No.: Date.:

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Kelas : B

Metoda Penentuan Potensial

3.1 Persamaan Laplace

$\oint \vec{E} \cdot d\vec{l} = 0$ untuk $\vec{E}$ konservatif

$\oint \vec{E} \cdot d\vec{P} = \int \nabla \times \vec{E} \cdot d\vec{A} = 0$

$\nabla \times \vec{E} = 0$

$\nabla E = \rho / \varepsilon_0$

$E = -\nabla V$

$\nabla \vec{E} = -\nabla \cdot \nabla V$

$\frac{1}{\varepsilon_0} \rho = -\nabla^2 V$

$\boxed{\nabla^2 V = -\frac{1}{\varepsilon_0} \rho}$ ... Hukum poisson

$\nabla^2 V = 0 \rightarrow$ Pers. Laplace

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{\partial^2 V}{\partial x^2} = 0 \; ; \; \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \; ; \; \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Menghitung potensial listrik dari rumus poisson

$\nabla^2 V = \frac{1}{\varepsilon_0} \rho$, ε_0 = konstan

$\boxed{\nabla^2 V = 0}$ → untuk satu dekawsi maka

$\frac{\partial^2 Vx}{\partial x^2} = 0$ maka

$$\frac{d^2 V_x}{dx^2} = \frac{d}{dx}\left(\frac{dV}{dx}\right) = 0$$

$$\frac{dV_x}{dx} = k$$

$$dV_x = k\,dx$$

$$V_x = kx + b$$

$$V(x) = kx + b$$

Untuk koordinat bola,

$$\nabla^2 V = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

$$r^2 \frac{\partial V}{\partial r} = r^2 \frac{\partial V}{\partial r} = k$$

$$dV = \frac{1}{r^2} k\,dr$$

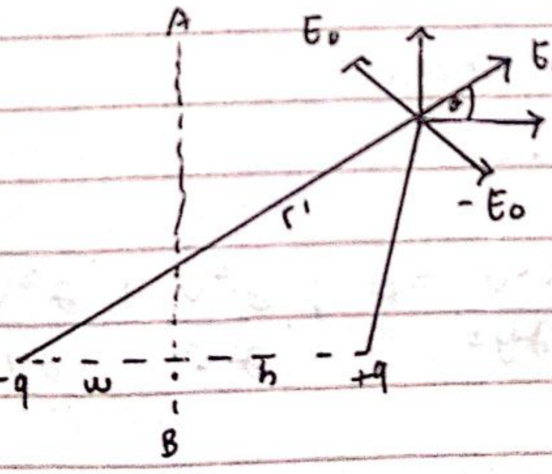
$$V_{(r)} = -\frac{1}{r} k + C$$

3.2 Metoda Bayangan

a). Muatan listrik oleh batang konduktor

$$E_p = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r^2} - \frac{(r + 2h\cos\theta)}{(r^2 + 4h^2 + 4hr\cos\theta)^{3/2}}\right]$$

$$E_\theta = \frac{q}{4\pi\varepsilon_0}\left[\frac{2h\sin\theta}{(r^2 + 4h^2 + 4hr\cos\theta)^{3/2}}\right]$$



$$E_z = E_r\cos\theta - E_\theta\sin\theta$$

$$E_z = \frac{q}{4\pi\varepsilon_0}\left[\frac{\cos\theta}{r^2} - \frac{r\cos\theta + 2h}{r'^3}\right]$$

Setiap titik pada bidang AB, $r = r' = a$

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$$E_z(a) = \frac{q}{4\pi\varepsilon_0}\left(\frac{\cos\theta}{a^2} - \frac{a\cos\theta + 2h}{a^3}\right)$$

$$= \frac{-q}{4\pi\varepsilon_0}\left(\frac{2h}{a^3}\right) = \frac{-qh}{2\pi\varepsilon_0 a^3} = \frac{1}{\varepsilon_0}\sigma(a)$$

$$\sigma(a) = \frac{-qh}{2\pi a^3}$$

σ = muatan induksi persatuan luas pada pelat konduktor AB.

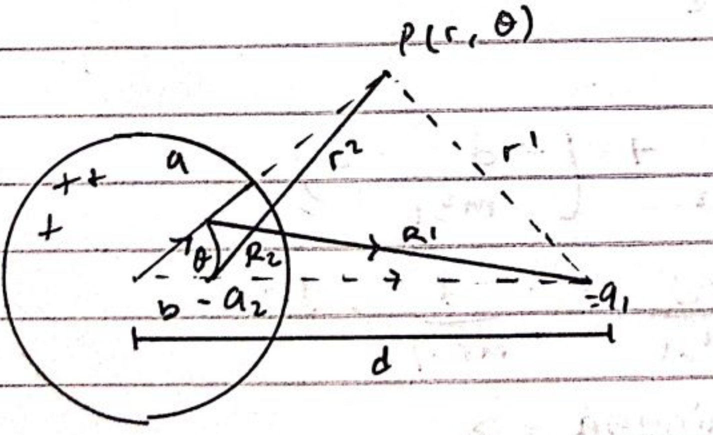
- Muatan induksi total pada pelat AB:

$$\int_0^\infty \sigma\, dA = \int_0^\infty \sigma\,(2\pi x\, dx) = -hq\int_0^\infty \frac{x}{a^3}\,dx$$

$$= -hq\int_0^\infty \frac{a}{a^3}\,da$$

$$= -q \rightarrow \text{bukti sebagai muatan bayangan}$$

b). Muatan titik dengan bola konduktor



$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1} - \frac{q_2}{R_2}\right) = 0$$

$$R_1^2 = a^2 + d^2 - 2ad\cos\theta$$

$$R_2^2 = a^2 + b^2 - 2ab\cos\theta$$

untuk $V = 0$

$$\frac{q'}{R_1} = \frac{q^2}{R_2} \rightarrow \frac{q_1^2}{q_2^2} = \frac{R_1^2}{R_2^2} = \frac{a^2+d^2-2ad\cos\theta}{a^2+b^2-2ab\cos\theta}$$

$$\left(\frac{q_1}{q_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^2 = \frac{d}{b}\left(\frac{a^2/d + d - 2a\cos\theta}{a^2/b + b - 2a\cos\theta}\right)$$

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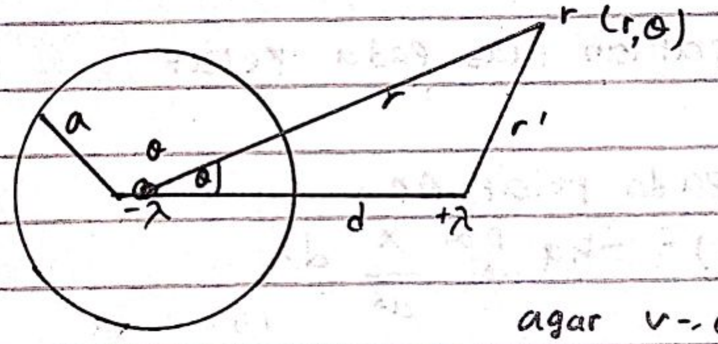
Hubungan diatas berlaku setiap θ bila

1). $a^2 = d\,b$

2). $\left(\frac{a_1}{a_2}\right) = \frac{d}{b}$, karena $d > b$ dan $a_1 > a_2$

c) Muatan garis dengan silinder bermuatan



dimana $V = 0$

V permukaan $= V = 0$

$$V_P = \frac{+\lambda}{4\pi\varepsilon_0} \ln r' - \frac{\lambda}{2\pi\varepsilon_0} \ln r + c$$

agar $V = 0$ maka

$c = -\frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{r'}{r}\right)^2$ V, semua permukaan, dimana

$$\left(\frac{r'}{r}\right)^2 = \frac{r^2 + d^2 - 2rd\cos\theta}{r^2} : m^2 = \text{konstan dg } 0 < m < \infty$$

$r^2 + d^2 - 2rd\cos\theta = m^2 r^2$ untuk $x = r\cos\theta$, $y = r\sin\theta$

maka :

$r^2 + d^2 - 2rd\cos\theta = m^2 r^2$

diperoleh

$$\left(x + \frac{a}{m^2-1}\right)^2 + y^2 = \frac{m^2 d^2}{(m^2-1)^2} \rightarrow \left(-\frac{d}{m^2-1}; 0\right)$$

dari-dari silinder $a = \frac{md}{m^2-1}$, $x = \frac{-d}{mr-1}$, $y = 0$

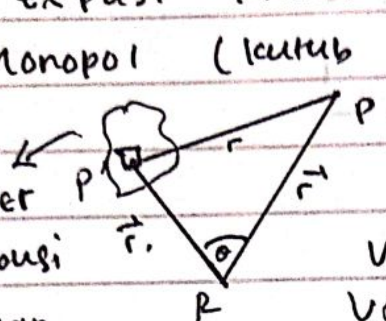
jarak sumbu silinder hingga $+\lambda$

$P = d + \frac{a}{m^2-1} = \frac{m^2 d}{m^2-1} = m.a$ dan jarak

$r = \frac{a}{m^2-1} = \frac{a}{m} = \frac{a^2}{r}$; $x = \frac{d}{p}$

3.3 Expasi Multipol

1). Monopol (kutub tunggal)



Sumber distribusi muatan

$$V_P = -\int_\infty^P \vec{E}\, dr \; . . . (1)$$

$$V_R = -\int_\infty^R \vec{E}\, dr \; . . . (2)$$

$$V_P - V_R = -\int_\infty^P E\, dr + \int_\infty^R \vec{E}\, dr$$

$$V_{RP} = -\int_R^r \vec{E}\, dr = -\frac{1}{4\pi\varepsilon_0}\int_R^P \frac{\rho'\, dt}{\pi}$$

Kalo muatan itu berupa titik

$$V_P = -\int_\infty^{P=r} \vec{E}\, dr$$

$$V_P = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Jadi $V_M(P) \sim \frac{1}{r}$; $V_{(M)}P$ = Potensial monopol di P

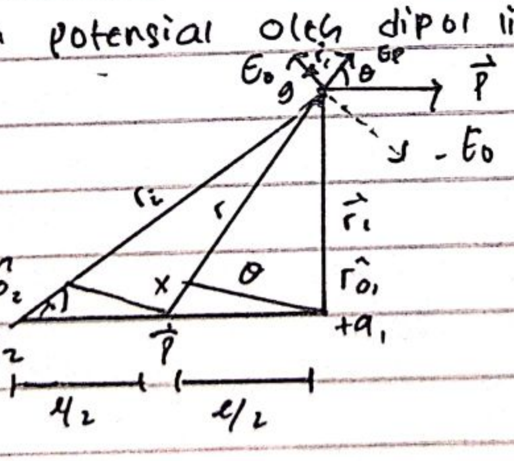
2) Dipole (dua kutub) listrik

Dipole adalah pasangan muatan listrik yang mempunyai muatan berlawanan ($\Sigma q_i = 0$)

Ada dipol, maka pada dipol tersebut akan timbul momen dipol. Besarnya ($\vec{p}$) adalah muatan kali jarak

$\vec{p} = q \cdot \vec{e}$ → artinya dari ⊖ ke ⊕

Besarnya potensial oleh dipol listrik



$VP = 0$

dimana $q_1 = -q_2 = q$

$$V(r,\theta) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

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$r_1 = r - \frac{\ell}{2}\cos\theta$, $r_2 = r + \frac{\ell}{2}\cos\theta$

$$V(r,\theta) = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\ell - \frac{1}{2}\cos\theta} - \frac{1}{r + \frac{\ell}{2}\cos\theta}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{\ell\cos}{r^2 - \frac{\ell^2}{4}\cos^2\theta}\right)$$

untuk $r >> \ell$ maka

$$V(r,\theta) = \frac{q\,\ell\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$V(r,\theta) = \frac{\vec{P}\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\vec{P}\cdot\vec{r_0}}{4\pi\varepsilon_0 r^2}$$

$$E(r,\theta) = -\nabla V$$

$$E(r,\theta) = \frac{\vec{P}}{4\pi\varepsilon_0 r^2}\left(2\cos\theta\cdot\vec{r_0} + \sin\theta\,\bar{\theta}_0\right)$$

$$E(r,\theta) = \frac{1}{4\pi\varepsilon_0 r^2}\left(2\vec{P}\cos\theta\right)\vec{r_0} + \left(\vec{P}\sin\theta\right)\bar{\theta}_0$$

Jadi

$$E(r,\theta) = \frac{1}{4\pi\varepsilon_0 r^2}\left[3\left(\vec{P}\cdot\vec{r_0}\right)\vec{r_0} - \vec{P}\right]$$