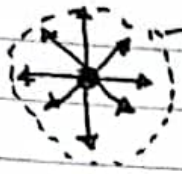
Date 25-03-2024

Kalkulus

Integral Lipat tiga

Medan listrik - Simetri bunga



→ Permukaan gauss

$$\Delta V \neq dv$$

$$\oint \vec{E}\, d\vec{a} = \int (\nabla \cdot \vec{E})\, d\vec{v}$$

$$\oint \vec{E}\, da = \oint \frac{q}{4\pi\varepsilon_0 r^2} r^2 \sin\theta\, d\theta$$

$$= \frac{q}{4\pi\varepsilon_0} \int_0^{\pi} \int_0^{2\pi} \sin\theta\, d\theta\, d\phi$$

$$= \frac{q(2\pi)}{4\pi\varepsilon_0} \int_0^{4} \sin\theta\, d\theta$$

$$= \oint \vec{E}\, da = \frac{4\pi q}{4\pi\varepsilon_0}$$

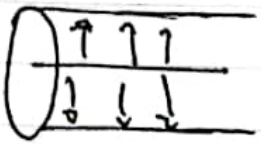
$$\vec{E} \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

∇ : divergensi
(dari kaluh kesatu - pembahasan)

$$\iiint \nabla E\, dv = \iiint \frac{\rho\, dv}{\varepsilon_0} \rightarrow \boxed{\nabla \cdot E = \frac{1}{\varepsilon_0}\rho}$$

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

Contoh soal



permukaan Gauss

$$\oint \vec{E} \cdot d\vec{o} = \frac{q}{\varepsilon_0} \rightarrow \pi . L$$

$$\oint \vec{E}\, d\vec{A} = \frac{\pi l}{\varepsilon_0}$$

$$E\,(2\pi r l) = \frac{\lambda l}{\varepsilon_0}$$

$$\vec{E} = \frac{\lambda}{2\pi \varepsilon_0 r}\, \hat{r}_0$$

2. Sebuah silinder pejal panjang sekali dengan jari-jari silinder R rapat muatannya ρ hitung medan listrik di dalam silinder!

hal (b1)

$\vec{E}(r)$ Pada $r < R$

$$\oint \vec{E} \cdot dA = \frac{q}{\varepsilon_0}$$

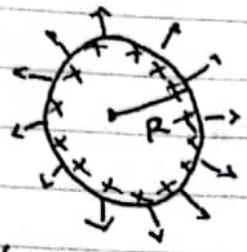
$$\vec{E}\,(2\pi r l) = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{2\pi \varepsilon_0 r l}\, r_0 \text{ - dimana } r < 0$$

Pada $r = R$ maka $\vec{E}\,(2\pi R l) = \frac{q}{\varepsilon_0}\, \hat{r}_0$

$$\vec{E} = \frac{q}{2\pi \varepsilon_0 R l}\, \hat{r}_0$$

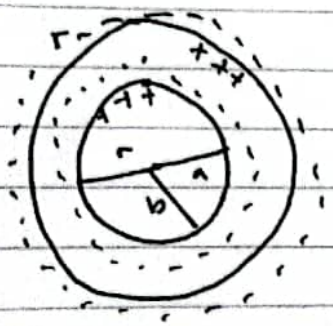
3. bola



Hitung E pada
a $r < R$
b $r > R$

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{incl}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q_{incl}}{\varepsilon_0} \qquad \vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}_0$$



$r < a \rightarrow \vec{E} = 0$

$a < r < b$

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{1}{\varepsilon_0}\iiint \rho \, dv$$

$$E(4\pi r^2) = \frac{4}{3}\frac{\pi \cdot a^3}{\varepsilon_0}$$

$$= \frac{1}{\varepsilon_0}\int_0^{\pi}\int_0^{r}\int_0^{2\pi} r \sin\theta \, d\theta \, d\phi \, dr$$

$$E = \frac{\rho a^3}{3\varepsilon_0 r^2}\hat{r}_0$$

$$= \frac{4\pi}{\varepsilon_0}\int_0^{a} r \, d\phi$$

$$\vec{E}(4\pi r^2) = \frac{4\pi}{\varepsilon_0}\int_0^{r} r \, dp$$

$$= \frac{4\pi}{\varepsilon_0}\left(\frac{1}{3}r^3\Big|_0^{r}\right.$$

* $\vec{E}$ → r > b

$$\oint \vec{E}\, d\vec{a} = \frac{Q}{\varepsilon_0}$$

$$E\,(4\pi r^2) = \frac{1}{\varepsilon_0} \int_a^b \int_0^{2\pi} \int_0^{\pi} \sin\theta \, d\theta \, d\phi \, dr$$

P > b

* $E\,(4\pi r^2) = \frac{\rho}{\varepsilon_0} 4\pi \int_a^b r^2 dr$

$$= \frac{\rho 4\pi}{\varepsilon_0} \left(\frac{1}{3} r^3 \Big|_a^b \right)$$

$$= \frac{\rho 4\pi}{3\varepsilon_0} (b^3 - a^3)$$

$$\vec{E} = \frac{\rho}{3\varepsilon_0 r^2} (b^3 - a^3)$$