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Metoda Penentu Potensial

3.1. Persamaan laplace

$\oint \vec{E} \cdot d\vec{\ell} = 0$ Untuk $\vec{E}$ konservatif

$\oint \vec{E} \cdot d\vec{P} = \int \nabla \times \vec{E} \cdot d\vec{A} = 0$

$\nabla \times \vec{E} = 0$

$\nabla \, E > \frac{\rho}{\varepsilon_0}$

$E = -\nabla V$

$\nabla \vec{E} = -\nabla \cdot \nabla V$

$\frac{1}{\varepsilon_0} \rho = -\nabla^2 V$

$\nabla^2 V = \frac{-1}{\varepsilon_0} \rho \longrightarrow$ Hukum Poisson

$\nabla^2 V = 0 \longrightarrow$ Pers. laplace

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{\partial^2 V}{\partial x^2} = 0 \; ; \; \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \; ; \; \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Menghitung Potensial Listrik dari rumus Poisson

$\nabla^2 V = \frac{1}{\varepsilon_0} \rho$, ε_0 = konstan

$\nabla^2 V = 0 \rightarrow$ untuk suatu delebusi maka $\frac{\partial^2 V_x}{\partial x^2} = 0$ maka :

$$\frac{d^2 V_x}{dx^2} = \frac{d}{dx}\left(\frac{dv}{dx}\right) = 0$$

$$\frac{dV_x}{dx} = K$$

$dV_x = K\,dx$

$V_x = Kx + b$

$V(x) = Kx + b$

Untuk koordinat bola :

$\nabla^2 V = 0$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dv}{dr}\right) = 0$$

$$r^2 \frac{dv}{dr} = r^2 \frac{\partial v}{\partial r} = K$$

$dv = \frac{1}{r^2} K\,dr$

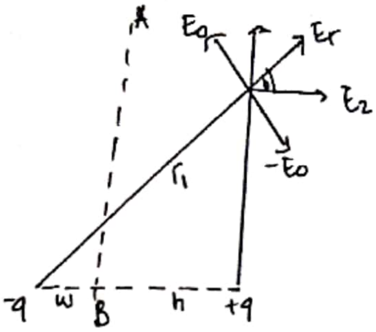
$V(r) = -\frac{1}{r} K + E$

3.2. Metoda bayangan

a). Muatan Listrik oleh batang konduktor

$$E_p = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r^2} - \frac{(r + 2h\cos\theta)}{(r^2 + 4h^2 + 4hr\cos\theta)^{3/2}}\right]$$

$$E_0 = \frac{q}{4\pi\varepsilon_0}\left[\frac{2h\sin\theta}{(r^2 + 4h^2 + 4hr\cos\theta)^{3/2}}\right]$$



$E_z = E_r \cos\theta - E_0 \sin\theta$

$$E_z = \frac{q}{4\pi\varepsilon_0}\left[\frac{\cos\theta}{r^2} - \frac{r\cos\theta + 2h}{r'^3}\right]$$

Setiap titik pada bidang AB, $r = r' = a$

$$E_z(a) = \frac{q}{4\pi\varepsilon_0}\left(\frac{\cos\theta}{a^2} - \frac{a\cos\theta + 2h}{a^3}\right)$$

$$= \frac{-q}{4\pi\varepsilon_0}\left(\frac{2h}{a^3}\right) = \frac{-qh}{2\pi\varepsilon_0 a^3} = \frac{1}{\varepsilon_0}(a)$$

$$\sigma(a) = \frac{-qh}{2\pi a^3}$$

σ = Muatan konduksi persatuan luas pada pelat konduktor AB

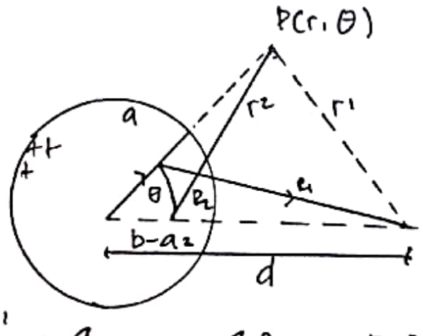
- Muatan induksi total pada pelat AB :

$$\int_0^\infty \sigma\, dA = \int_0 \sigma(2\pi x\, dx) = -hq\int_0^\infty \frac{x}{a^3}dx$$

$$= -hq\int_0^\infty \frac{a}{a^3}\, da$$

$$= -q \rightarrow \text{bukti sebagai muatan bayangan}$$

b). Muatan titik dengan bola konduktor



$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1} - \frac{q_2}{r_2}\right) = 0$$

$R_1^2 = a^2 + d^2 - 2ad\cos\theta$

$R_2^2 = a^2 + b^2 - 2ab\cos\theta$

Untuk $V = 0$

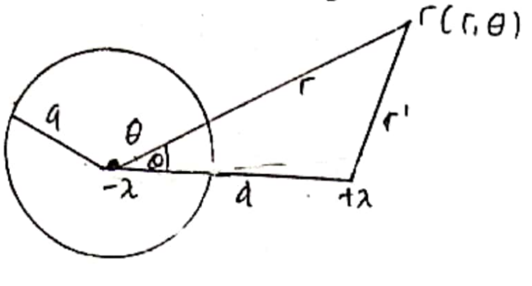
$$\frac{q'}{R_1} \neq \frac{q_2}{R_2} \rightarrow \frac{q_1^2}{q_2^2} = \frac{R_1^2}{R_2^2} = \frac{a^2 + d^2 - 2ad\cos\theta}{a^2 + b^2 - 2ab\cos\theta}$$

$$\left(\frac{q_1}{q_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^2 = \frac{d}{b}\left(\frac{\frac{a^2}{d} + d - 2a\cos\theta}{\frac{a^2}{b} + b - 2a\cos\theta}\right)$$

Hubungan diatas berlaku setiap θ bila

- $a^2 = db$
- $\left(\frac{q_1}{q_2}\right) = \frac{d}{b}$, karena $d > b$ dan $q_1 > q_2$

c). Muatan garis dengan silinder bermuatan



dimana $V=0$

V Permukaan $=V=0$

$V_p = \frac{+\lambda}{4\pi\varepsilon_0} \ln r' - \frac{\lambda}{4\pi\varepsilon_0} \ln r + c$

agar $V=0$ maka :

$c = \frac{-\lambda}{4\pi\varepsilon_0} \ln \left(\frac{r_1}{r}\right)^2 V$, Semua permukaan dimana .

$\left(\frac{r'}{r}\right)^2 = \frac{r^2+d^2-2rd\cos\theta}{r^2} = m^2 =$ konstan dengan $0 < m < \infty$

$r^2+d^2-2rd\cos\theta = m^2r^2$

Maka : untuk $x = r\cos\theta$, $y = r\sin\theta$

$r^2+d^2-2rd\cos\theta = m^2r^2$

diperoleh

$\left(x+\frac{a}{m^2-1}\right)^2 + y^2 = \frac{m^2d^2}{(m^2-1)^2} + \left(\frac{-d}{m^2-1}, 0\right)$

Jari² silinder $a = \frac{md}{m^2-1}$, $x = \frac{-d}{m^2-1}$, $y=0$

Jarak sumbu silinder hingga $+\lambda$

$P = d + \frac{a}{m^2-1} = \frac{m^2d}{m^2-1} = m.a$ dan jarak

$x = \frac{a}{m^2-1} = \frac{a}{m} = \frac{a^2}{r}$ $r\,x = \frac{d}{P}$

3.3. Expansi Multipol

1). Monopol (kutub tunggal)

$V_P = -\int_\infty^P \vec{E}\,dr \;\dots (1)$

$V_R = -\int_\infty^R \vec{E}\,dr \;\dots (2)$

$V_P - V_R = -\int_0^\infty \vec{E}\,dr + \int_\infty^R \vec{E}\,dr$

$V_{RP} = -\int_s^r \vec{E}\,dr = -\frac{1}{4\pi\varepsilon_0} \int_R^P \frac{\rho' d_t}{\pi}$

Jika muatan itu berupa titik

$V_P = -\int_\infty^{P=r} \vec{E}\,dr$

$V_P = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$

Jadi, $U_m(P) \sim \frac{1}{r}$; $V(m)\,P =$ Potensial monopol di P

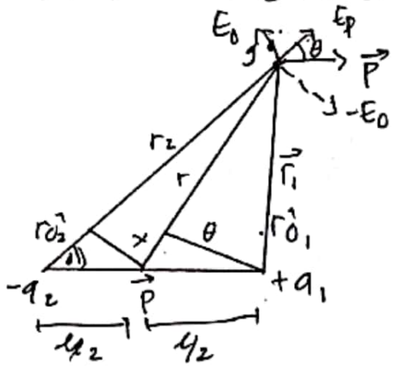
2). dipole (dua kutub) Listrik

dipole adalah Pasangan muatan listrik yang mempunyai muatan berlawanan $(\Sigma q_i = 0)$

adi dipol, maka pada dipol tersebut timbul momen dipol, besarnya $(\vec{P})$ adalah muatan kali

$\vec{P} = q \cdot \vec{e} \longrightarrow$ artinya dari $\ominus$ ke $\oplus$

Besarnya potensial oleh dipol listrik



$VP = 0$ dimana $q_1 = q_2 = q$

$$V(r,\theta) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$r_1 = r - \frac{l}{2}\cos\theta$, $r_2 = r + \frac{l}{2}\cos\theta$

$$V(r,\theta) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r - \frac{l}{2}\cos\theta} - \frac{1}{r + \frac{l}{2}\cos\theta}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{l\cos}{r^2 - \frac{l^2}{4}\cos^2\theta}\right)$$

untuk $r >> l$ maka

$$V(r,\theta) = \frac{q\, l\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$V(r,\theta) = \vec{P}\frac{\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\vec{P}\cdot\vec{r_0}}{4\pi\varepsilon_0 r^2}$$

$$E(r,\theta) = -\nabla V$$

$$E(r,\theta) = \frac{\vec{P}}{4\pi\varepsilon_0 r^2}(2\cos\theta \cdot \hat{r_0} + \sin a\hat{\theta_0})$$

$$E(r,0) = \frac{1}{4\pi\varepsilon_0 r^2}(2\vec{p}\cos\theta)\hat{r_0} + (p\sin\theta)\vec{\theta_0})$$

Jadi,

$$E(r_0) = \frac{1}{4\pi\varepsilon_0 r^2}\left[3(\vec{P}\cdot\vec{r_0})\hat{r_0} - \vec{P}\right]$$