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BAB III

Metoda Penentuan Potensial

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3.1 Persamaan Laplace

$\oint \vec{E} \cdot d\vec{\ell} = 0$. untuk $\vec{E}$ konservatif

$\oint \vec{E} \cdot d\vec{p} = \int \nabla \times \vec{E} \cdot d\vec{A} = 0$

$\nabla \times \vec{E} = 0$

$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$

$E = -\nabla V$

$\nabla \vec{E} = -\nabla \cdot \nabla V$

$\frac{1}{\varepsilon_0} \rho = -\nabla^2 V$

$\boxed{\nabla^2 V = \frac{1}{\varepsilon_0} \rho}$ Hukum Poisson

$\nabla^2 V = 0 \rightarrow$ persamaan Laplace.

$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$

$\frac{\partial^2 V}{\partial x^2} = 0$; $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$;

$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$

Pembahasan kali ini di batasi pada persamaan Laplace satu dimensi, untuk menghitung potensial listrik. Dari rumus Poisson.

$\nabla^2 V = \frac{1}{\varepsilon_0} \rho$

Dalam pers. Laplace $\rho = 0$, ε_0 = konstan

$\nabla^2 V = 0$; untuk satu dimensi maka

$\frac{d^2 V_x}{dx^2} = \frac{d}{dx}\left(\frac{dv}{dx}\right) = 0$

$\frac{dV_x}{dx} = k$, k = konstan

$dV_x = k\, dx$

$\begin{cases} V_x = kx + b \\ V(x) = kx + b \end{cases}$, b = bilangan konstanta.

Untuk koordinat bola, maka pers. Laplace satu dimensi :

$\nabla^2 V = 0$

$\frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$

$r^2 \frac{\partial V}{\partial r} = r^2 \frac{dv}{dr} = k$

$dv = \frac{1}{r^2} k\, dr$

$V_\varphi = -\frac{1}{r} k + c$

$V_a = 10\ V$
$V_b = 20\ V$
$x_a = 0$
$x_b = 10$
$x_p = 5$

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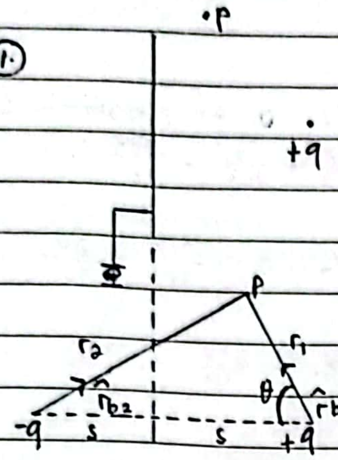
3.2 Metodo Bayangan dengan

a.) muatan titik oleh batang konduktor

(1.)



$V_p = ?\ldots;\ E_r = -\frac{\partial v}{\partial r};\ E_\theta = -\frac{1}{r}\frac{\partial V}{\partial r}$

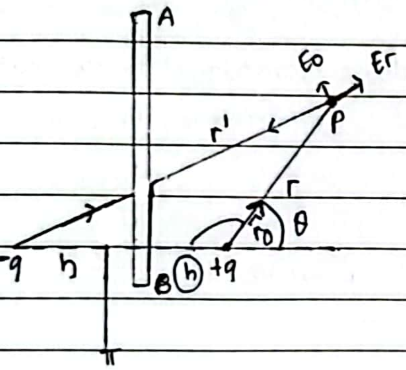
$V_p = \frac{1}{4\pi\varepsilon_0}\frac{+q}{r_1} + \frac{-q}{4\pi\varepsilon_0 r_2}$

$\vec{E}_\rho = -\frac{\partial V}{\partial \rho}$

$= \frac{q}{4\pi\varepsilon_0}\left(\frac{r_2 - r_1}{r_1 - r_2}\right)$

$E_\theta = -\frac{1}{\rho}\frac{\partial V}{\partial \theta}$

(2) Penjumlahan biasa



$V_p = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r'}\right)$

$V_p = \frac{q}{4\pi\varepsilon_0}\left(\frac{r' - r}{r \cdot r'}\right)$

$V_p = V(r, \theta)$.

Perhatikan gambar

$V(r,\theta) = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{(r^2 + 4h^2 + 4hr\cos\theta)^{\frac{1}{2}}}\right]$

$E_r = \frac{\partial V}{\partial r}$:

$\hookrightarrow \frac{\partial}{\partial r}\left(\frac{1}{r} - \frac{1}{(r^2 + 4h^2 + 4hr\cos\theta)^{\frac{1}{2}}}\right)$

$= -\frac{1}{r^2} + \frac{\frac{1}{2}(r^2 + h^2 + 4hr\cos\theta)^{-\frac{1}{2}}(2r + 4h\cos\theta)}{(r^2 + 4h^2 + 4hr\cos\theta)}$

$= -\frac{1}{r^2} + \frac{(r + 2h\cos\theta)}{(r^2 + 4h^2 + 4hr\cos\theta)^{3/2}}$

$\therefore E_r = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r^2} + \frac{(r + 2h\cos\theta)}{(r^2 + 4h^2 + 4hr\cos\theta)^{3/2}}\right]$

$\therefore E_\theta = \frac{q}{4\pi\varepsilon_0 r}\left[\frac{2h\sin\theta}{(r^2 + 4h^2 + 4hr\cos\theta)^{3/2}}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{2h\sin\theta}{4r^2}\right]$

$\vec{E}_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta}$

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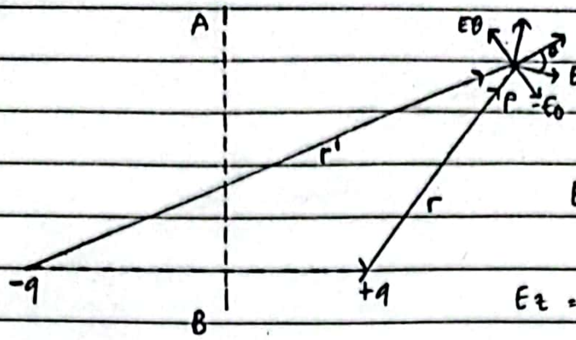

$$E_z = E_r \cos\theta - E_\theta \sin\theta$$

$$E_z = \frac{q}{4\pi\varepsilon_0}\left[\frac{\cos\theta}{r^2} - \frac{r\cos\theta + 2h}{r'^3}\right]$$

Untuk setiap titik pada bidang AB, $r = r' = a$, sehingga

$$E_z(a) = \frac{q}{4\pi\varepsilon_0}\left(\frac{\cos\theta}{a^2} - \frac{a\cos\theta + 2h}{a^3}\right)$$

$$= \frac{-q}{4\pi\varepsilon_0}\left(\frac{2h}{a^3}\right) = \frac{-qh}{2\pi\varepsilon_0 a^3} = \frac{1}{\varepsilon_0}\sigma(a)$$

$\sigma(a) = \frac{-qh}{2\pi a^3}$ $\sigma(a)$ = muatan induksi persatuan luas pada pelat konduktor AB.

muatan induksi total pada pelat AB :

$$\int_0^\infty \sigma\, dA = \int_0^\infty \sigma(2\pi x\, dx) = -hq\int_0^\infty \frac{x}{a^3}\, dx$$

$$= -hq\int_0^\infty \frac{a}{a^3}\, da$$

$= -q$ → ini bukti sebagai muatan bayangan.