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Date

## Hukum Gauss and applied

$\tau \to V$ (rho)

$$\oint_S \vec{E} \cdot d\vec{A} = \int_V (\nabla \cdot \vec{E})\, d\tau$$

substrace of Gauss

$q$

integral lipat 2

$$\oint E \cdot dA = \oint \frac{q}{4\pi \varepsilon_0 r^2}\, r^2 \sin\theta \, d\theta \, d\phi$$

$$= \frac{q}{4\pi \varepsilon_0} \int_0^{\pi} \int_0^{2\pi} \sin\theta \, d\theta \, d\phi$$

$$= \frac{4\pi q}{4\pi \varepsilon_0}$$

$$\oint_S \vec{E} \cdot da = \frac{q}{\varepsilon_0}$$ → Persamaan Gauss dalam bentuk diferensial

E → Fluks (medan yang muncul dari sumber luaran

divergensi → Perubahan fluks yang muncul dari luaran

$$\int_V (\nabla \cdot \vec{E})\, d\tau$$ → $$\oint \vec{E} \cdot da = \int_V (\nabla \cdot \vec{E})\, d\tau$$

$$\oint Q = \rho \, d\tau$$ → $$Q = \rho \, d\tau$$

$$\int_V (\nabla \cdot \vec{E})\, d\tau = \frac{Q}{\varepsilon_0}$$ ini dapat dari

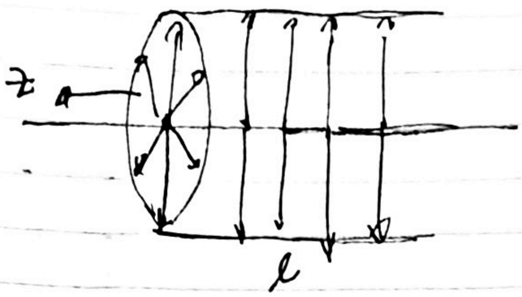
integral lipat 3 itu dari V = Volume

$$= \frac{1}{\varepsilon_0} \int_V \rho \, d\tau$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$ → Persamaan Gauss dalam bentuk diferensial

Contoh soal !

1)




$$\oint \vec{E} \cdot da = \frac{q}{\varepsilon_0}$$

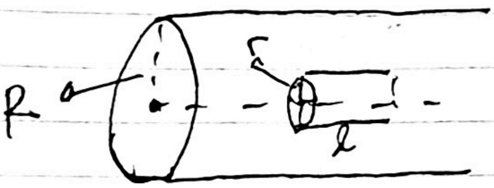
• rapat muatannya $\lambda$

$$E \cdot (2\pi z \cdot \cancel{l}) = \frac{\lambda \cdot \cancel{l}}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi \varepsilon_0 z} \hat{z_0}$$

Hukum Gauss berlaku pada simetri tinggi, artinya kalo bulat berarti bulat banget, kalo panjang berarti panjang banget.

2) Sebuah silinder pejal panjang sekali, dengan jari-jari silinder R, rapat muatannya q. hitung medan listrik di dalam silinder !.




$$\oint \vec{E}\, da = \frac{Q}{\varepsilon_0}$$

$$\vec{E}\,(2\pi r l) = \frac{Q}{\varepsilon_0}$$

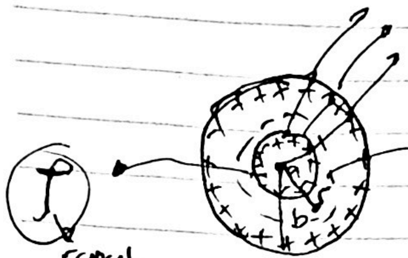
• Q karena muatan rapatnya Q bukan lamda ($\lambda$).

r kecil karena dibagian kecilnya

$$E = \frac{Q}{2\pi \varepsilon_0 r l} \hat{r_0}$$

Soal !

1) Di daerah $a \le r \le b$. Cari medan listrik di tiga wilayah:
(i) $r < a$
(ii) $a < r < b$
(iii) $r > b$
Plot $|E|$ sebagai fungsi dari r.




fluks nya di r

rapat muatan

(i) $\vec{E} = 0$

(ii) $\oint \vec{E}\, da = \dfrac{Q}{\varepsilon_0}$

$$\oint \vec{E} \cdot da = \frac{Q}{\varepsilon_0}$$

$$\vec{E}\,(4\pi r^2) = \frac{1}{\varepsilon_0} \iiint \rho \, d\tau$$

r nya dari fluksnya cuma sampe r

$$= \frac{1}{\varepsilon_0} \int_0^a \int_0^{2\pi} \int_0^{\pi} \rho r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$= \frac{4\pi\rho}{\varepsilon_0} \int_0^a r^2 dr$$

sampe a itu karena muatannya cuma sampe di a

$$= \frac{4\pi\rho}{\varepsilon_0} \cdot \frac{r^3}{3}\Big|_0^a$$

$$= \frac{4\pi\rho}{\varepsilon_0} \cdot \frac{a^3}{3}$$

$$= \frac{4\pi\rho a^3}{3\varepsilon_0}$$

$$= \frac{4\pi\rho a^3}{3\varepsilon_0}$$

(iii) $r > b$

$$\oint \vec{E} \cdot da = \frac{1}{\varepsilon_0} \iiint \rho \, d\tau$$

$$= \frac{1}{\varepsilon_0} \iiint \rho r^2 \sin\theta \, d\theta \, d\phi \, dr$$

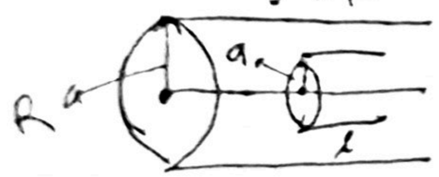
$$= \frac{4\pi\rho}{\varepsilon_0} \int_a^b r^2 \, d\rho$$

$$= \frac{4\pi r}{\varepsilon_0} \left( \tfrac{1}{3} r^3 \Big|_a^b \right)$$

$$= \frac{4\pi r}{\varepsilon_0} \left( \frac{b^3}{3} - \frac{a^3}{3} \right)$$

$$= \frac{4\pi r}{\varepsilon_0} \; \frac{b^3 - a^3}{3}$$

2) $\rho = kr^2$




(i) $r < R$

(ii) $r > R$

(i) $$\oint \vec{E} \cdot da = \frac{1}{\varepsilon_0} \iiint \rho \, d\tau$$

$$\vec{E}(2\pi r \cdot l) = \frac{1}{\varepsilon_0} \int_0^{2\pi}\int_0^{l}\int_0^{R} (kr^2) \quad r \, d\phi \, dz \, dr$$

$$= \frac{k(\pi)}{\varepsilon_0} \int_0^l dz \int_0^R r^3 \, dr$$

$$= \frac{k(2\pi)}{\varepsilon_0} \; z\Big|_0^l \cdot \frac{r^3}{3}\Big|_0^R$$

$$= \frac{k(2\pi)}{\varepsilon_0} \cdot l \cdot \frac{R^3}{3}$$

$$= \frac{2\pi k \, l \, R^3}{\varepsilon_0 \cdot 3}$$

- Potensial : disebabkan oleh muatan listrik
- beda potensial : disebabkan oleh arus listrik

$$W = F \cdot s$$

$$V = \frac{W}{Q}$$

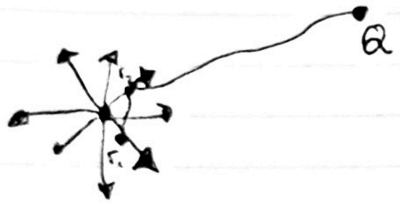
$$= \frac{F \cdot s}{Q}$$

$$= \frac{K \frac{q_1 \cdot q_2}{r} \cdot s}{q_2}$$ — $s = r$

$$V = K \frac{q_1}{r}$$

$$V = -\int \vec{E}\, dr$$

$-$ karena melawan dari sumber muatan




$$dV = -\int_{r_1}^{r_2} \vec{E}\, dr$$

$$= -\int_{r_1}^{r_2} \frac{q}{4\pi\varepsilon_0 r^2}\, dr$$

$$= -\frac{q_1}{4\pi\varepsilon_0}\int_{r_1}^{r_2} r^{-2}\, dr$$

$$\frac{1}{-2+1} r^{-2+1} = \frac{1}{-1} r^{-1} = -r^{-1} = -\frac{1}{r^1}$$

$$= -\frac{q_1}{4\pi\varepsilon_0}\left(-\frac{1}{r}\Big|_{r_1}^{r_2}\right)$$

$$= -\frac{q_1}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$

$$V = K\frac{q_1}{r}$$