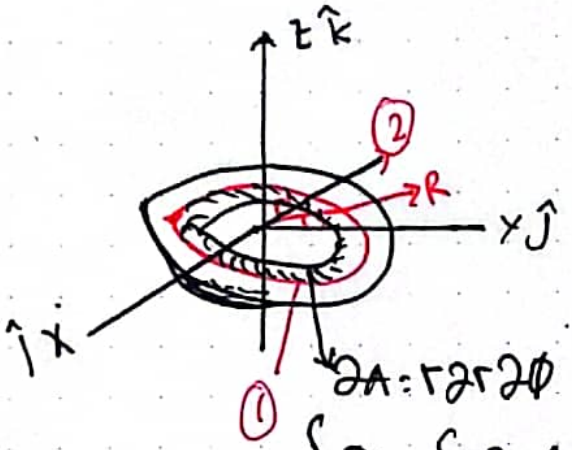
26. Februari 2024

Kalisma

* Sistem Koordinat



$\partial A = r\partial r\partial\phi$

$\int \partial A = \int r\partial r \int \partial\phi$

$A = \frac{1}{2}\left[r^2\right]_{r_1}^{r_2}\left(\phi\right]_0^{2\pi}$

$A = \frac{1}{2}\left[r_2^2 - r_1^2\right](2\pi - 0)$

$A = \frac{1}{2}\left(r_2^2 - r_1^2\right)2\pi$

$A = \frac{2}{2}\pi\left(\frac{r_2^2}{\pi} - r_1^2\right)$

$A = \pi(r_2^2 - r_1^2)$

$A = A_2 - A_1 \quad A_2 = \pi r_2^2$

$A_1 = \pi r_1^2$

Rumus lingkaran

$A = \pi r^2$

$A = \pi r^2$

$$A = \frac{1}{2}|r^2| \int_{r_1}^{r_2} (\theta)_0^{2\pi}$$

$$A = \frac{1}{2}(r_2^2 - r_1^2)(\theta_2 - \theta_1)$$

$$A = \frac{1}{2}((6-4)\,cm^2)\left(\frac{1}{2}\pi - \frac{1}{2}\pi\right)$$

$$A = \frac{1}{2}(\quad)^2 \frac{1}{2}\pi$$

$$A = 3\pi\,cm^2$$

$$A = 3.14\,cm^2$$

$R = 6\,cm$ $\quad 360^\circ = 2\pi$

$r_2 = 4\,cm$ $\quad 180^\circ = \pi$

$r_1 = 2\,cm$ $\quad 90^\circ = \frac{1}{2}\pi$

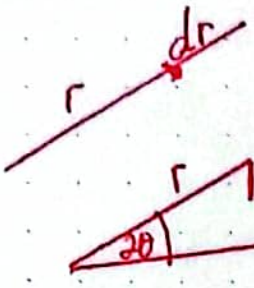
$\theta_2 = 90^\circ$ $\quad 60^\circ = \frac{1}{3}\pi$

$\theta_1 = 60^\circ$

Koordinat Polar

Rumus Volume Bola

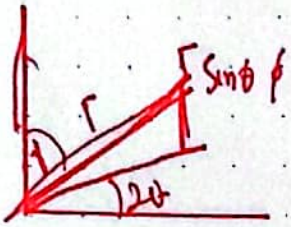
$$V = \frac{4}{3}\pi R^3$$



$$\partial r = \partial r$$

$$\partial \theta = \partial \theta$$

$$\partial \phi = r \sin\theta \, \partial \phi$$



$$\int \partial \tau = \partial r \cdot r\partial\theta \cdot r\sin\theta \, \partial\phi$$

$$V = \int \partial \tau = \int \partial r \cdot \int r\partial\theta \cdot \int r\sin\theta \, \partial\phi$$

$$= \partial\tau \int_{r=0}^{r=R} \partial r \int_{\theta=0}^{\theta=\pi} \partial\theta \int_{\phi=0}^{\phi=2\pi} r\sin\theta \, \partial\phi$$

$$\int_{\tau=0}^{\tau=V} \partial\tau \cdot \int_{r=0}^{r=R} r^2 \partial r \cdot \int_{\theta=0}^{\theta=\pi} r\partial\theta \int_{\phi=0}^{\phi=2\pi} \sin\theta \, \partial\phi$$

$$\left[t\right]_0^V = \frac{1}{3}\left[r^3\right]_0^R \left(-\cos\theta\right)_0^{\pi} \left[\phi\right]_0^{2\pi}$$

$$V - 0 = \frac{1}{3}R^3 \left(-\cos\pi - (-\cos 0)\right) 2\pi$$

$$V = \frac{1}{3}R^3 \cdot 2 \cdot 2\pi$$

Vektor satuan

Gradient

$$\nabla T = \frac{\partial T}{\partial r}\hat{r} + \frac{\partial (T)}{r \partial \theta}\hat{\theta} + \frac{\partial (T)}{r \sin\theta \partial \phi}\hat{\phi}$$

Divergent

$$\nabla \cdot \nabla = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 V_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\sin\theta V_\theta) + \frac{1}{r \sin\theta}\frac{\partial V_\theta}{\partial \phi}$$

Curl

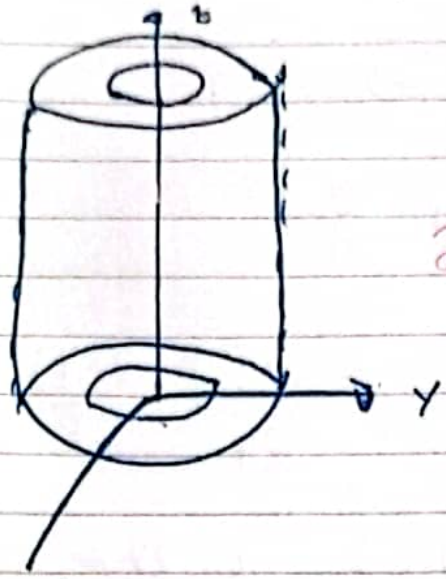
$$\nabla \cdot V = \frac{1}{r \sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta V_\phi) - \frac{\partial V_\theta}{\partial \phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial V_r}{\partial \phi} - \frac{\partial}{\partial r}(r V_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r V_\theta) - \frac{\partial V_r}{\partial \theta}\right]\hat{\phi}$$

Laplacian

$$\nabla^2 T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}$$

No
Date

Sistem Koordinat Silinder



Satuan Vektor :

$\hat{r}$. $\hat{\phi}$. $\hat{z}$
∂r $r\partial\phi$ ∂z

Vektor satuan r

$$\partial \tau = \partial r \cdot \hat{r} + r \cdot \partial\phi\, \hat{\phi} + \partial z\, \hat{z}$$

Volume tabung Silinder

$$\partial \tau = \partial r \cdot r \cdot \partial\phi \cdot \partial z$$

$$\int_{\tau_0=0}^{\tau=V} \partial \tau = \int_0^R r \cdot \partial r \int_{\phi=0}^{\phi=2\pi} \partial\phi \cdot \int_1^z \partial z$$

$$(\tau)_0^V = \left[\frac{1}{2} r^2\right]_0^R [\phi]_0^{2\pi} [z]^z.$$

$$V - 0 = \frac{1}{2} R^2 \cdot 2\pi \cdot z$$

$$\boxed{V = \pi r^2 z}$$

No
Date

Gradient

$$\nabla = \frac{\partial}{\partial r}\hat{r} + \frac{\partial}{r\partial\phi}\hat{\phi} + \frac{\partial}{\partial z}\hat{z}$$

$$\nabla = \frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial\phi}\hat{\phi} + \frac{\partial}{\partial z}\hat{z}$$

$$\nabla T = \frac{\partial (T)}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial (T)}{\partial\phi}\hat{\phi} + \frac{\partial (T)}{\partial z}\hat{z}$$

$$\nabla \cdot V = \frac{1}{r}\frac{\partial}{\partial r}(rV_r) + \frac{1}{r}\frac{\partial V_\phi}{\partial\phi} + \frac{\partial V_z}{\partial z}$$

$$\nabla \times V = \left(\frac{1}{r}\frac{\partial V_z}{\partial\phi} - \frac{\partial V_\phi}{\partial z}\right)\hat{r} + \left(\frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r}\right)\hat{\phi}$$

0 Lines, 6 mm

No
Date

Soal!

1. Berapa Volume spesimen atau Potongan bola yang dibentuk pada koordinat bola
Potongan bangun yang dibentuk pada koordinat bola jika memenuhi kriteria sebagai berikut

Jawab :

Diket : R : 8 cm
r : 6 cm
θ_2 : 90°
θ_1 : 45°
ϕ : 90°
ϕ : 45°

$$\int 2\,\mathcal{C} = 2r \cdot r2\theta \cdot r\sin\theta\, 2\phi$$

$$(\mathcal{C})_0^v = \frac{1}{3}(r^3)_0^R\,(-\cos\theta)_0^{\theta}\,(\phi)_{\phi_0}^{\phi}$$

$$V = \int 2\pi = \int 2r$$

$$\frac{1}{3}(6^3)_0^{\theta}\,(-\cos\phi^{\circ})_{45^\circ}^{90^\circ}\,(\phi)_{45^\circ}^{90^\circ}$$