 dr$

$$Q = \int_0^R \rho(r) \cdot 4\pi r^2 dr$$

$$= \int_0^R \left(5K\varepsilon_0 r^2\right) \cdot 4\pi r^2 dr$$

$$= 20\pi K\varepsilon_0 \int_0^R r^4 dr$$

$$= 20\pi K\varepsilon_0 \cdot \frac{R^5}{5}$$

$$= 4\pi K\varepsilon_0 R^5$$

$$\int_0^R r^4 dr = \left[\frac{r^5}{5}\right]_0^R = \frac{R^5}{5}$$