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Kelas B

BAB III
Metode Penentuan Potensial

3.1 Persamaan Laplace

$\oint \vec{E} \cdot d\vec{\ell} = 0$ untuk $\vec{E}$ konservatif

$$\oint \vec{E} \cdot d\vec{P} = \int \nabla \times \vec{E} \cdot d\vec{A} = 0$$

$$\nabla \times E = 0$$

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

$$E = -\nabla V$$

$$\nabla \cdot \vec{E} = -\nabla \cdot \nabla V$$

$$\frac{1}{\varepsilon_0} \rho = -\nabla^2 V$$

$$\boxed{\nabla^2 V = -\frac{1}{\varepsilon_0} \rho} \longrightarrow \text{Hukum Poisson}$$

$\nabla^2 V = 0 \Rightarrow$ Persamaan Laplace

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{\partial^2 V}{\partial x^2} = 0 \; ; \; \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Untuk menghitung potensial listrik dari rumus Poisson :

$$\nabla^2 V = \frac{1}{\varepsilon_0} \rho$$

Dalam pers. Laplace $\rho = 0$, ε_0 = konstan

$\nabla^2 V = 0$; untuk satu dimensi maka

$\frac{\partial^2 V_x}{\partial x^2} = 0 \Rightarrow$ karena V fungsi dari x saja, maka

$$\frac{d^2 V_x}{dx^2} = \frac{d}{dx}\left(\frac{dv}{dx}\right) = 0$$

$\frac{dV_x}{dx} = k$, k = konstan

$$dV_x = k\,dx$$

$V_x = kx + b$; b = bilangan konstanta

Untuk koordinat bola, maka pers. Laplace satu dimensi :

$$\nabla^2 V = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

$$r^2 \frac{\partial V}{\partial r} = r^2 \frac{dV}{dr} = k$$

$$dV = \frac{1}{r^2} k \, dr$$

$$V(r) = -\frac{1}{r^2} k + C$$

$V_a = 10\ V$

$V_b = 20 V$

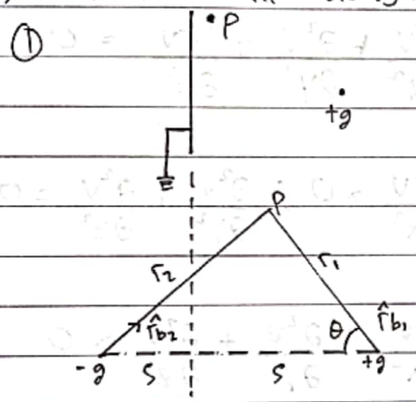
$X_a = 0$

$X_b = 10$

$Y_p = 5$

3.2. Metode Bayangan

a.) Muatan titik dengan batang konduktor

① 

$V_p = \ldots ?$ $E_r = -\frac{\partial V}{\partial r}$; $E_\theta = -\frac{1}{r}\frac{\partial V}{\partial r}$

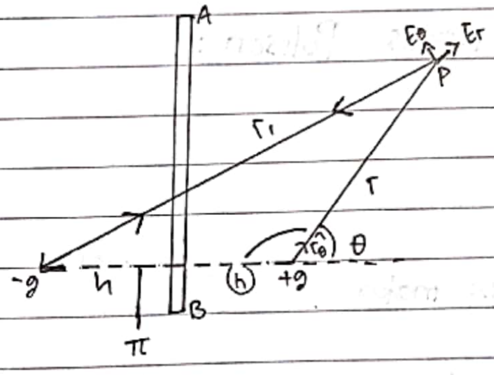
$$V_p = \frac{1}{4\pi\varepsilon_0}\frac{+q}{r_1} + \frac{-q}{4\pi\varepsilon_0 r_2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{r_2 - r_1}{r_1 - r_2}\right)$$

$$\vec{E}_0 = -\frac{\partial V}{\partial p}$$

$$\vec{E}_\theta = -\frac{1}{p}\frac{\partial V}{\partial \theta}$$

② Penjumlahan Biasa



$$V_p = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r'}\right)$$

$$V_p = \frac{q}{4\pi\varepsilon_0}\left(\frac{r' - r}{r \cdot r'}\right)$$

$$V_p = V(r, \theta)$$

Perhatikan Gambar.

$$V(r,\theta) = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{(r^2 + 4h^2 + 4hr\cos\theta)^{1/2}}\right]$$

$$E_r = -\frac{\partial V}{\partial r}$$

$$= \frac{\partial}{\partial r}\left(\frac{1}{r} - \frac{1}{(r^2 + 4h^2 + 4hr\cos\theta)^{1/2}}\right)$$

$$= -\frac{1}{r^2} + \frac{\frac{1}{2}(r^2+4h^2+4hr\cos\theta)^{1/2}(2r+4h\cos\theta)}{(r^2+4h^2+4hr\cos\theta)}$$

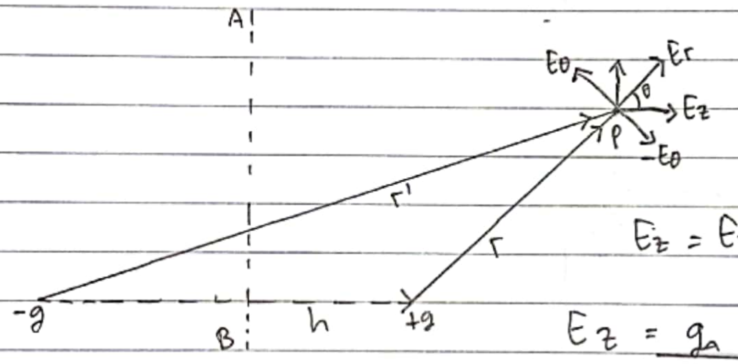
$$= -\frac{1}{r^2} + \frac{(r+2h\cos\theta)}{(r^2+4h^2+4hr\cos\theta)^{3/2}}$$

$$\therefore E_r = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r^2} + \frac{(r+2h\cos\theta)}{(r^2+4h^2+4hr\cos\theta)^{3/2}}\right]$$

$$\therefore E_\theta = \frac{q}{4\pi\varepsilon_0 r}\left[\frac{2\sin\theta}{(r^2+4h^2+4hr\cos\theta)^{3/2}}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{2h\sin\theta}{4r^2}\right]$$

$$\vec{E}_\theta = -\frac{1}{r}\frac{\partial V}{\partial\theta}$$



$$E_z = E_r\cos\theta - E_\theta\sin\theta$$

$$E_z = \frac{q}{4\pi\varepsilon_0}\left[\frac{\cos\theta}{r^2} - \frac{r\cos\theta+2h}{r'^3}\right]$$

Untuk setiap titik pada bidang AB, $r = r' = a$, sehingga,

$$E_z(a) = \frac{q}{4\pi\varepsilon_0}\left(\frac{\cos\theta}{a^2} - \frac{a\cos\theta+2h}{a^3}\right)$$

$$= -\frac{q}{4\pi\varepsilon_0}\left(\frac{2h}{a^3}\right)$$

$$= -\frac{qh}{2\pi\varepsilon_0 a^3}$$

$$= \frac{1}{\varepsilon_0}\sigma(a)$$

$$\sigma(a) = -\frac{qh}{2\pi\varepsilon_0 a^3}$$

$\sigma(a)$ = muatan induksi persatuan luas pada pelat konduktor AB

Muatan induksi total pada pelat AB

$$\int_0^\infty \sigma(a) = \int_0^\infty \sigma(2\pi x\,dx) = -hq\int_0^\infty \frac{x}{a^3}\,dx$$

$$= -hq\int_0^\infty \frac{a}{a^3}\,da$$

$$= \boxed{-q}$$ ⇒ bukti sebagai muatan bayangan