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2) Dik : Panjang batang : L

Rapat muatan linear = λ

Titik P berjarak z dari : a) ujung kawat

b) Tengah kawat

Ditanya : a. kuat medan listrik di titik P yang berjarak z diatas ujung kawat

b. kuat medan listrik di titik P yang berjarak z diatas tengah "

Jawab :

$$dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r^2}$$

a. koordinat dengan kuadrat dari $x=0$, $x=L$, dan titik P. berada diatas $x=0$, sejauh z

$$r = \sqrt{x^2 + z^2}$$

$$dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda dx}{x^2 + z^2} = \frac{z}{\sqrt{x^2 + z^2}} = \frac{1}{4\pi\epsilon_0} \cdot \lambda z \cdot \frac{dx}{(x^2 + z^2)^{3/2}}$$

Integrasi dari $x=0$, hingga $x=L$

$$E = \frac{\lambda z}{4\pi\epsilon_0} \int_0^L \frac{dx}{(x^2 + z^2)^{3/2}}$$
$$= \int_0^L \frac{dx}{(x^2 + z^2)^{3/2}} = \left[\frac{x}{z^2 \sqrt{x^2 + z^2}} \right]_0^L = \frac{1}{z^2 \sqrt{L^2 + z^2}}$$

$$E = \frac{\lambda}{4\pi\epsilon_0} \cdot \frac{L}{z \sqrt{L^2 + z^2}}$$

b) kawat dari $x = -\frac{L}{2}$, hingga $x = \frac{L}{2}$, titik P diatas titik tengahnya

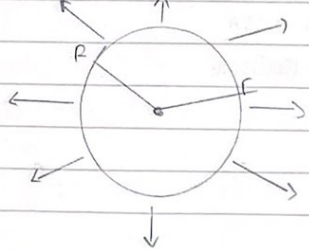
$$dE = \frac{1}{4\pi\epsilon_0} \cdot \lambda z \cdot \frac{dx}{(x^2 + z^2)^{3/2}}$$

Integrasi dari $-\frac{L}{2}$ sampai $\frac{L}{2}$

$$E = \frac{\lambda z}{4\pi\epsilon_0} \int_{-L/2}^{L/2} \frac{dx}{(x^2 + z^2)^{3/2}}$$

3) Hitung medan listrik dan rapat muatan σ dan jari-jari R

• Besar medan listrik diluar bola ($r > R$)



$$F = \frac{k q_1 q_2}{r^2} \quad E = \frac{q}{4\pi\epsilon_0 r^2}$$

$$E = F/q$$

$$E = k \frac{q}{r^2}$$

$$k = \frac{1}{4\pi\epsilon_0}$$

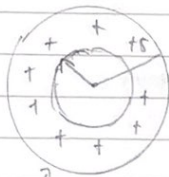
R = jari-jari bola

r = jari-jari permukaan Gauss

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

q = total muatan listrik

• Besar medan listrik di permukaan bola ($r = R$)



Gaussian surface

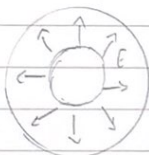
$$E = \frac{\sigma}{\epsilon_0}$$

σ = rapat permukaan

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

• Besar medan listrik didalam bola ($r < R$)

$E = 0$ karena muatan hanya berada dipermukaan



$E = 0$ (muatan dipermukaan konduktor)

4) Jawab :

2. Rapat muatan

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$= \frac{1}{r^2} \frac{d}{dr} (r^2 E_r) ; E_r = k r^2$$

$$r^2 E_r = r^2 \cdot k r^2 = k r^4$$

$$\frac{d}{dr} (k r^4) = 4 k r^3$$

$$\nabla \cdot \vec{E} = \frac{1}{r^2} 4 k r^3 = 4 k r$$

maka :

$$\rho(r) = \epsilon_0 \nabla \cdot \vec{E} = 4 k \epsilon_0 r^2$$

b = muatan total

$$Q = \int_V \rho(r) dv$$

Pada koordinat bola, $dv = r^2 \sin \theta dr d\theta d\phi$

$$Q = \int_0^{2\pi} \int_0^\pi \int_0^R (\epsilon \epsilon_0 \rho(r)) \cdot r^2 \sin \theta dr d\theta d\phi$$

$$= \epsilon \epsilon_0 \int_0^{2\pi} d\phi \int_0^\pi \sin \theta d\theta \int_0^R r^2 dr$$

$$\int_0^{2\pi} d\phi = 2\pi$$

$$\int_0^\pi \sin \theta d\theta = 2$$

$$\int_0^R r^2 dr = \frac{R^3}{3}$$

$$= Q = \epsilon \epsilon_0 \cdot 2\pi \cdot 2 \cdot \frac{R^3}{3} = 4\pi \epsilon_0 \epsilon \rho R^3$$

$$\rightarrow Q = 4\pi \epsilon_0 \epsilon \rho R^3$$

5) Medan listrik vertikal E_z yaitu

$$E_z = \frac{2ke\tau}{z} \sin \theta \text{ dimana } ke = \frac{1}{4\pi\epsilon_0}$$

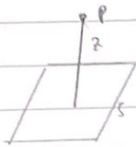
$\tau = \text{muatan}$

total medan 4 sisi

$$\theta = \tan^{-1} \left(\frac{3/2}{z} \right)$$

$$E_{\text{total}, z} = 4 E_{\text{sisi}}, z = \frac{4 \cdot 2ke\tau}{z} \sin \theta = \frac{8ke\tau}{z} \sin \theta$$

$$\text{dengan } \sin \theta = \frac{3/2}{\sqrt{z^2 + (3/2)^2}} = \frac{3}{2\sqrt{z^2 + 9/4}}$$



sehingga

$$E_z = \frac{8ke\tau}{z} \cdot \frac{3}{2\sqrt{z^2 + 9/4}} = \frac{4ke\tau}{z\sqrt{z^2 + 9/4}}$$

$$ke = \frac{1}{4\pi\epsilon_0}$$

$$E_z = \frac{\tau}{\pi\epsilon_0 z \sqrt{z^2 + 9/4}}$$