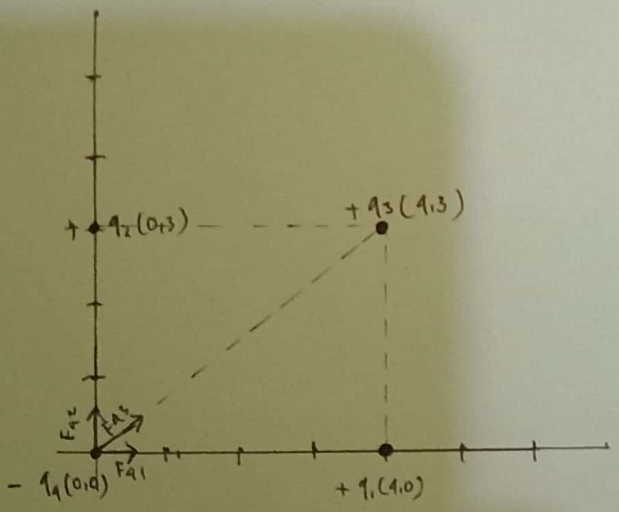
بسم الله الرحمن الرحيم

Nama : Rabiatul Adawiah
Npm : 2213022073
Kelas : C
Matkul : Kelistrikan dan Kemagnetan

"UTS"

1. Empat partikel bermuatan listrik terdistribusi sebagai berikut:
$q_1(4,0) = 2$ stat C, $q_2(0,3) = 5$ stat C, $q_3(4,3) = 10$ stat C dan $q_4(0,0) = -1$ stat C. Tentukan Medan listrik pada posisi (0,0), dimana jarak dalam centimeter dengan $K = 1$ dyne.cm^2/stat.C^2.
Hitunglah:
a. Gaya Coulomb yang dialami oleh muatan q_a
b. Medan listrik pada titik (0,0) atau muatan q_a
c. Potensial listrik pada titik (0,0).

Jawab:



$q_1(4,0) = 2$ stat.C
$q_2(0,3) = 5$ stat.C
$q_3(4,5) = 10$ stat.C
$q_4(0,0) = -1$ stat.C
$K = 1$ dyne.cm^2/stat.C^2
$1 = 9.10^9$

a. Gaya Coulomb

$$\vec{F} = \vec{E}_q$$

$$\vec{F} = k\frac{Qq}{r^2}\hat{r} \quad \text{(dua muatan)}$$

$$\vec{F} = k\frac{Qq_1}{r_1^2}\hat{r}_1 + k\frac{Qq_2}{r_2^2}\hat{r}_2 + \ldots \quad (>\text{dua muatan})$$

$$\vec{F} = q_4 k\left\{\frac{q_2}{r_{02}^2}\hat{r}_{02} + \frac{q_3}{r_{03}^2}\hat{r}_{03} + \frac{q_1}{r_{01}^2}\hat{r}_{01}\right\}$$

Perhatikan arah vektor

$$\vec{F} = (-1)\cdot(9.10^6)\left\{\frac{5}{9}(3\hat{j}) + \frac{10}{25}\left(\frac{3\hat{j}+4\hat{i}}{5}\right) + \frac{2}{16}(4\hat{i})\right\}$$

$$= (-1)\,9.10^6\left\{\left(\frac{5}{9}+\frac{30}{125}\right)\hat{j} + \left(\frac{2}{16}+\frac{40}{125}\right)\hat{i}\right\}$$

$$= (-1)(9.10^6)\left\{0,795\,\hat{j} + 0,445\,\hat{i}\right\}$$

$$= -9.10^6\,(0,795\,\hat{j} + 0,445\,\hat{i})$$

Jadi besarnya gaya $F_x = -9.10^6\,((0,795)^2+(0,445)^2)^{1/2}$ dalam arah $-\text{arc tg}\,(0,795/0,445)$

b. Medan Listrik.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0 r^2}\frac{q}{r^2}\hat{r} \quad \text{dengan} \quad \hat{r} = \frac{\vec{r}}{|r|}$$

- Medan listrik oleh q_2

$$\vec{E}_2 = k\frac{q_2}{r_2^2}\cdot\hat{r} = 9\cdot10^6\,\frac{5}{9}(3\hat{j})$$

- Medan listrik oleh q_3

$$\vec{E}_3 = k\frac{q_3}{r_3^2}\hat{r} = 9.10^6\,\frac{10}{25}\left(\frac{4\hat{i}+3\hat{j}}{5}\right) = \left(\frac{40}{125}\hat{i}+\frac{30}{125}\hat{j}\right)9.10^6$$

- Medan listrik oleh q_1

$$\vec{E}_1 = k\frac{q_1}{r_1^2}\hat{r} = 9.10^6\,\frac{2}{16}(4\hat{i})$$

- Medan listrik oleh q_1, q_2 dan q_3

$$\vec{E} = 9.10^6\left(\frac{2}{16}+\frac{40}{125}\right)\hat{i} + 9.10^6\left(\frac{5}{9}+\frac{30}{125}\right)\hat{j}$$

$$= 9.10^6\,(0,445)\,\hat{i} + 9.10^6\,(0,795)\,\hat{j}$$

$$E = 9.10^6\sqrt{(0,445)^2+(0,795)^2}$$

$$= 9.10^6\,(0,445+0,795)$$

Jadi medan listrik pada titik (0,0) adalah $9.10^6\sqrt{(0,445)^2+(0,795)^2}$ dyne cm^2/body pada arah arc. tg (0,795 / 0,445).

c. Potensial Listrik

$$V = k\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}$$

$$V = 9.10^6\left(\frac{2}{4} + \frac{5}{3} + \frac{10}{5}\right)$$

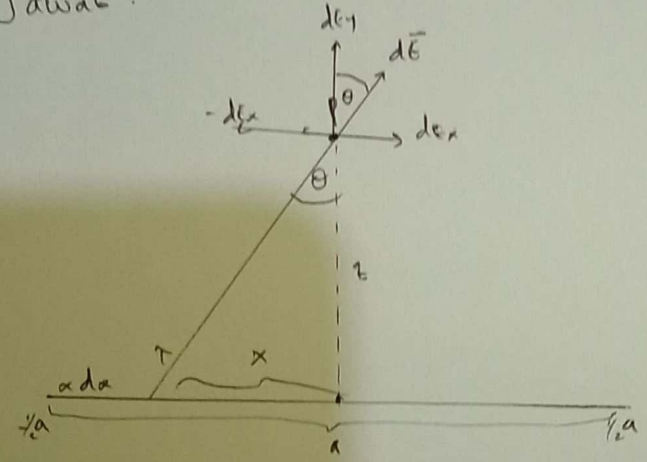
$$= 9.10^6\,(4,167)$$

$$= 3,7503 \cdot 10^7$$

Jadi potensial pada titik 0,0 adalah 4,167 erg/statC = 3,7503 stat.V

2. Sebuah kawat lurus panjangnya a, membawa muatan sebesar λ. Hitunglah medan listrik di titik P di atas kawat yang berjarak z. di tengah - tengah kawat.

Jawab:



$d\vec{E} = dE_y + dE_x$ → Saling meniadakan.

$d\vec{E} = dE_y$

$d\vec{E}_y = dE\cos\theta$

misalkan $a = 2L$

$-\frac{1}{2}a = -L$

$\frac{1}{2}a = L$

$$E = \frac{1}{4\pi\varepsilon_0}\int_{-L}^{+L}\frac{\lambda\,dL}{r^2}\cos\theta$$

$$= \frac{2}{4\pi\varepsilon_0}\int_0^L \frac{\lambda\,dx}{(x^2+z^2)}\cos\theta$$

$$= \frac{2\lambda z}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(x^2+z^2)^{3/2}}$$

$$= \frac{2\lambda z}{4\pi\varepsilon_0}\left(\frac{x}{z^2\sqrt{x^2+z^2}}\right)\Big|_0^L$$

$$= \frac{2\lambda z}{4\pi\varepsilon_0}\left(\frac{L}{L^2\sqrt{L^2+z^2}}\right) - 0$$

$$\int\frac{dx}{(x^2+z^2)^{3/2}} = \frac{x}{z^2\sqrt{x^2+z^2}}$$

Jadi nilai medan listrik di titik P adalah

$$\vec{E} = \frac{2\lambda z}{4\pi\varepsilon_0}\left(\frac{\frac{1}{2}a}{(\frac{1}{2}a)^2\sqrt{(\frac{1}{2}a)^2+z^2}}\right)$$

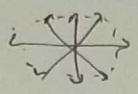
3. Apa yang anda ketahui tentang Hukum Gauss (Gambarkan permukaan gauss bloknya secara simbolik (rumus) sesuai definisi. Dimana diterapkan dan beri contoh !

Jawab : Fluks yang muncul pada luasan daerah tertutup karena ada muatan.

Hukum gauss adalah besarnya fluks atau garis gaya listrik yg keluar dari suatu permukaan tertutup sebanding muatan yang dilingkupi oleh luasan tertutup.

Hukum Gauss → integral lipat 3, bisa dihitung dengan lipat 2 Simetri tinggi.
Fluks terjadi dari bawah ke atas.

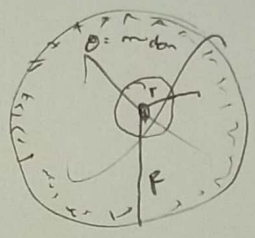
$\Delta V \neq dV$



$\oint_s \vec{E}\, d\vec{a} = \int_v (\nabla \cdot \vec{E})\, dv$

$\oint \vec{E}\, d\vec{a} = \frac{q}{\varepsilon_0}$

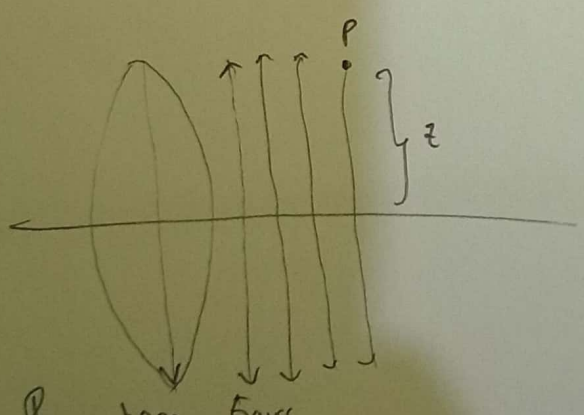
↓
Persamaan Lipat.



$\iiint \nabla E\, dv = \iiint \frac{\rho\, dv}{\varepsilon_0}$

$\nabla \vec{E} = \frac{\rho}{\varepsilon_0}$

↓
divergensi

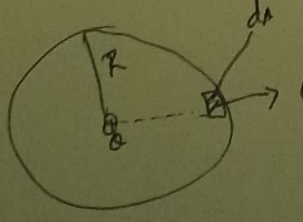


$\oint \vec{E}\, da = \frac{q}{\varepsilon_0} \cdot \lambda L$

$\oint \vec{E}\, d\vec{a} = \frac{\lambda L}{\varepsilon_0}$

$\vec{E}(2\pi r \cdot L) = \frac{\lambda L}{\varepsilon_0}$

$\vec{E} = \frac{\lambda}{2\pi \varepsilon_0 r} \cdot \hat{r}_0$



$E_n = \frac{kQ}{R^2}$

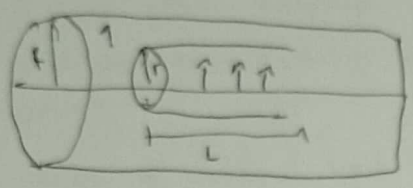
Q = muatan
---- = garis medan listrik melintasi a
Flus = $E_n \cdot 4\pi R^2$

Flus tot = $\oint E_n$
↳ Permukaan yg dilalui Flus = dA

R = Jarak muatan ke Permukaan gauss

3. Hukum Gauss diterapkan pada : - Gaya listrik distribusi
- Mesin Foto Copy
- Peroping Asap
- Printer
- Penangkal listrik

Contoh



$$\oint \vec{E}\, d\vec{a} = \frac{q}{\varepsilon_0}$$

UNTUK $r < R$

$$\oint\oint \vec{E} \cdot da = \frac{q}{\varepsilon_0}$$

$$E(2\pi rL) = \frac{1}{\varepsilon_0}$$

$$\vec{E} = \frac{q}{2\pi\varepsilon_0 rL}\,\hat{r}_0$$

UNTUK $R = r$

$$\vec{E}(2\pi rL) = \frac{q}{\varepsilon_0}\,r_0$$

$$E = \frac{q}{2\pi\varepsilon_0 R}\,\hat{r}_0$$

4. a. $V_1 = x^2\hat{i} + 3xz^2\hat{j} - 2xz\hat{k}$

b. $V_2 = xy\hat{i} + 2yz\hat{j} + 3xz\hat{k}$

cari curl

Jawab :

a.) $V_1 = x^2\hat{i} + 3xz^2\hat{j} - 2xz\hat{k}$

$$\nabla \cdot V = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \times \left(x^2\hat{i} + 3xz^2\hat{j} - 2xz\hat{k}\right)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & 3xz^2 & -2xz \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial(-2xz)}{\partial y} - \frac{\partial 3xz^2}{\partial z}\right) - \hat{j}\left(\frac{\partial(-2xz)}{\partial x} - \frac{\partial x^2}{\partial z}\right) + \hat{k}\left(\frac{\partial 3xz^2}{\partial x} - \frac{\partial x^2}{\partial y}\right)$$

$$= \hat{i}(0 - 6xz) + \hat{j}(-2z - 0) + \hat{k}(3z^2 - 0)$$

$$= -6xz(\hat{i}) - 2z(\hat{j}) + 3z^2(\hat{k})$$

b.) $V_2 = xy\hat{i} + 2yz\hat{j} + 3xz\hat{k}$

$$\nabla \cdot V = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \times \left(xy\hat{i} + 2yz\hat{j} + 3xz\hat{k}\right)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & 3xz \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial 3xz}{\partial y} - \frac{\partial 2yz}{\partial z}\right) + \hat{j}\left(\frac{\partial xy}{\partial z} - \frac{\partial 3xz}{\partial x}\right) + \hat{k}\left(\frac{\partial 2yz}{\partial x} - \frac{\partial xy}{\partial y}\right)$$

$$= \hat{i}(0 - 2y) + \hat{j}(0 - 3z) + \hat{k}(0 - x)$$

$$= -2y(\hat{i}) - 3z(\hat{j}) - x(\hat{k})$$