

Nama : Isna Zakriyah Solehah
 NPM : 2213022067
 Kelas : 22C

UTS KALISMA

①. Diketahui :

$$q_1 (4,0) = 2 \text{ Stat C}$$

$$q_2 (0,3) = 5 \text{ Stat C}$$

$$q_3 (4,3) = 10 \text{ Stat C}$$

$$q_4 (0,0) = -1 \text{ Stat C}$$

$$k = 1 \text{ dyne} \cdot \text{cm}^3 / \text{Stat C}$$

a). Menghitung Gaya Coulomb
 oleh muatan q_4

Rumus Gaya Coulomb

$$F = \frac{k |q_1 q_2|}{r^2}$$

- Mencari jarak

$$q_4 \text{ dan } q_1 = \sqrt{(0-4)^2 + (0-0)^2}$$

$$= 4$$

$$q_4 \text{ dan } q_2 = \sqrt{(0-0)^2 + (0-3)^2}$$

$$= 3$$

$$q_4 \text{ dan } q_3 = \sqrt{(0-4)^2 + (0-3)^2}$$

$$= 5$$

- Gaya Coulomb

$$q_4 \text{ dan } q_1 = \frac{k |2(-1)|}{4^2}$$

$$= 0,125 \text{ dyne}$$

$$q_4 \text{ dan } q_2 = \frac{k |5(-1)|}{3^2}$$

$$= 0,556 \text{ dyne}$$

$$q_4 \text{ dan } q_3 = \frac{k |10(-1)|}{5^2}$$

$$= 0,4 \text{ dyne}$$

Jadi, total gaya Coulomb yg
 dialami oleh muatan q_4 =
 $0,125 + 0,556 + 0,4 = 1,081 \text{ dyne}$

b). Menghitung medan listrik pada
 titik (0,0)

$$\text{Rumus } E = \frac{k |q_1|}{r^2}$$

- Jarak antara

$$(0,0) \text{ dan } q_1 = \sqrt{(0-4)^2 + (0-0)^2}$$

$$= 4 \text{ cm}$$

$$(0,0) \text{ dan } q_2 = \sqrt{(0-0)^2 + (0-3)^2}$$
$$= 3 \text{ cm}$$

$$(0,0) \text{ dan } q_3 = \sqrt{(0-4)^2 + (0-3)^2}$$
$$= 5 \text{ cm}$$

$$(0,0) \text{ dan } q_4 = \sqrt{(0-0)^2 + (0-0)^2}$$
$$= 0 \text{ cm}$$

Medan listrik

$$(0,0) = k \left(\frac{|q_1|}{4^2} + \frac{|q_2|}{3^2} + \frac{|q_3|}{5^2} + \frac{|q_4|}{0^2} \right)$$
$$= 1,667 \text{ dyne/stat C}$$

c) Menghitung potensial listrik pada titik (0,0)

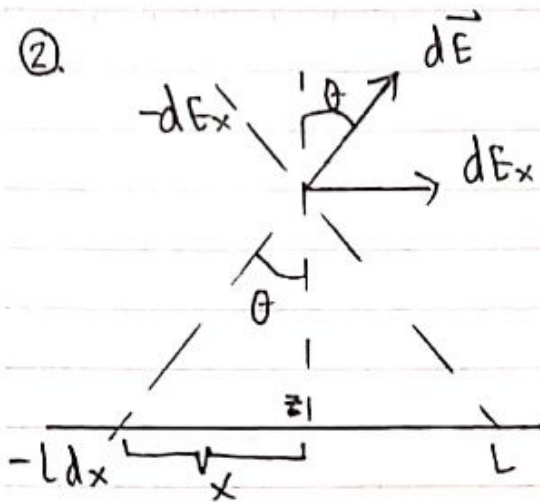
$$V = k \sum \frac{q_i}{r}$$

Jarak Antara

$$(0,0) \text{ dan } q_1 = \sqrt{(0-4)^2 + (0-0)^2}$$
$$= 4 \text{ cm}$$

$$(0,0) \text{ dan } q_2 = \sqrt{(0-0)^2 + (0-0)^2}$$
$$= 0 \text{ cm}$$

②



$$= \square \left[\frac{\lambda}{z^2 \sqrt{L^2 + z^2}} \right]$$

$$r = (x^2 + z^2)^{\frac{1}{2}}$$

$$d\vec{E} = dE_y + dE_x$$

$$d\vec{E} = dE_y$$

$$dE_y = dE \cos \theta$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int_{-L}^L \frac{\lambda dx}{r^2} \cos \theta$$

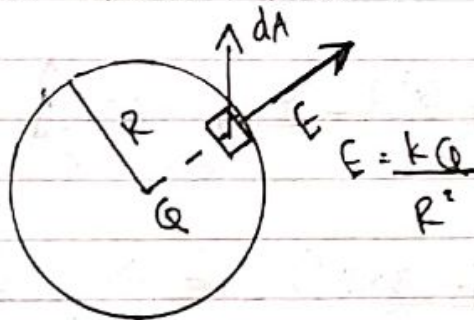
$$= \frac{2}{4\pi\epsilon_0} \int_0^L \frac{\lambda dx}{(x^2 + z^2)} \cos \theta$$

$$\vec{E} = \frac{2\lambda z}{4\pi\epsilon_0} \int_0^L \frac{dx}{(x^2 + z^2)^{3/2}}$$

$$\vec{E} = \frac{2\lambda z}{4\pi\epsilon_0} \left(\frac{x}{z^2 \sqrt{x^2 + z^2}} \right) \Big|_0^L$$

$$= \square \left[\frac{1}{z^2 \sqrt{L^2 + z^2}} - \frac{0}{z^2 \sqrt{0 + z^2}} \right]$$

③. Hukum Gauss Merupakan Suatu permukaan bola berjari-jari R yang pusatnya terletak ~~di~~ dimuat titik Q . Medan listrik di sebarang tempat pada permukaan tegak lurus terhadap permukaan tersebut dan memiliki besar.

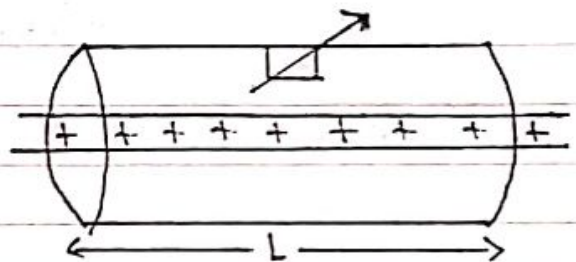


Suatu permukaan berbentuk bola yang melingkupi muatan titik Q . Jumlah garis medan listrik yang melintasi permukaan ini akan bergerak melintasi semua bagian yg melingkupi Q . fluksnya dapat dengan mudah dihitung untuk permukaan berbentuk bola. Fluksnya ini besarnya adalah E kali luas permukaan $4\pi R^2$. Secara matematika dirumuskan

$\oint E \cdot dA = 4\pi kQ \rightarrow$ pers Gauss
dim Integral

$$\text{atau } \oint E \cdot dA = \frac{Q_{in}}{\epsilon_0}$$

Contoh Aplikasi hukum Gauss
 $\Rightarrow$ pada kawat lurus panjang, ds.
jari-jari R



Contoh Soal :



Hitung $\vec{E}$ pada

a). $r < R$

b). $r > R$

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$\vec{E} = (4\pi r^2) = \frac{Q_{enc}}{\epsilon_0}$$

Medan $\leq a = 0 \rightarrow \vec{E}$
 $a < r < b$

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{1}{\epsilon_0} \iiint \rho d\tau$$

$$= \frac{1}{\epsilon_0} \int_0^a \int_0^\pi \int_0^{2\pi} r^2 \sin \theta$$

$$d\theta d\phi dr$$

$$\vec{E}(4\pi r^2) = \frac{4\pi}{\epsilon_0} \int_0^a r^2 dr$$

$$= \frac{4\pi}{\epsilon_0} \left(\frac{1}{3} r^3 \right)_0^a$$

$$\vec{E}(4\pi r^2) = \frac{4\pi a^3}{3}$$

$$\vec{E} = \frac{\epsilon_0}{3a^3} \hat{r}$$

$$\vec{E} \rightarrow r > b$$

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q}{\epsilon_0}$$

$$\vec{E} \cdot (4\pi r^2) = \frac{1}{\epsilon_0} \int_a^b \int_0^\pi \int_0^{2\pi} \rho \sin \theta d\theta d\phi dr$$

$$r > b$$

④. Menghitung Curl atau rotasi dari fungsi Vektor :

a). $V_1 = x^2 \hat{i} + 3xz^2 \hat{j} - 2xz \hat{k}$

b). $V_2 = xy \hat{i} + 2yz \hat{j} + 3xz \hat{k}$

Jawab :

a). Curl terhadap V_1

$$\nabla \times \vec{V}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

$$= \left(\frac{\partial(-2xz)}{\partial y} - \frac{\partial(3xz^2)}{\partial z} \right) \hat{i} + \left(\frac{\partial(x^2)}{\partial z} - \frac{\partial(-2xz)}{\partial x} \right) \hat{j}$$

$$+ \left(\frac{\partial(3xz^2)}{\partial x} - \frac{\partial(x^2)}{\partial y} \right) \hat{k}$$

$$= (0 - 6xz) \hat{i} + (0 + 2z) \hat{j} + (3z^2 - 0) \hat{k}$$

$$= -6xz \hat{i} + 2z \hat{j} + 3z^2 \hat{k}$$

b). Curl terhadap V_2

$$\nabla \times \vec{V}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

$$\nabla \times \vec{V}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & 3xz \end{vmatrix}$$

$$= \left(\frac{\partial(3xz)}{\partial y} - \frac{\partial(2yz)}{\partial z} \right) \hat{i} + \left(\frac{\partial(xy)}{\partial z} - \frac{\partial(3xz)}{\partial x} \right) \hat{j} + \left(\frac{\partial(2yz)}{\partial x} - \frac{\partial(xy)}{\partial y} \right) \hat{k}$$

$$= (0 - 2y) \hat{i} + (0 - 3z) \hat{j} + (0 - x) \hat{k} = -2y \hat{i} - 3z \hat{j} - x \hat{k}$$