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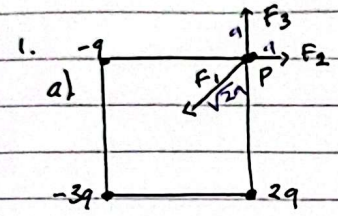
UTS Kelistrikan dan kemagnetan

1. a)



b) * $F_1 = k \cdot \frac{q^2}{a^2}$

* $F_2 = k \cdot \frac{3q^2}{2a^2}$ (sudut 45° terhadap sumbu x dan y)

$$F_{2x} = -\frac{3q^2}{2a^2} \cdot \cos 45^\circ = -\frac{3q^2}{2a^2} \cdot \frac{\sqrt{2}}{2} = -\frac{3q^2\sqrt{2}}{4a^2}$$

$$F_{2y} = -\frac{3q^2}{2a^2} \cdot \sin 45^\circ = -\frac{3q^2\sqrt{2}}{4a^2}$$

* $F_3 = k \cdot \frac{2q^2}{a^2}$

Total komponen gaya

$$F_x = -\frac{q^2}{a^2} - \frac{3q^2\sqrt{2}}{4a^2} = -\frac{q^2}{a^2}\left(1 + \frac{3\sqrt{2}}{4}\right)$$

$$F_y = \frac{2q^2}{a^2} - \frac{3q^2\sqrt{2}}{4a^2} = \frac{q^2}{a^2}\left(2 - \frac{3\sqrt{2}}{4}\right)$$

$$F_{total} = \frac{q^2}{a^2} \cdot k \cdot \sqrt{\left(1 + \frac{3\sqrt{2}}{4}\right)^2 + \left(2 - \frac{3\sqrt{2}}{4}\right)^2}$$

c) $F_x = -\frac{q^2}{a^2}\left(1 + \frac{3\sqrt{2}}{4}\right)$

$F_y = \frac{q^2}{a^2}\left(2 - \frac{3\sqrt{2}}{4}\right)$

$\Rightarrow \tan\theta = \frac{F_y}{|F_x|} = \frac{2 - \frac{3\sqrt{2}}{4}}{1 + \frac{3\sqrt{2}}{4}}$

Gunakan $\sqrt{2} \approx 1{,}414$

$\tan\theta = \frac{2 - 1{,}061}{1 + 1{,}061} = \frac{0{,}939}{2{,}061} \approx 0{,}456$

$\theta = \tan^{-1}(0{,}456) \approx 24{,}5^\circ$

* $F_x < 0$ (ke kiri)

$F_y > 0$ (ke atas)

Maka gaya berada di kuadran II, sudutnya:

θ dari sumbu x positif = 180° - 24,5°

= 155,5°

(ke atas)

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d) dari 1 6, besar gaya listrik total:

$$F_{tot} = \frac{q^2}{a^2} \cdot k \cdot \sqrt{\left(1 + \frac{3\sqrt{2}}{4}\right)^2 + \left(2 - \frac{3\sqrt{2}}{4}\right)^2}$$

maka, medan listrik di titik P:

$$E_{tot} = \frac{F_{tot}}{q} = \frac{q}{a^2} \cdot k \cdot \sqrt{\left(1 + \frac{3\sqrt{2}}{4}\right)^2 + \left(2 - \frac{3\sqrt{2}}{4}\right)^2}$$

arah medan listrik:

$$\tan\theta = \frac{F_y}{|F_x|} = \frac{2 - \frac{3\sqrt{2}}{4}}{1 + \frac{3\sqrt{2}}{4}}$$

$$\theta = \tan^{-1}\left(\frac{2 - \frac{3\sqrt{2}}{4}}{1 + \frac{3\sqrt{2}}{4}}\right)$$

Arah medan listrik : $180^\circ - \theta$, kuadran II (karena $F_x < 0$, $F_y > 0$)

e) Tidak terjadi kesetimbangan gaya, karena $F_{total} \neq 0$

2. a) $dE_z = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda_z \, dx}{(x^2 + z^2)^{3/2}}$

$$E_z = \frac{\lambda_z}{4\pi\epsilon_0} \int_0^L \frac{dx}{(x^2 + z^2)^{3/2}}$$

$$= \frac{\lambda z}{4\pi\epsilon_0} \left\{ \frac{dx}{(x^2 + z^2)^{3/2}} \right|_0^L$$

$$= \frac{\lambda z}{4\pi\epsilon_0} \cdot \frac{L}{z\sqrt{L^2 + z^2}}$$

b) $dE_z = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda_z \, dx}{(x^2 + z^2)^{3/2}}$

$$E_z = \frac{\lambda_z}{4\pi\epsilon_0} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dx}{(x^2 + z^2)^{3/2}}$$

$$= \frac{\lambda_z}{2\pi\epsilon_0} \int_0^{\frac{L}{2}} \frac{dx}{(x^2 + z^2)^{3/2}}$$

$$E = \frac{\lambda}{2\pi\epsilon_0} \cdot \frac{L}{2z\sqrt{(L^2/4) + z^2}}$$

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3. Medan listrik $\vec{E}$ akibat bola konduktor bermuatan homogen tergantung dari posisi titik pengamat:

- Di luar bola ($r > R$)

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2}$$

karena $Q = 4\pi R^2 \sigma$, maka:

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{4\pi R^2 \sigma}{r^2}$$

$$= \frac{R^2 \sigma}{\epsilon_0 r^2}$$

- Di permukaan bola ($r = R$):

$$E = \frac{\sigma}{\epsilon_0} \quad \text{(hukum gauss)}$$

- Di dalam bola ($r < R$)

$$E = 0$$

Kesimpulannya

$$E(r) \begin{cases} 0 & \text{untuk } r < R \\ \frac{\sigma}{\epsilon_0} & \text{untuk } r = R \\ \frac{R^2 \sigma}{\epsilon_0 r^2} & \text{untuk } r > R \end{cases}$$

4a) $\nabla \cdot E = \frac{\rho}{\epsilon_0}$

$$\nabla \cdot E = \frac{1}{r^2} \frac{d}{dr}(r^2 E_r)$$

dengan $E_r = kr^3$, maka:

$$\nabla \cdot E = \frac{1}{r^2} \cdot \frac{d}{dr}(r^2 \cdot kr^3)$$

$$= \frac{1}{r^2} \cdot \frac{d}{dr}(kr^5)$$

$$= \frac{1}{r^2} \cdot 5kr^4 = 5kr^2$$

$\nabla \cdot E = \frac{\rho}{\epsilon_0}$

$\frac{\rho}{\epsilon_0} = 5kr^2$

$\rho(r) = 5k\epsilon_0 r^2$

b) $Q = \int_V \rho(r)\, dV$

$dV = 4\pi r^2 dr$

$$Q = \int_0^R \rho(r) \cdot 4\pi r^2 dr$$

$$= \int_0^R (5k\epsilon_0 r^2) \cdot 4\pi r^2 dr$$

$$= 20\pi k\epsilon_0 \int_0^R r^4 dr$$

$$= 20\pi k\epsilon_0 \cdot \frac{R^5}{5}$$

$$= 4\pi k\epsilon_0 R^5$$

$\int_0^R r^4 dr = \left[\frac{r^5}{5}\right]_0^R = \frac{R^5}{5}$

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5. $dq = \lambda dx$, jaraknya ke titik P : $r = \sqrt{x^2 + (\frac{s}{2})^2 + z^2}$

$$dE_z = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq \cos\theta}{r^2}, \text{ dimana } \cos\theta = \frac{z}{r}$$

$$dE_z = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda\, dx \cdot z}{(x^2 + (\frac{s}{2})^2 + z^2)^{3/2}}$$

$$= \frac{4\lambda z}{4\pi\epsilon_0} \int_{-\frac{s}{2}}^{\frac{s}{2}} \frac{dx}{(x^2 + (\frac{s}{2})^2 + z^2)^{3/2}}$$

ubah:

$$a^2 = (\frac{s}{2})^2 + z^2 \Rightarrow E_z = \frac{\lambda z}{\pi\epsilon_0} \int_{-s/2}^{s/2} \frac{dx}{(x^2 + a^2)^{3/2}}$$

integral ini standar:

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2 + a^2}} + C$$

Evaluasi dari $-\frac{s}{2}$ ke $\frac{s}{2}$:

$$E_z = \frac{\lambda z}{\pi\epsilon_0} \left[\frac{x}{a^2\sqrt{x^2 + a^2}} \Big|_{-\frac{s}{2}}^{\frac{s}{2}} \right]$$

$$= \frac{\lambda z}{\pi\epsilon_0} \left[\frac{\frac{s}{2}}{a^2\sqrt{(\frac{s}{2})^2 + a^2}} - \left(-\frac{\frac{s}{2}}{a^2\sqrt{(\frac{s}{2})^2 + a^2}} \right) \right]$$

$$= \frac{\lambda z}{\pi\epsilon_0} \left(\frac{s}{a^2\sqrt{s^2/4 + a^2}} \right)$$

$$E_z = \frac{4\lambda z}{4\pi\epsilon_0} \int_{-\frac{s}{2}}^{\frac{s}{2}} \frac{dx}{(x^2 + (\frac{s}{2})^2 + z^2)^{3/2}}$$

atau

$$E_z = \frac{4\lambda z s}{\pi\epsilon_0 \left[(s^2 + 4z^2)\sqrt{s^2 + 2z^2} \right]}$$

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