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5.1 Persamaan Laplace

$\oint \vec{E} \cdot d\vec{E} = 0$ untuk $\vec{E}$ konservatif

$$\oint \vec{E} \cdot d\vec{r} = \int \nabla \times \vec{E} \cdot d\vec{a} = 0$$

$$\nabla \times \vec{E} = 0$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{E} = -\nabla V$$

$$\vec{E} = -\nabla V$$

$$\frac{1}{\epsilon_0} \rho = -\nabla^2 V$$

$\nabla^2 V = 0 \rightarrow$ Pers Laplace

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 ;$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{\epsilon_0} \rho \rightarrow \text{Hk. Poisson}$$

Pembahasan ini dibatasi pada pers Laplace satu dimensi, untuk menghitung potensial listrik, dari rumusan Poisson.

$$\nabla^2 V = \frac{1}{\epsilon_0} \rho$$

Dalam pers Laplace $\rho = 0$, $\epsilon_0 = \text{konstan}$

$\nabla^2 V = 0$; untuk 1 dimensi, maka

$$\frac{d^2 V}{dx^2} = k, k = \text{konstan}$$

$$dV = k dx$$

$$\int V dx = kx + b, b = \text{bil. konstan}$$

Untuk koordinat bola, pers Laplace 1 dimensi

$$\nabla^2 V = 0$$

$$\frac{1}{r^2} \cdot \frac{2}{2r} \left(r^2 \frac{\partial V}{\partial r} \right) = 0$$

$$r^2 \frac{\partial V}{\partial r} = r^2 \frac{dV}{dr} = k$$

$$dV = \frac{1}{r^2} k dr$$

$$V(r) = \frac{1}{r} k + c$$

$$V_A = 10V$$

$$V_B = 20V$$

$$x_A = 0$$

$$x_B = 10$$

$$x_P = 5$$

