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Pendidikan fisika 20 B

METODA PENENTUAN POTENSIAL

3.1 Persamaan Laplace

$\oint \vec{E} \cdot d\vec{e} = 0$, untuk $\vec{E}$ konservatif

$\oint \vec{E} \cdot d\vec{p} = \int \nabla \cdot \vec{E} \cdot dA = 0$

$\nabla \cdot \vec{E} = 0$

$\nabla E = \frac{\rho}{\varepsilon}$

$E = -\nabla V$

$\nabla \vec{E} = -\nabla \cdot \nabla V$

$\frac{1}{\varepsilon_0} \rho = -\nabla^2 V$

$\nabla^2 V = -\frac{1}{\varepsilon_0} \rho$ (Hukum Poisson)

$\nabla^2 V = 0$ (Persamaan Laplace)

$$\frac{d^2 V}{dx^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{\partial^2 V}{\partial x^2} = 0 ; \quad \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 ; \quad \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\nabla^2 V = \frac{1}{\varepsilon_0} \rho$$

Dalam persamaan Laplace $\rho = 0$, ε_0 = konstan

$\nabla^2 V = 0$; untuk satu dimensi, maka

$\frac{\partial^2 V_x}{\partial x^2} = 0$, karena V fungsi dari x saja, maka

$\frac{d^2 V_x}{dx^2} = \frac{d}{dx} \left(\frac{dV}{dx} \right) = 0$

$\frac{dV_x}{dx} = k$, k = konstan

$dV_x = k \, dx$

$V_x = kx + b$, b = konstan

$V_{(x)} = kx + b$

Untuk koordinat bola, maka persamaan Laplace satu dimensi

$\nabla^2 V = 0$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

$$r^2 \frac{\partial V}{\partial r} = r^2 \frac{dV}{dr} = k$$

$$dV = \frac{1}{r^2} k \, dr$$

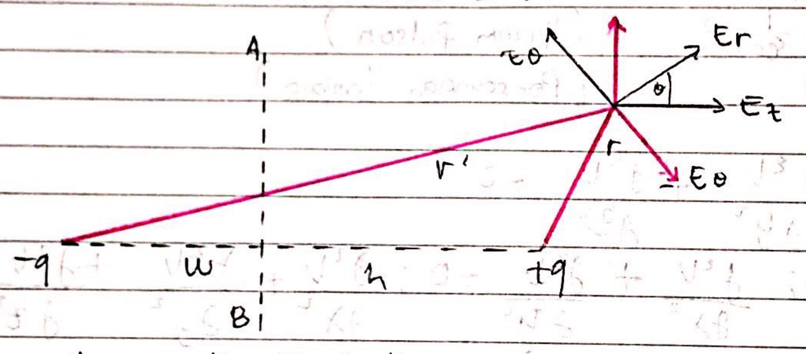
$$V(p) = -\frac{1}{r} k + C$$

3.2 Metoda bayangan

a) Muatan titik dengan batang konduktor

$$\therefore E_p = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r^2} - \frac{(r + 2h\cos\theta)}{(r^2 + 4h^2 + 4hr\cos\theta)^{\frac{3}{2}}}\right]$$

$$\therefore E_\theta = \frac{q}{4\pi\varepsilon_0}\left[\frac{2h\sin\theta}{(r^2 + 4h^2 + 4hr\cos\theta)^{\frac{3}{2}}}\right]$$



$$E_z = E_r \cos\theta - E_\theta \sin\theta$$

$$E_z = \frac{q}{4\pi\varepsilon_0}\left[\frac{\cos\theta}{r^2} - \frac{r\cos\theta + 2h}{r^3}\right]$$

Untuk setiap titik pada bidang AB, $r = r' = a$, sehingga

$$E_z(a) = \frac{q}{4\pi\varepsilon_0}\left(\frac{\cos\theta}{a^2} - \frac{a\cos\theta + 2h}{a^3}\right)$$

$$= -\frac{q}{4\pi\varepsilon_0}\left(\frac{2h}{a^3}\right) = \frac{-qh}{2\pi\varepsilon_0 a^3} = \frac{1}{\varepsilon_0}\sigma(a)$$

$$\sigma(a) = -\frac{qh}{2\pi a^3}$$ → σ = muatan induksi persatuan luas pada pelat konduktor AB

Muatan induksi total pada pelat AB :

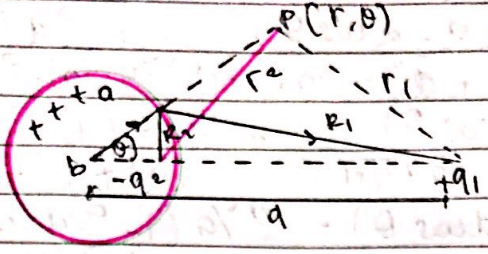
$$\int_0^\infty \sigma \, dA = \int_0^\infty \sigma \, (2\pi x \, dx) = -hq\int_0^\infty \frac{x}{a^3}\,dx$$

$$= -hq\int_0^\infty \frac{a}{a^3}\,da$$

$= -q$ → bukti sebagai muatan bayangan.

b) Muatan titik dengan bola konduktor

kita menempatkan muatan $-q$ sedemikian rupa sehingga V dipermukaan bola $V = 0$



$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1} - \frac{q_2}{R_2}\right) = 0$$

$R_1^2 = a^2 + d^2 - 2ad\cos\theta$

$R_2^2 = a^2 + b^2 - 2ab\cos\theta$

untuk $V = 0$ maka,

$$\frac{q_1}{R_1} = \frac{q_2}{R_2} \rightarrow \frac{q_1^2}{q_2^2} = \frac{R_1^2}{R_2^2} = \frac{a^2 + d^2 - 2ad\cos\theta}{a^2 + b^2 - 2ab\cos\theta}$$

$$\left(\frac{q_1}{q_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^2 = \frac{d}{b}\left(\frac{a^2/d + d - 2a\cos\theta}{a^2/b + b - 2a\cos\theta}\right)$$

dengan hubungan diatas berlaku untuk setiap θ, bila

1) $a^2 = ab$

2) $\left(\frac{q_1}{q_2}\right) = \frac{d}{b}$, karena $d > b$ dan $q_1 > q_2$

bola konduktor dengan jari-jari a dengan $V = 0$, dengan muatan q, sejarak x dari pusat bola, maka bola tersebut tepat digantungkan dengan muatan titik $(-q)$

$$\frac{q_1^2}{q_2^2} = \frac{d}{b} \quad ; -q_2 = \pm\sqrt{\frac{b^2}{d}} \cdot q_1 \quad ; \text{untuk } b = \frac{a^2}{d}$$

$$q_2 = -\frac{a}{d} q_1$$

Jadi $V(r, \theta) = V_p$

$$V_p = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} - \frac{q_2}{r_2}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} - \frac{a/d}{r_2}\right)$$

$r_1 = (r^2 + d^2 - 2rd\cos\theta)^{\frac{1}{2}}$

$r_2 = (r^2 + b^2 - 2rb\cos\theta)^{\frac{1}{2}}$

$$V_p = \frac{q_1}{4\pi\varepsilon_0}\left(\frac{1}{(r^2 + a^2 - 2rd(\cos\theta))^{\frac{1}{2}}} - \frac{a/d}{\left[r^2 + \frac{a^4}{d^2} - 2r\left(\frac{a^2}{d}\right)\cos\theta\right]^{\frac{1}{2}}}\right)$$

$$E_r = -\frac{\partial V}{\partial r} \quad ; \quad E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta}$$

khusus untuk, permukaan bola : dimana $r = a$

$$E_r = -\frac{q_1}{4\pi\varepsilon_0}\left[\frac{-(a - d\cos\theta)}{(a^2+d^2-2ad\cos\theta)^{3/2}} + \frac{a/d\,(a - \frac{a^2}{d}\cos\theta)}{(a^2+\frac{a^3}{d^2}-2a(\frac{a^2}{d})\cos\theta)} + \frac{a^2/d\,(1 - a/d\cos\theta)}{a^3/d^3\,(d^2+a^2-2ad\cos\theta)^{3/2}} + \frac{d^2/a\,(1 - a/d\cos\theta)}{(a^2+d^2-2ad\cos\theta)^{3/2}}\right]$$

$$E_r = \frac{-q_1}{4\pi\varepsilon_0}\left[\frac{-(a-d\cos\theta) + d^2/a\,(1 - a/d\cos\theta)}{(a^2+d^2-2ad\cos\theta)^{3/2}}\right]$$

$$E_r = \frac{-q_1}{4\pi\varepsilon_0 a}\left(\frac{d^2-a^3}{(a^3+d^2-2ad\cos\theta)^{3/2}}\right)$$

Dengan cara yang sama

$E_\theta = 0$

Disini terjadi pergeseran listrik D

$D = \varepsilon_0 E \quad -D = \sigma_f = \sigma$

$\sigma = \varepsilon_0 E_r$

$$\sigma = \frac{-q_1}{4\pi a}\left(\frac{d^2-a}{(a^2+a^2-2ad\cos\theta)^{3/2}}\right)$$

σ = muatan induksi persatuan luas pada bola konduktor

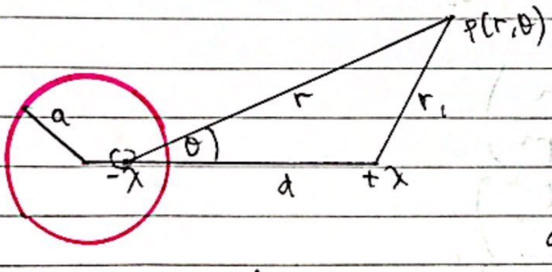
F = (gaya antara bola dengan q_1) adalah

$$F = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{(d-b)^2}$$

$$F = -\frac{1}{4\pi\varepsilon_0}\frac{a q_1^2}{a(d-b)^2}$$

c) Muatan garis dengan silinder bermuatan :

Ada 2 muatan garis saling sejajar terpisah jarak d



Dimana $V = 0$

V permukaan $= V = 0$

$$V_p = \frac{+\lambda}{4\pi\varepsilon_0}\ln r' - \frac{\lambda}{2\pi\varepsilon_0}\ln r + C$$

agar $V=0$ maka :

$$C = -\frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{r_1}{r}\right)^2,$$ V, semua permukaan, dimana :

$$\left(\frac{r_1}{r}\right)^2 = \frac{r^2+d^2-2rd\cos\theta}{r^2} = m^2 = \text{konstanta dengan } 0 < m < \infty$$

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$r^2 + d^2 - 2rd\cos\theta = m^2 r^2$ untuk $x = r\cos\theta$, $y = r\sin\theta$ maka:

$r^2 + d^2 - 2rd\cos\theta = m^2 r^2$

diperoleh :

$$\left(x + \frac{a}{m^2-1}\right)^2 + y^2 = \frac{m^2 d^2}{(m^2-1)^2} \rightarrow \left(\frac{-d}{m^2-1} ; 0\right)$$

Jari-jari silinder $a = \frac{md}{m^2-1}$, $x = \frac{-d}{mr-1}$, $y = 0$

Untuk $m > 1$ letak sumbu silinder dengan harga $V = 0$ adalah

pada jarak $\frac{a}{m^2-1} = m \cdot a$

dan jarak : $x = \frac{d}{m^2-1} = \frac{a}{m} = \frac{a^2}{r}$

$$x = \frac{\sigma}{p}$$