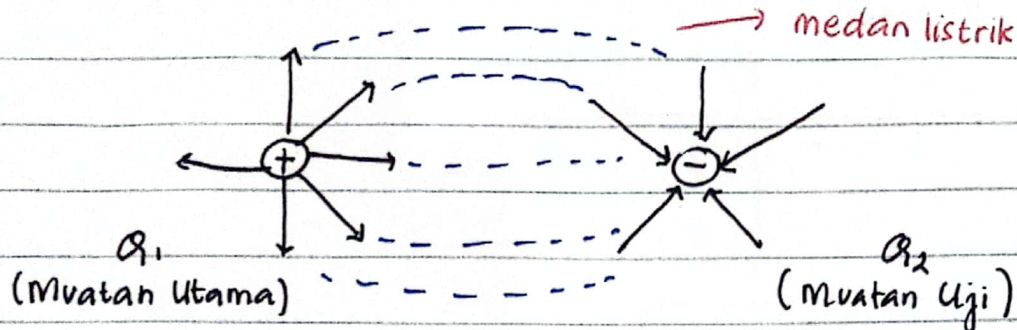


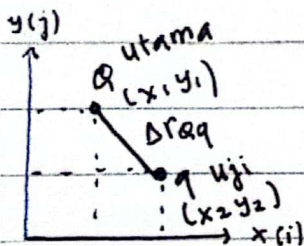
Medan Listrik

↳ Daerah yang masih dipengaruhi oleh gaya listrik / gaya colomb



* Muatan utama menghasilkan medan listrik pada muatan uji

Persamaan.



$$\vec{F}_{21} = Q_2 \vec{E}_{21}$$

$$\rightarrow \vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{r_{21}^2} \vec{r}_{21}$$

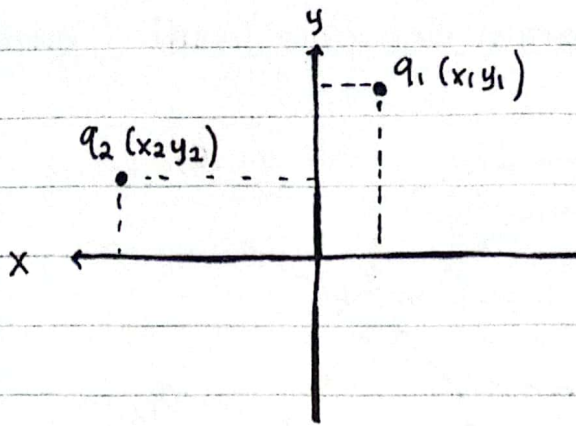
$$\vec{E}_{21} = Q_2 \frac{1}{4\pi\epsilon_0} \frac{Q_1}{r_{21}^2} \vec{r}_{21}$$

$$\boxed{\vec{E}_{21} = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{r_{21}^2} \vec{r}_{21}} \rightarrow \vec{r}_{21} = \frac{\Delta x \hat{i} + \Delta y \hat{j}}{\sqrt{\Delta x^2 + \Delta y^2}} = \frac{\vec{r}_{21}}{|\vec{r}_{21}|}$$

$\hookrightarrow |\vec{r}_{21}| = r_{12} = \sqrt{\Delta x^2 + \Delta y^2}$

$$\boxed{\vec{E}_{21} = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{r_{21}^3} \vec{r}_{21}}$$

Misalkan :



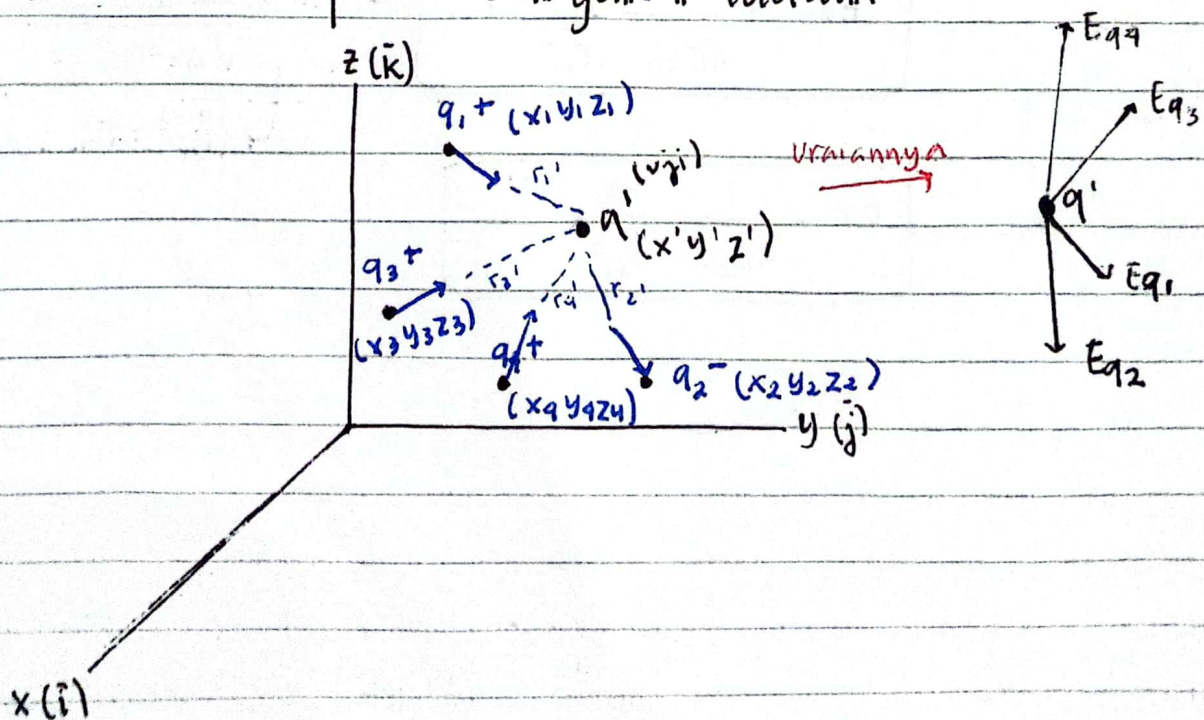
$$\vec{E}_{q_1} = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{12}^2} \hat{r}_{12}$$

→ jika yg ditanya q_1 , maka jadikan q_1 sebagai muatan utama

$$\vec{E}_{q_2} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_{21}^2} \hat{r}_{21}$$

→ jika yg ditanya q_2 , maka jadikan q_2 sebagai muatan utama

Medan Listrik pada Banyak Muatan



$$E_{\text{total}} = E_q' = E_{q_1} + E_{q_2} + E_{q_3} + E_{q_4}$$

$$* E_{q_1} = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1'^2} \hat{r}_1'$$

$$\hat{r}_1' = \frac{\vec{r}_1'}{|\vec{r}_1'|} = \frac{\Delta x_1' \hat{i} + \Delta y_1' \hat{j} + \Delta z_1' \hat{k}}{\sqrt{\Delta x_1'^2 + \Delta y_1'^2 + \Delta z_1'^2}}$$

$$* E_{q_2} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_2'^2} \hat{r}_2'$$

$$r_1' = |\vec{r}_1'| = (\Delta x_1'^2 + \Delta y_1'^2 + \Delta z_1'^2)^{\frac{1}{2}}$$

$$* E_{q_3} = \frac{1}{4\pi\epsilon_0} \frac{q_3}{r_3'^2} \hat{r}_3'$$

$$\begin{matrix} \downarrow & \downarrow & \downarrow \\ (x' - x_1) & (y' - y_1) & (z' - z_1) \end{matrix}$$

$$* E_{q_4} = \frac{1}{4\pi\epsilon_0} \frac{q_4}{r_4'^2} \hat{r}_4'$$

Sehingga menjadi ,

$$* E_{q_1} = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1'^2} \hat{r}_1'$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_1}{(\Delta x_1'^2 + \Delta y_1'^2 + \Delta z_1'^2)^{\frac{1}{2}}^2} \cdot \frac{\Delta x_1' \hat{i} + \Delta y_1' \hat{j} + \Delta z_1' \hat{k}}{(\Delta x_1'^2 + \Delta y_1'^2 + \Delta z_1'^2)^{\frac{1}{2}}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_1}{(\Delta x_1'^2 + \Delta y_1'^2 + \Delta z_1'^2) (\Delta x_1'^2 + \Delta y_1'^2 + \Delta z_1'^2)^{\frac{1}{2}}} \cdot (\Delta x_1' \hat{i} + \Delta y_1' \hat{j} + \Delta z_1' \hat{k})$$

$$* E_{q_2} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_2'^2} \hat{r}_2'$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_2}{((\Delta x_2'^2 + \Delta y_2'^2 + \Delta z_2'^2)^{\frac{1}{2}})^2} \cdot \frac{\Delta x_2' \hat{i} + \Delta y_2' \hat{j} + \Delta z_2' \hat{k}}{(\Delta x_2'^2 + \Delta y_2'^2 + \Delta z_2'^2)^{\frac{1}{2}}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_2}{(\Delta x_2'^2 + \Delta y_2'^2 + \Delta z_2'^2) (\Delta x_2'^2 + \Delta y_2'^2 + \Delta z_2'^2)^{\frac{1}{2}}} \cdot (\Delta x_2' \hat{i} + \Delta y_2' \hat{j} + \Delta z_2' \hat{k})$$

$$* E_{q_3} = \frac{1}{4\pi\epsilon_0} \frac{q_3}{r_3'^2} \hat{r}_3'$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_3}{((\Delta x_3'^2 + \Delta y_3'^2 + \Delta z_3'^2)^{1/2})^2} \cdot \frac{\Delta x_3' \hat{i} + \Delta y_3' \hat{j} + \Delta z_3' \hat{k}}{(\Delta x_3'^2 + \Delta y_3'^2 + \Delta z_3'^2)^{1/2}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_3}{(\Delta x_3'^2 + \Delta y_3'^2 + \Delta z_3'^2)(\Delta x_3'^2 + \Delta y_3'^2 + \Delta z_3'^2)^{1/2}} \cdot \Delta x_3' \hat{i} + \Delta y_3' \hat{j} + \Delta z_3' \hat{k}$$

$$* E_{q_1} = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1'^2} \hat{r}_1'$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_1}{((\Delta x_1'^2 + \Delta y_1'^2 + \Delta z_1'^2)^{1/2})^2} \cdot \frac{\Delta x_1' \hat{i} + \Delta y_1' \hat{j} + \Delta z_1' \hat{k}}{(\Delta x_1'^2 + \Delta y_1'^2 + \Delta z_1'^2)^{1/2}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_1}{(\Delta x_1'^2 + \Delta y_1'^2 + \Delta z_1'^2)(\Delta x_1'^2 + \Delta y_1'^2 + \Delta z_1'^2)^{1/2}} \cdot \Delta x_1' \hat{i} + \Delta y_1' \hat{j} + \Delta z_1' \hat{k}$$

$$E_{\text{total}} = E_{q'} = E_{q_1} + E_{q_2} + E_{q_3} + E_{q_4}$$

$$E_{q'} = \left(\frac{1}{4\pi\epsilon_0} \frac{q_1}{(\Delta x_1'^2 + \Delta y_1'^2 + \Delta z_1'^2)(\Delta x_1'^2 + \Delta y_1'^2 + \Delta z_1'^2)^{1/2}} (\Delta x_1' \hat{i} + \Delta y_1' \hat{j} + \Delta z_1' \hat{k}) \right) + \left(\frac{1}{4\pi\epsilon_0} \frac{q_2}{(\Delta x_2'^2 + \Delta y_2'^2 + \Delta z_2'^2)(\Delta x_2'^2 + \Delta y_2'^2 + \Delta z_2'^2)^{1/2}} (\Delta x_2' \hat{i} + \Delta y_2' \hat{j} + \Delta z_2' \hat{k}) \right) + \left(\frac{1}{4\pi\epsilon_0} \frac{q_3}{(\Delta x_3'^2 + \Delta y_3'^2 + \Delta z_3'^2)(\Delta x_3'^2 + \Delta y_3'^2 + \Delta z_3'^2)^{1/2}} (\Delta x_3' \hat{i} + \Delta y_3' \hat{j} + \Delta z_3' \hat{k}) \right) + \left(\frac{1}{4\pi\epsilon_0} \frac{q_4}{(\Delta x_4'^2 + \Delta y_4'^2 + \Delta z_4'^2)(\Delta x_4'^2 + \Delta y_4'^2 + \Delta z_4'^2)^{1/2}} (\Delta x_4' \hat{i} + \Delta y_4' \hat{j} + \Delta z_4' \hat{k}) \right)$$

$$+ \left(\frac{1}{4\pi\epsilon_0} \frac{q_2}{(\Delta x_2'^2 + \Delta y_2'^2 + \Delta z_2'^2)(\Delta x_2'^2 + \Delta y_2'^2 + \Delta z_2'^2)^{1/2}} (\Delta x_2' \hat{i} + \Delta y_2' \hat{j} + \Delta z_2' \hat{k}) \right) + \left(\frac{1}{4\pi\epsilon_0} \frac{q_3}{(\Delta x_3'^2 + \Delta y_3'^2 + \Delta z_3'^2)(\Delta x_3'^2 + \Delta y_3'^2 + \Delta z_3'^2)^{1/2}} (\Delta x_3' \hat{i} + \Delta y_3' \hat{j} + \Delta z_3' \hat{k}) \right) + \left(\frac{1}{4\pi\epsilon_0} \frac{q_4}{(\Delta x_4'^2 + \Delta y_4'^2 + \Delta z_4'^2)(\Delta x_4'^2 + \Delta y_4'^2 + \Delta z_4'^2)^{1/2}} (\Delta x_4' \hat{i} + \Delta y_4' \hat{j} + \Delta z_4' \hat{k}) \right)$$

$$+ \left(\frac{1}{4\pi\epsilon_0} \frac{q_3}{(\Delta x_3'^2 + \Delta y_3'^2 + \Delta z_3'^2)(\Delta x_3'^2 + \Delta y_3'^2 + \Delta z_3'^2)^{1/2}} (\Delta x_3' \hat{i} + \Delta y_3' \hat{j} + \Delta z_3' \hat{k}) \right) + \left(\frac{1}{4\pi\epsilon_0} \frac{q_4}{(\Delta x_4'^2 + \Delta y_4'^2 + \Delta z_4'^2)(\Delta x_4'^2 + \Delta y_4'^2 + \Delta z_4'^2)^{1/2}} (\Delta x_4' \hat{i} + \Delta y_4' \hat{j} + \Delta z_4' \hat{k}) \right)$$

$$+ \left(\frac{1}{4\pi\epsilon_0} \frac{q_4}{(\Delta x_4'^2 + \Delta y_4'^2 + \Delta z_4'^2)(\Delta x_4'^2 + \Delta y_4'^2 + \Delta z_4'^2)^{1/2}} (\Delta x_4' \hat{i} + \Delta y_4' \hat{j} + \Delta z_4' \hat{k}) \right)$$

$$\begin{aligned}
 E_{q'} = \frac{1}{4\pi\epsilon_0} & \left(\left(\frac{q_1}{(\Delta x_1'^2 + \Delta y_1'^2 + \Delta z_1'^2)(\Delta x_1'^2 + \Delta y_1'^2 + \Delta z_1'^2)^{1/2}} (\Delta x_1' \hat{i} + \Delta y_1' \hat{j} + \Delta z_1' \hat{k}) \right) + \right. \\
 & \left(\frac{q_2}{(\Delta x_2'^2 + \Delta y_2'^2 + \Delta z_2'^2)(\Delta x_2'^2 + \Delta y_2'^2 + \Delta z_2'^2)^{1/2}} (\Delta x_2' \hat{i} + \Delta y_2' \hat{j} + \Delta z_2' \hat{k}) \right) + \\
 & \left(\frac{q_3}{(\Delta x_3'^2 + \Delta y_3'^2 + \Delta z_3'^2)(\Delta x_3'^2 + \Delta y_3'^2 + \Delta z_3'^2)^{1/2}} (\Delta x_3' \hat{i} + \Delta y_3' \hat{j} + \Delta z_3' \hat{k}) \right) + \\
 & \left. \left(\frac{q_4}{(\Delta x_4'^2 + \Delta y_4'^2 + \Delta z_4'^2)(\Delta x_4'^2 + \Delta y_4'^2 + \Delta z_4'^2)^{1/2}} (\Delta x_4' \hat{i} + \Delta y_4' \hat{j} + \Delta z_4' \hat{k}) \right) \right)
 \end{aligned}$$

Medan Listrik yg dihasilkan Distribusi Muatan

	Cartesian (x, y, z)	Cylindrical (ρ, φ, z)	Spherical (r, θ, φ)
dl	dx, dy, dz	dρ, ρdφ, dz	dr, r dθ, r sin θ dφ
dA	dx dy, dy dz, dz dx	dρ dz, ρ dφ dz, ρ dφ dρ	r dr dθ, r sin θ dr dφ, r ² sin θ dθ dφ
dV	dx dy dz	ρ dρ dφ dz	r ² sin θ dr dθ dφ

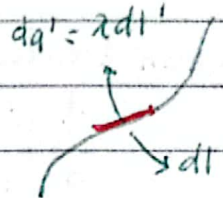
a. Rapat Muatan

↳ Kuantitas muatan yg didistribusikan pada bidang tertentu (space tertentu)

a) Rapat muatan panjang (ds') atau (dl')

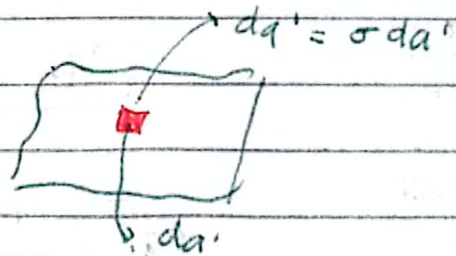
$$\lambda = \frac{dq'}{dl'}$$

$$\lambda = \frac{dq'}{ds'}$$



b) Rapat muatan luas (da')

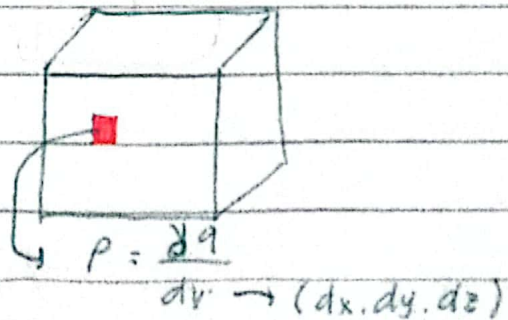
$$\sigma = \frac{dq'}{da'}$$



c) Rapat Muatan Volume (dτ') atau (dv')

$$\rho = \frac{dq'}{dv'}$$

$$\rho = \frac{dq'}{dv'}$$



Medan Listrik pada Muatan Cincin

$$\vec{E}_{dqx} = \vec{E}_{px}$$

$$\vec{E}_{py} = \vec{E}_{dqy}$$

$$\sin \theta = \frac{r}{(r^2 + x^2)^{1/2}}$$

$$\cos \theta = \frac{x}{(r^2 + x^2)^{1/2}}$$

$$b = \sqrt{r^2 + x^2}$$

jarak $\leftarrow b$

dq

Jika cincin diambil sampel

$$dl = 2\pi r$$

$$ds' = 2\pi r$$

muatannya di sekitar cincin

Kerapatan muatan cincin (muatan per satuan panjang)

$$\lambda = \frac{q}{l} \rightarrow \lambda = \frac{q}{k}$$

$$\lambda = \frac{dq}{2\pi r} \rightarrow \text{kel. lingkaran}$$

$$\lambda = \frac{q}{l}$$

$$\lambda = \frac{dq}{dl}$$

$$\lambda = \frac{q}{2\pi r}$$

$$dq = \lambda \cdot dl$$

$$q = \lambda 2\pi r$$

$$E_{dq} = \frac{1}{4\pi\epsilon_0} \frac{dq}{b^2}$$

$$E_{dq} = \frac{1}{4\pi\epsilon_0} \frac{\lambda dl}{r^2 + x^2}$$

$$\vec{E}_{ay} = E_{aq} \cdot \sin \theta = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda \, dl}{(r^2 + x^2)} \sin \theta$$

$$= \frac{1}{4\pi\epsilon_0} \int \frac{\lambda \cdot dl}{(r^2 + x^2)} \frac{r}{(r^2 + x^2)^{1/2}}$$

$$\vec{E}_{ax} = E_{aq} \cos \theta = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda \, dl}{(r^2 + x^2)} \cos \theta$$

$$\vec{E}_{px} = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda \, dl}{(r^2 + x^2)} \frac{x}{(r^2 + x^2)^{1/2}}$$

$$\vec{E}_p = \frac{1}{4\pi\epsilon_0} \frac{\lambda x}{(r^2 + x^2)^{3/2}} \int dl$$

$$\vec{E}_p = \frac{1}{4\pi\epsilon_0} \frac{\lambda x}{(r^2 + x^2)^{3/2}} \int_0^r 2\pi \, dr$$

$$\vec{E}_p = \frac{1}{4\pi\epsilon_0} \frac{\lambda x}{(r^2 + x^2)^{3/2}} 2\pi \int_0^r dr$$

$$\vec{E}_p = \frac{1}{4\pi\epsilon_0} \frac{\lambda x}{(r^2 + x^2)^{3/2}} 2\pi (r)_0^r$$

$$\vec{E}_p = \frac{1}{4\pi\epsilon_0} \frac{(\lambda 2\pi r) x}{(r^2 + x^2)^{3/2}}$$

$$\boxed{\vec{E}_p = \frac{1}{4\pi\epsilon_0} \frac{qx}{(r^2 + x^2)^{3/2}}}$$

$$\vec{E}_p = \frac{1}{4\pi\epsilon_0} \frac{qx}{(r^2+x^2)^{3/2}}$$

$$\vec{E}_p = \frac{1}{4\pi\epsilon_0} \frac{qx}{((r^2+x^2)^{1/2})^3}$$

$$\vec{E}_p = \frac{1}{4\pi\epsilon_0} \frac{qx}{b^3}$$

$$\vec{E}_p = \frac{1}{4\pi\epsilon_0} q \frac{1}{b^2} \frac{x}{b}$$

$$\vec{E}_p = \frac{1}{4\pi\epsilon_0} q \frac{1}{b^2} \frac{x}{b} \frac{r^2}{r^2}$$

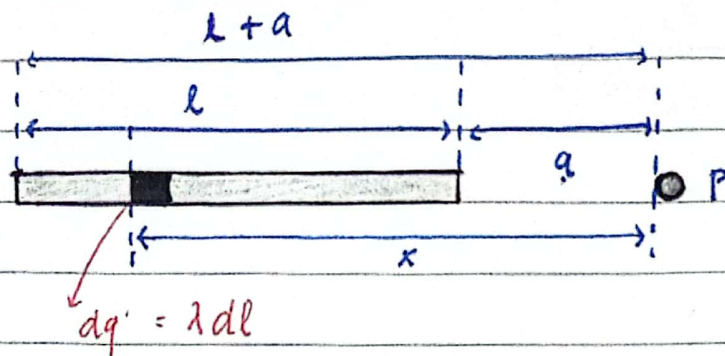
$$\vec{E}_p = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \frac{r^2}{b^2} \frac{x}{b}$$

$$\rightarrow \sin \theta = \frac{r}{b}, \quad \cos \theta = \frac{x}{b}$$

$$\boxed{\vec{E}_p = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \sin^2 \theta \cos \theta}$$

$$\sin^2 \theta = \frac{r^2}{b^2}$$

Medan Listrik oleh Muatan Batang



$$\vec{E}_P = k \int \frac{dq}{r^2}$$

$$\vec{E}_P = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda dl}{x^2}$$

$$\vec{E}_P = \frac{1}{4\pi\epsilon_0} \lambda \int_a^{l+a} \frac{dx}{x^2}$$

$$\vec{E}_P = \frac{1}{4\pi\epsilon_0} \lambda \int_a^{l+a} \left(\frac{1}{x^2} \right) dx$$

$$\vec{E}_P = \frac{1}{4\pi\epsilon_0} \lambda \left[-\frac{1}{x} \right]_a^{l+a}$$

$$\vec{E}_P = \frac{1}{4\pi\epsilon_0} \lambda \left[\left\{ -\frac{1}{l+a} \right\} - \left\{ -\frac{1}{a} \right\} \right]$$

$$\vec{E}_P = \frac{1}{4\pi\epsilon_0} \lambda \left[\left\{ \frac{1}{a} \right\} - \left\{ \frac{1}{l+a} \right\} \right]$$

$$\vec{E}_P = \frac{1}{4\pi\epsilon_0} \lambda \left\{ \frac{(l+a)-a}{a(l+a)} \right\}$$

$$\vec{E}_P = \frac{1}{4\pi\epsilon_0} \lambda \left\{ \frac{l}{a(l+a)} \right\}$$

$$\vec{E}_P = \frac{1}{4\pi\epsilon_0} \frac{\lambda l}{(a(l+a))}$$

$$\vec{E}_P = \frac{1}{4\pi\epsilon_0} \frac{q}{(a(l+a))}$$