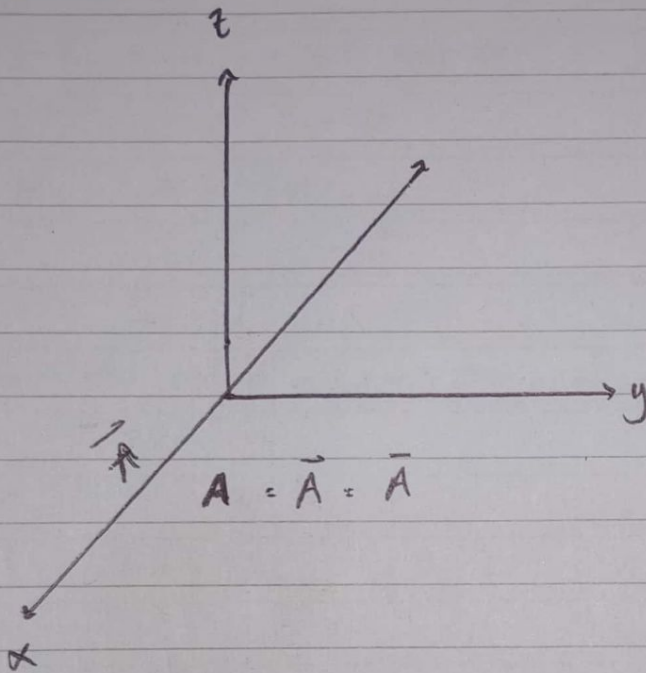


KELISTRIKAN DAN KEMAGNETAN

Analisis Vektor



$$\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$|\vec{A}| = \sqrt{x^2 + y^2 + z^2}$$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

$$= \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$$

Contoh :

$$\vec{A} = 4\hat{i} + 5\hat{j} + 2\hat{k}$$

$$\vec{B} = 2\hat{i} + \hat{j} + 3\hat{k}$$

Pengjumlahan : $\vec{A} + \vec{B}$

$$6\hat{i} + 4\hat{j} + 5\hat{k}$$

Pertalian Vektor

a.) dot (.)

$$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cdot \cos \theta$$

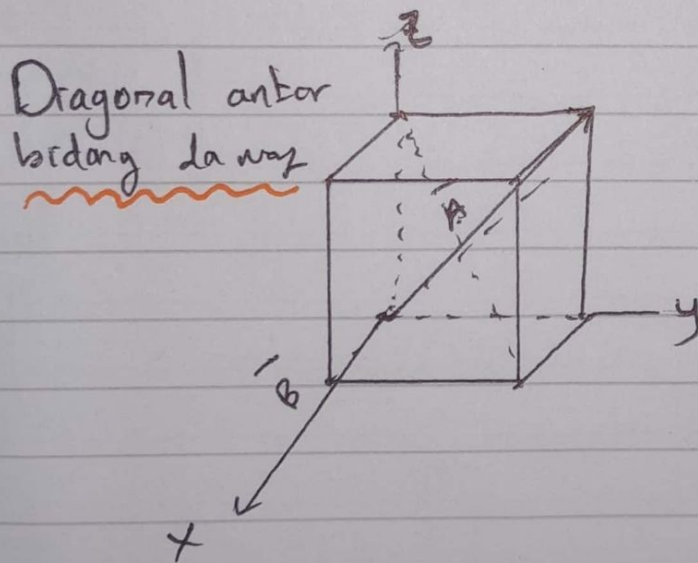
$$\begin{aligned}\vec{A} \cdot \vec{B} &= (4\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (2\hat{i} + \hat{j} + 3\hat{k}) \\ &= 8 + 3 + 6 \\ &= 17\end{aligned}$$

$$|\vec{A}| = \sqrt{16 + 9 + 4}$$

$$|\vec{B}| = \sqrt{4 + 1 + 9}$$

$$17 = \sqrt{29} \cdot \sqrt{14} \cos \theta$$

$$\theta = \cos^{-1} \frac{17}{\sqrt{29} \cdot \sqrt{14}}$$



$$\begin{aligned}\vec{A} &= 0\hat{i} + \hat{j} + \hat{k} \\ \vec{B} &= 1\hat{i} + 0\hat{j} + 0\hat{k}\end{aligned}$$

$$\begin{aligned}\vec{A} \cdot \vec{B} &= (0\hat{i} + \hat{j} + \hat{k}) \cdot (1\hat{i} + 0\hat{j} + 0\hat{k}) \\ &= 0 + 0 + 1 \\ &= 1\end{aligned}$$

$$|\vec{A}| = \sqrt{0+1+1} = \sqrt{2}$$

$$|\vec{B}| = \sqrt{1+0+1} = \sqrt{2}$$

$$\vec{A} \cdot \vec{B} = \sqrt{2} \cdot \sqrt{2} \cos \theta$$

$$1 = 2 \cos \theta$$

$$\theta = \arccos \frac{1}{2}$$

Hitunglah sudut antara diagonal ruang

$$\vec{A} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{B} = -\hat{i} + \hat{j} + \hat{k}$$

$$\vec{A} \cdot \vec{B} = (\hat{i} + \hat{j} + \hat{k}) \cdot (-\hat{i} + \hat{j} + \hat{k})$$

$$= -1 - 1 + 1$$

$$= -1$$

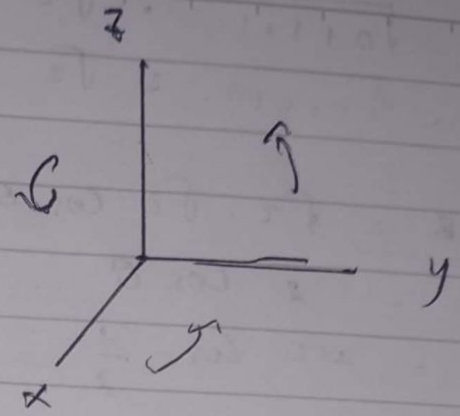
$$-1 = \sqrt{3} \cdot \sqrt{3} \cos \theta$$

$$\theta = \arccos \frac{-1}{\sqrt{3} \cdot \sqrt{3}}$$

$$\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$$

$$\vec{B} = 3\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 3 & 2 & 2 \end{vmatrix}$$



$$= \hat{i}(6-2) - \hat{j}(4-3) + \hat{k}(9-4)$$

Operator Nabla (∇)

$$\nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$$

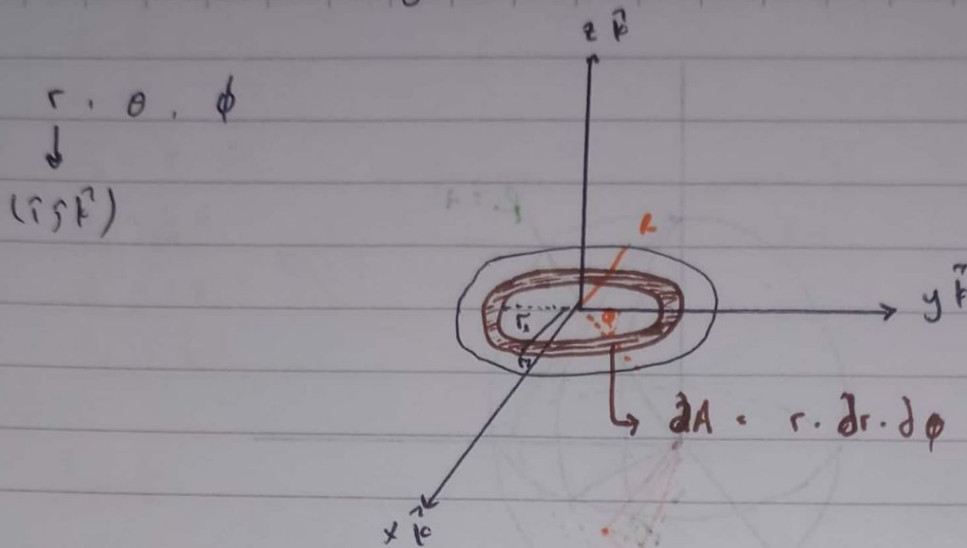
$$T = x + 2y + 3z^2 \quad (\text{skalar})$$

$$\begin{aligned} \nabla \cdot T &= \hat{i} + \hat{j}(2) + \hat{k}(6z) \\ &= \hat{i} + 2\hat{j} + 6z\hat{k} \end{aligned}$$

$$\vec{V} = 3x^2\hat{i} + 2xy\hat{j} + 4xyz\hat{k}$$

$$\nabla \cdot \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & 2xz & -2xz \end{vmatrix}$$

Kelistrikan dan Kemagnetan



$$\int_A dA = \int_{r_1}^{r_2} r \, dr \cdot \int_{\phi_1}^{\phi_2} d\phi$$

$$[A]_{A_1}^{A_2} = \left[\frac{1}{2} r^2 \right]_{r_1}^{r_2} \cdot [\phi]_{\phi_1}^{\phi_2}$$

$$(A_2 - A_1) = \frac{1}{2} (r_2^2 - r_1^2) - (\phi_2 - \phi_1)$$

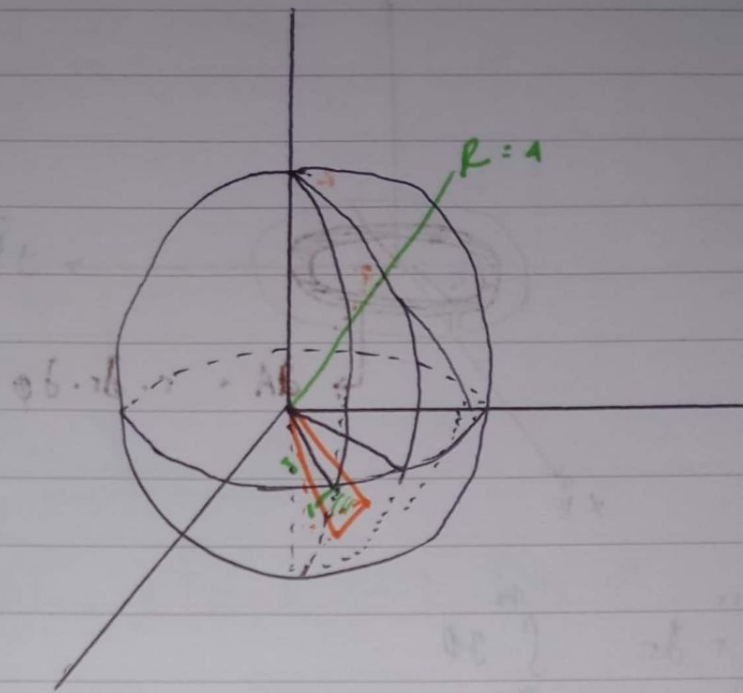
$$A = \frac{1}{2} \cdot r^2 \cdot 2\pi$$

$$= \pi r^2$$

• Rapat muat luas (σ)

$$\sigma = \frac{\partial q}{\partial A}$$

$$= \frac{q}{A}$$



$$\partial L_r = \partial r$$

$$\partial L_\theta = r \cdot \partial \theta$$

$$\partial L_\phi = r \cdot \sin \theta \cdot \partial \phi$$

$$\begin{aligned} \partial \tau &= \partial L_r \cdot \partial L_\theta \cdot \partial L_\phi \\ &= \partial r \cdot r \cdot \partial \theta \cdot r \sin \theta \cdot \partial \phi \end{aligned}$$

$$\begin{aligned} V &= \int \partial \tau \\ &= \int_0^R r \partial r \int_0^\pi r \partial \theta \cdot \int_0^{2\pi} \sin \theta \partial \phi \\ &= \int_0^R r^2 \partial r \int_0^\pi \sin \theta \partial \theta \int_0^{2\pi} \partial \phi \\ &= \frac{1}{3} r^3 (-\cos \theta) \Big|_0^\pi \cdot (\phi) \Big|_0^{2\pi} \\ &= \frac{1}{3} r^3 (-\cos \pi - (-\cos 0)) \cdot (2\pi - 0) \\ &= \frac{1}{3} r^3 \cdot 2 \cdot 2\pi \\ &= \frac{4}{3} \pi r^3 \end{aligned}$$

$$R_2 = 4$$

$$R_1 = 3$$

$$\theta_2 = \pi$$

$$\theta_1 = \frac{1}{2}\pi$$

$$\phi_2 = 2\pi$$

$$\phi_1 = \frac{1}{2}\pi$$

Berapa volume sampel dari
bola yg beraturan $R=4$
jika $\frac{1}{2}\pi$

Jawab :

$$V = \partial T$$

$$= \int r \, dr \cdot \int r \, d\theta \cdot \int \sin \theta \, d\phi$$

$$= \int_3^4 r^2 \, dr \cdot \int_{\frac{1}{2}\pi}^{\pi} \sin \theta \, d\theta \cdot \int_{\frac{1}{2}\pi}^{2\pi} d\phi$$

$$V = \left(\frac{1}{3} R^3 \right) \Big|_3^4 \cdot \left(-\cos \theta \right) \Big|_{\frac{1}{2}\pi}^{\pi} \cdot \left(\phi \right) \Big|_{\frac{1}{2}\pi}^{2\pi}$$

$$= \frac{1}{3} (4^3 - 3^3) \cdot \left((-\cos 180^\circ - (-\cos 90^\circ)) \right) \cdot \left(2\pi - \frac{1}{2}\pi \right)$$

$$= \left(\frac{1}{3} \cdot (64 - 27) \right) (1 - 0) \left(\frac{3}{2}\pi \right)$$

$$= \frac{1}{2} \cdot 37$$

$$= 18.5 \pi$$

$$V_{\text{tot}} = 267.9$$

$$\nabla = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}$$

$$\nabla = \frac{\partial}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \hat{\phi}$$

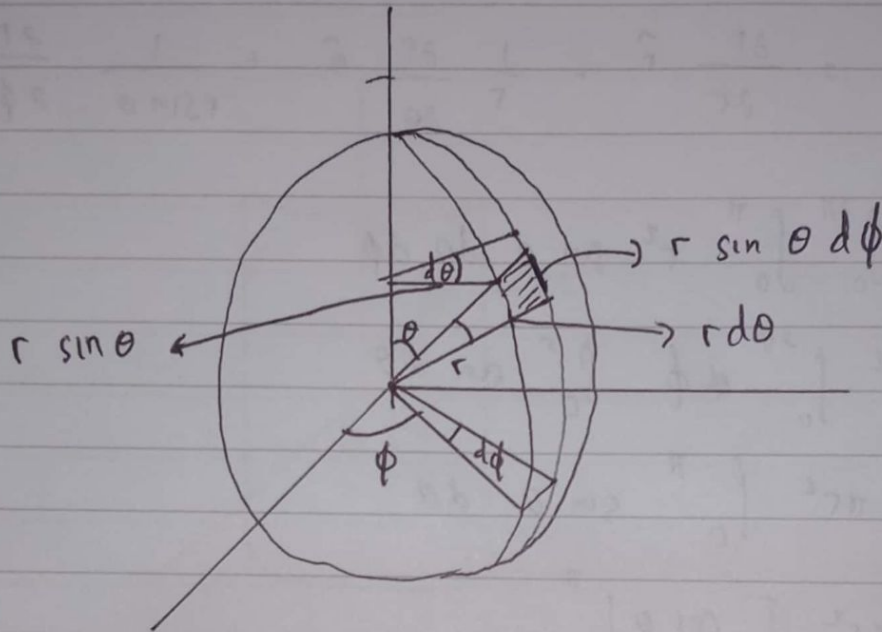
$$\nabla T = \frac{\partial T}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial T}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial T}{\partial \phi} \hat{\phi}$$

$$PR = 55 \text{ (hal gram)}$$

$$\frac{5\pi}{2}$$

Kelistrikan dan Kemagnetan

Sistem Koordinat Bola



$$\sin \theta = \frac{x}{r}$$

$$x = r \sin \theta$$

• Satuan luas

$$da = dp \cdot dl$$

$$= r d\theta \cdot r \sin \theta d\phi$$

$$= r^2 \sin \theta d\theta d\phi$$

$$da = r^2 \sin \theta d\theta d\phi$$

• Satuan volume

$$dV = dp \cdot dl \cdot dr$$

$$= r^2 \sin \theta d\theta d\phi dr$$

Gradien :

$$\begin{aligned}\nabla T &= \frac{\partial T}{\partial x} \hat{i} + \frac{\partial T}{\partial y} \hat{j} + \frac{\partial T}{\partial z} \hat{k} \\ &= \frac{\partial T}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial T}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial T}{\partial \phi} \hat{\phi}\end{aligned}$$

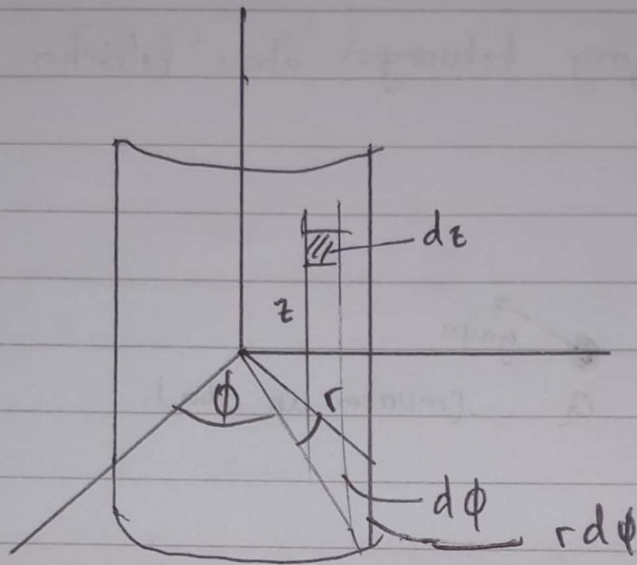
Luas :

$$\begin{aligned}A &= \int_0^{2\pi} \int_0^{\pi} r^2 \sin \theta \, d\theta \, d\phi \\ &= r^2 \int_0^{2\pi} d\phi \int_0^{\pi} \sin \theta \, d\theta \\ &= 2\pi r^2 \int_0^{\pi} \sin \theta \, d\theta \\ &= 2\pi r^2 \left[\cos \theta \right]_0^{\pi} \\ &= 2\pi r^2 (1 - (-1)) \\ &= 4\pi r^2\end{aligned}$$

Volume :

$$\begin{aligned}V &= \int_0^{\pi} \int_0^{2\pi} \int_0^r r^2 \sin \theta \, d\theta \, d\phi \, dr \\ &= \left(\int_0^{\pi} \sin \theta \, d\theta \right) \left(\int_0^r r^2 \, dr \right) \left(\int_0^{2\pi} d\phi \right) \\ &= \left(\cos \theta \Big|_0^{\pi} \right) \left(\frac{1}{3} r^3 \right) (2\pi) \\ &= 2 \left(\frac{r^3}{3} \right) (2\pi) \\ &= \frac{4}{3} \pi r^3\end{aligned}$$

1.1 Sistem Koordinat Silinder



Luas :

$$dA = d\phi \cdot r \cdot dr$$

$$= r d\phi dr$$

Volume :

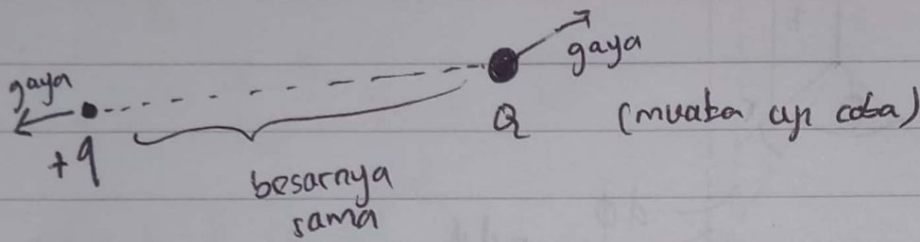
$$dV = r d\phi dz dr$$

11 | Gaya Listrik

• Muatan Listrik

Ion adalah atom yang kekurangan atau kelebihan elektron

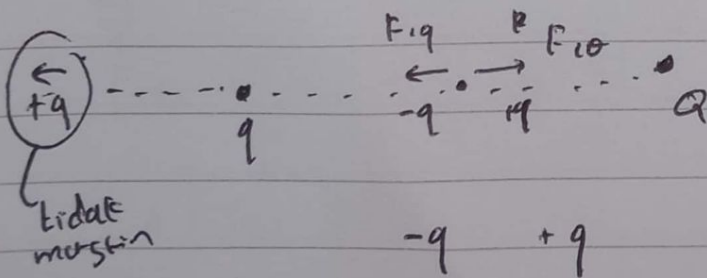
$$q = \pm Ne$$



$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \cdot \hat{r}_0$$

$$\hat{r}_0 = \frac{\vec{r}}{|\vec{r}|}$$

Contoh :



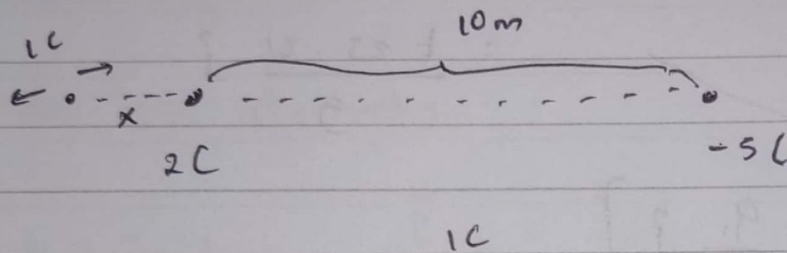
$$F_{1q} = F_{1Q}$$

$$k \cdot \frac{q \cdot Q}{(r-x)^2} = k \cdot \frac{q \cdot q}{(x^2)} \quad x = \dots ?$$

$$\frac{Q}{(P^2 - 2Px + x^2)} = \frac{q}{x^2}$$

$$Qx^2 = q(P^2 - 2Px + x^2)$$

Contoh :



$$F_{12} = F_{13}$$

$$\frac{1C}{x^2} = \frac{1C}{(10+x)^2}$$

$$\frac{3}{x^2} = \frac{3}{x^2 + 20x + 100}$$

$$3x^2 = 2x^2 + 40x + 200$$

$$3x^2 - 2x^2 - 40x = 200$$

$$x^2 - 40x = 200$$

$$x^2 - 40x - 200 = 0$$

$$() ()$$

Buatkan rumus

$$x = -b \pm \sqrt{\frac{b^2 - 4ac}{2a}}$$

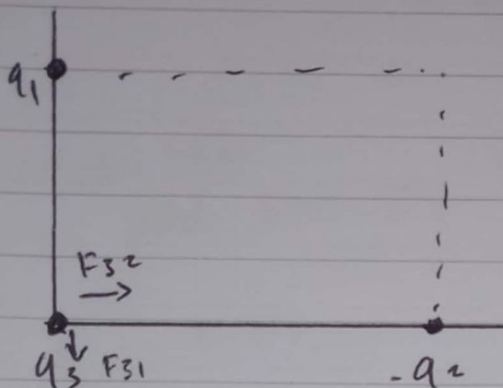
$$x = -40 \pm \sqrt{\frac{(-40)^2 - (4 \cdot 1 \cdot 200)}{2}}$$

$$x = -40 \pm \sqrt{400}$$

$$= -40 \pm 20$$

$$(+) = -40 + 20 = -20$$

$$(-) = -40 - 20 = -60$$



$$F_{32} = \frac{k q_3 q_2}{4^2} \frac{3\hat{i} + 0\hat{j}}{\sqrt{3^2 + 0^2}}$$

$$= k \frac{q_3 q_2}{16} \hat{i}$$

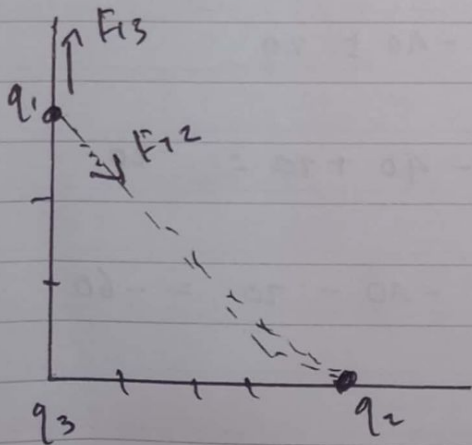
$$F_{31} = \frac{k q_3 q_1}{3^2} (-\hat{j})$$

$$= \frac{k 23 \cdot 21}{9} \hat{j}$$

$$\vec{F} = k q_3 \left[\frac{q_2}{16} \hat{i} - \frac{q_1}{9} \hat{j} \right]$$

$$|\vec{F}| = k q_3 \sqrt{\left(\frac{q_2}{16}\right)^2 + \left(\frac{q_1}{9}\right)^2}$$

Hitunglah gaya yg dialami q_1



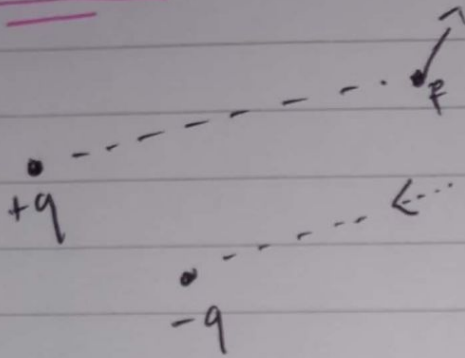
$$F_{13} = \frac{k q_1 q_3}{3^2} \hat{j}$$

$$F_{12} = \frac{k q_1 q_2}{5^2} \left(\frac{q_1 \hat{i} - 3\hat{j}}{5} \right)$$

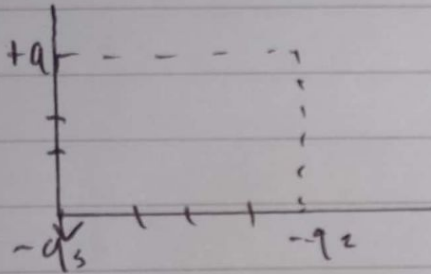
$$= \frac{4k q_1 q_2}{125} \hat{i} - \frac{3k q_1 q_2}{125} \hat{j}$$

$$\vec{F} = k q_1 \left[\left(\frac{4q_2}{125} \right) \hat{i} + \left(\frac{q_3}{9} - \frac{3q_2}{125} \right) \hat{j} \right]$$

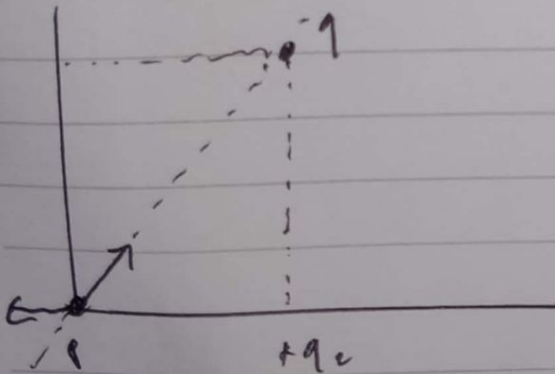
Medan Listrik



medan berya satu muatan



$$\begin{aligned}\vec{E}_{q_3} &= \frac{F_{32}}{-q_3} \\ &= k \frac{q_3 q_2}{q_3} \hat{j} \\ F &= k \frac{q}{r^2} \hat{j}\end{aligned}$$



$$\begin{aligned}\vec{E}_1 &= k \frac{q_1}{r^2} (-\hat{i}) \\ &= k \frac{q_1}{9} \hat{i} \\ \vec{E}_2 &= k \frac{q_2}{r^2} \left(\frac{3\hat{i} + 1\hat{j}}{5} \right) \\ &= 3k \frac{q_2}{125} \hat{i} + 4k \frac{q_2}{125} \hat{j} \\ \vec{E} &= k \left[\left(\frac{3q_2}{125} - \frac{q_1}{9} \right) \hat{i} + \frac{4q_2}{125} \hat{j} \right]\end{aligned}$$