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UTS

1. Diketahui :

q_1 (4,0) = 2 stat C

q_2 (0,3) = 5 stat C

q_3 (4,3) = 10 stat C

q_4 (0,0) = -1 stat C

K = 1 dyne . cm^3/stat C

a. menghitung gaya Coulumb oleh muatan q_4

Rumus gaya Couloumb

$$F = \frac{K |q_1 q_2|}{r^2}$$

→ mencari jarak

$$q_4 \ \& \ q_1 = \sqrt{(0-4)^2 + (0-0)^2}$$
$$= 4$$

$$q_4 \ \& \ q_2 = \sqrt{(0-0)^2 + (0-3)^2}$$
$$= 3$$

$$q_4 \ \& \ q_3 = \sqrt{(0-4)^2 + (0-3)^2}$$
$$= 5$$

- Gaya Couloumb

$q_4 \& q_1 = \frac{k\,|2(-1)|}{4^2} = 0,125$ dyne

$q_4 \& q_2 = \frac{k\,|5(-1)|}{3^2} = 0,556$ dyne

$q_4 \& q_3 = \frac{k\,|10(-10)|}{5^2} = 0,4$ dyne

Jadi total gaya Couloumb yang dialami oleh muatan $q_4 = 0,125 + 0,556 + 0,4 = 1.081$ dyne.

b. Menghitung medan listrik pada titik (0,0)

Rumus $E = \frac{k \Sigma |q_1|}{r^2}$

- Jarak antara

(0,0) dan $q_1 = \sqrt{((0-4)^2 + (0-0)^2)} = 4$ cm

(0,0) dan $q_2 = \sqrt{((0-0)^2 + (0-3)^2)} = 3$ cm

(0,0) dan $q_3 = \sqrt{((0-4)^2 + (0-3)^2)} = 5$ cm

(0,0) dan $q_4 = \sqrt{((0-0)^2 + (0-0)^2)} = 0$ cm

Medan listri

$(0,0) = k\left(\frac{|2|}{4^2} + \frac{|5|}{3^2} + \frac{|10|}{5^2} + \frac{|1-0|}{0^2}\right)$

$= 1,667$ dyne / stat C

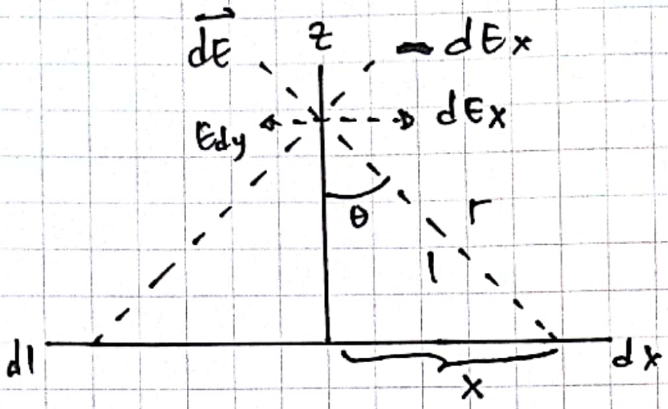
c. Menghitung Potensial listrik pada titik (0,0)

$V = k \Sigma \frac{q_1}{r}$

Jarak antara : $(0,0) \& q_1 = \sqrt{((0-4)^2 + (0-0)^2} = 4$ cm

$(0,0) \& q_2 = \sqrt{(0-0)^2} = 0$ cm

2.



turunan

$\cos\theta = \frac{z}{r} \rightarrow r = \frac{z}{\cos\theta}$

$\vec{E} = \vec{E_x} + \vec{E_y}$

$= \frac{1}{4\pi\varepsilon_0}\int_{-\infty}^{\infty} \frac{\lambda\, dx}{r^2} \cos\theta$

$\cos\theta = \frac{z}{r} \rightarrow r = \frac{z}{\cos\theta}$

$r = (x^2 + z^2)^{\frac{1}{2}}$

$\vec{dE} = dE_y + dE_x$

$\vec{dE} = dE_y$

$\vec{dE_y} = dE \cos\theta$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{\lambda\, dl}{r^2} \cos\theta$$

$$= \frac{2}{4\pi\varepsilon_0}\int_0^L \frac{\lambda\, dx}{(x^2+z^2)} \cos\theta$$

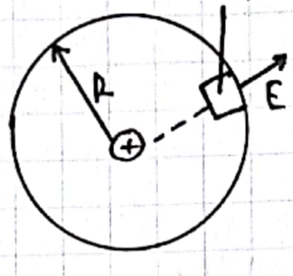
$$\vec{E} = \frac{2\lambda z}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(x^2+z^2)^{3/2}} \rightarrow \int \frac{dx}{(x^2+z^2)^{3/2}} = \frac{x}{z^2\sqrt{x^2+z^2}}$$

$$= \frac{4\lambda z}{4\pi\varepsilon_0}\left(\frac{x}{z^2\sqrt{x^2+z^2}}\right)\Bigg|_0^L$$

$$= \frac{2\lambda z}{4\pi\varepsilon_0}\left[\frac{1}{z^2\sqrt{x^2+y^2}} - \frac{0}{z^2\sqrt{0+z^2}}\right]$$

$$= \frac{2\lambda z}{4\pi\varepsilon_0}\left[\frac{L}{z^2\sqrt{L^2+z^2}}\right]$$

3. Hukum Gauss merupakan bola berjari-jari R yang Pusatnya terletak di muatan titik Q. Medan listrik disembarang tempat pada permukaan tegak lurus terhadap permukaan tersebut dan memiliki besar.



$$E = \frac{kQ}{R^2}$$

Suatu permukaan berbentuk bola yang melingkupi muatan titik Q. Jumlah garis medan listrik yang melintasi permukaan ini akan bergerak melintasi semua bagian permukaan yang melingkupi Q. Fluksnya dapat dengan mudah dihitung untuk permukaan berbentuk bola. Fluks ini besarnya adalah E_n kali luas permukaan $4\pi R^2$

Secara matematika dirumuskan

$$\oint E_n \, dA = 4\pi k \, Q_{dalam}$$ → Dalam bentuk integral

$$\oint E_n \, dA = \frac{Q_{in}}{\varepsilon_0}$$

E_n : Fluks total

dA : luas permukaan yang dilalui fluks

Q_{dalam} : Muatan yang dilingkupi oleh benda

R : Jarak antara muatan dan permukaan gauss.

Bentuk Integral di jabarkan dalam bentuk diferensial

$$\oint_{Surface} E \cdot da = \int_{Surface} (\nabla \cdot E)\, d\tau$$

$$Q_{enc} = \int_{Surface} \rho\, d\tau$$

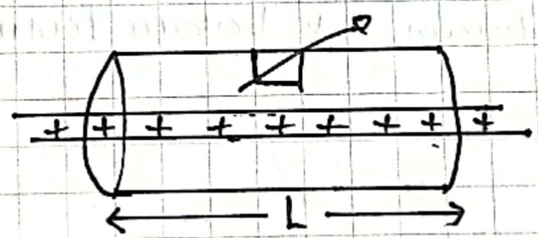
Selanjutnya

$$\int_{Volume} (\nabla \cdot E)\, d\tau = \int_{Volume} \left(\frac{1}{\varepsilon_0} \rho\right) d\tau$$

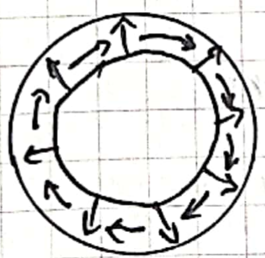
$$\nabla \cdot E = \frac{1}{\varepsilon_0} \rho$$

Contoh aplikasi Hukum Gauss

→ Pada kawat lurus panjang, dr jari-jari R



Contoh Soal :



Hitung $\vec{E}$ Pada

a) $r < R$

b) $r > R$

$$\oint \vec{E}\, dA = \frac{Q_{inc}}{\varepsilon_0}$$

$$\vec{E} = (4\pi r^2) = \frac{Q_{inc}}{\varepsilon}$$

Medan $< a = 0 \rightarrow \vec{E}$

$a < r < b$

$$\oint \vec{E} \cdot da = \frac{Q}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{1}{\varepsilon_0} \iiint \rho \, d\sigma$$

$$= \frac{1}{\varepsilon_0} \int_0^a \int_0^{\pi} \int_{\pi}^{2\pi} r \sin\theta$$

$$d\theta \, d\phi \, d\sigma$$

$$\vec{E}(4\pi r^2) = \frac{4\pi}{\varepsilon_0} \int_0^a r^2 \, dr$$

$$= \frac{4\pi}{\varepsilon_0} \left(\frac{1}{3} r^3 \right)_0^a$$

$$\vec{E}(4\pi r^2) = \frac{\frac{4}{3}\pi a^3}{\varepsilon_0}$$

$$= \frac{\rho a^3}{3\varepsilon r^2} \hat{r}$$

$\vec{E} \rightarrow r > b$

$$\oint \vec{E} \cdot \vec{da} = \frac{Q}{\varepsilon_0}$$

$$E \cdot (4\pi r^2) = \frac{1}{\varepsilon_0} \int_a^b \int_0^{3\pi}$$

$$\int_0^{\pi} \rho \sin\theta \, d\theta$$

$$d\phi \, dr$$

$\boxed{r > b}$

4. Menghitung Curl atau Rotasi dari fungsi Vektor

a). $V_1 = x^2\hat{i} + 3xz^2\hat{j} - 2xz\hat{k}$

b). $V_2 = xy\hat{i} + 2yz\hat{j} + 3xz\hat{k}$

Jawab:

a). Curl terhadap V1

Solusi : $\nabla \times \vec{V_1} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$

$$= \left(\frac{\partial(-2xz)}{\partial y} - \frac{\partial(3xz^2)}{\partial z}\right)\hat{i} + \left(\frac{\partial(x^2)}{\partial z} - \frac{\partial(-2xz)}{\partial x}\right)\hat{j}$$

$$+ \left(\frac{\partial(3xz^2)}{\partial x} - \frac{\partial(x^2)}{\partial y}\right)\hat{k}$$

$$= (0 - 6xz)\hat{i} + (0 + 2z)\hat{j} + (3z^2 - 0)\hat{k}$$

$$= -6xz\hat{i} + 2z\hat{j} + 3z^2\hat{k}$$

b). Curl terhadap V_2

$$\nabla \times \vec{V_2} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

$$\nabla \times \vec{V_2} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & 3xz \end{vmatrix}$$

$$= \left(\frac{\partial (3xz)}{\partial y} - \frac{\partial (2yz)}{\partial z} \right) \hat{i} + \left(\frac{\partial (xy)}{\partial z} - \frac{\partial (3xz)}{\partial x} \right) \hat{j}$$

$$+ \left(\frac{\partial (2yz)}{\partial x} - \frac{\partial (xy)}{\partial y} \right) \hat{k}$$

$$= (0 - 2y)\hat{i} + (0 - 3z)\hat{j} + (0 - x)\hat{k}$$

$$= -2y\hat{i} - 3z\hat{j} - x\hat{k}$$