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Date :

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Bab iii Metode penentuan potensial

3.1 Persamaan Laplace

$\oint \vec{E} \cdot d\vec{e} = 0$, untuk $\vec{E}$ konservatif

$\oint \vec{E} \cdot d\vec{p} = \int \nabla, \vec{E} \cdot dA = 0$.

$\nabla \times E = 0$

$\nabla E = \frac{\rho}{\varepsilon}$.

$E = -\nabla V$

$\nabla \vec{E} = -\nabla \cdot \nabla V$

$\frac{1}{\varepsilon_0}\rho = -\nabla^2 V$

$\nabla^2 V = -\frac{1}{\varepsilon_0}\rho$ (Hukum Poisson)

$\nabla^2 V = 0$ (Persamaan Laplace)

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 .$$

$$\frac{\partial^2 V}{\partial x^2} = 0 , \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 , \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 .$$

→ Untuk menghitung potensial listrik dan rumus poson.

$\nabla^2 V = \frac{1}{\varepsilon_0}\rho$

Dalam persamaan Laplace $\rho = 0$, ε_0 = konstan

$\nabla^2 V = 0$; untuk satu dimensi maka

$\frac{\partial^2 V_x}{\partial x^2} = 0$ → Karena V fungsi dari x saja, maka

$\frac{d^2 V_x}{dx^2} = \frac{d}{dx}\left(\frac{dV}{dx}\right) = 0$.

$\frac{dV_x}{dx} = k$; k = konstan

$dV = k\,dx$

$V_x = kx + b$; b = konstanta

$V(x) = kx + b$.

→ Untuk koordinat bola, maka persamaan Laplace satu dimensi

$\nabla^2 V = 0$

$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$.

$$r^2 \frac{\partial V}{\partial r} = r^2 \frac{dV}{dr} = k .$$

$$dV = \frac{1}{r^2} k\,dr$$

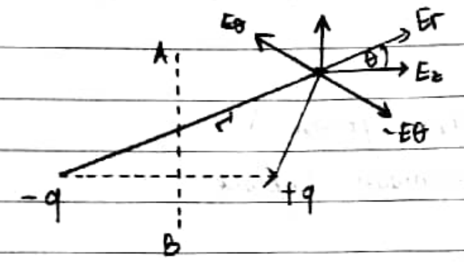
$$V(p) = -\frac{1}{r}k + c$$

3.2 Metoda Bayangan

a) Muatan titik dengan batang konduktor

•) $E_P = \dfrac{q}{4\pi\varepsilon_0}\left[\dfrac{1}{r^2} - \dfrac{(r+2h\cos\theta)}{(r^2+4h^2+4hr\cos\theta)^{3/2}}\right]$

•) $E_\theta = \dfrac{q}{4\pi\varepsilon_0}\left[\dfrac{2h\sin\theta}{(r^2+4h^2+4hr\cos\theta)^{3/2}}\right]$



$$E_z = E_r\cos\theta - E_\theta \sin\theta$$

$$E_z = \frac{q}{4\pi\varepsilon_0}\left[\frac{\cos\theta}{r^2} - \frac{r\cos\theta+2h}{r^3}\right]$$

Untuk setiap titik pada bidang AB, $r=r'=a$, sehingga

$$E_z(a) = \frac{q}{4\pi\varepsilon_0}\left(\frac{\cos\theta}{a^2} - \frac{a\cos\theta+2h}{a^3}\right)$$

$$= \frac{-q}{4\pi\varepsilon_0}\left(\frac{2h}{a^3}\right) = \frac{-qh}{2\pi\varepsilon_0 a^3} = \frac{1}{\varepsilon_0}\sigma(a) .$$

$\sigma(a) = \dfrac{-qh}{2\pi a^3}$; σ = muatan induksi persatuan luas pada pelat konduktor AB.

Muatan induksi total pada pelat AB :

$$\int_0^\infty \sigma\,dA = \int_0^\infty \sigma\,(2\pi x\,dx) = -hq\int_0^\infty \frac{x}{a^3}\,dx$$

$$= -hq \int_0^\infty \frac{a}{a^3}\,da$$

$= -q$ ⇒ Bukti muatan bayangan.

b) Muatan titik dengan bola konduktor

Menempatkan muatan $-q$ sedemikian rupa sehingga V dipermukaan benda/bola $V=0$.

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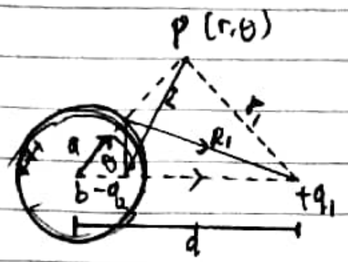


untuk itu; $V = \frac{q}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1} - \frac{q_2}{R_2}\right) = 0$.

$R_1^2 = q^2 + d^2 - 2ad\cos\theta$.

$R_2^2 = q^2 + b^2 - 2ab\cos\theta$.

untuk $V=0$ maka :

$\frac{q_1}{R_1} = \frac{q_2}{R_2} \rightarrow \frac{q_1^2}{q_2^2} = \frac{R_1^2}{R_2^2} = \frac{q^2+d^2+2ad\cos\theta}{q^2+b^2-2ab\cos\theta}$.

$\left(\frac{q_1}{q_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^2 = \frac{d}{b}\left(\frac{q^2/d + d - 2a\cos\theta}{q^2/b + b - 2a\cos\theta}\right)$

dengan hubungan diatas benar untuk setiap θ, bila ;

1) $q^2 = ab$

2) $(q_1/q_2) = d/b$, karena $d > b$ dan $q_1 > q_2$.

bola konduktor dengan jari-jari a dengan $V=0$ dengan muatan q_1 sejarak d dari pusat bola, maka bola tersebut dapat digantungkan dengan muatan titik $(-q)$

$\frac{q_1^2}{q_2^2} = \frac{d}{b}$; $-q_2 = +\sqrt{\frac{b^2}{q}}\cdot q_1$; untuk $b = q^2/d$.

$q_2 = -\frac{q}{d}\, q_1$.

Jadi, $V(r,\theta) = V_p$

$V_p = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} - \frac{q_2}{r_2}\right)$

$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} - \frac{q/d}{r_2}\right)$.

$r_1 = (r^2 + d^2 - 2rd\cos\theta)^{\frac{1}{2}}$

$r_2 = (r^2 + b^2 - 2rb\cos\theta)^{\frac{1}{2}}$.

$V_p = \frac{q_1}{4\pi\varepsilon_0}\left(\frac{1}{(r^2+d^2-2rd\cos\theta)^{\frac{1}{2}}} - \frac{a/d}{\left(r^2 + a^4/d^2 - 2r\left(a^2/d\right)\cos\theta\right)^{\frac{1}{2}}}\right)$

$E_r = -\frac{\partial V}{\partial r}$; $E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta}$.

khusus untuk permukaan bola : dimana $r=a$.

$E_r = -\frac{a}{4\pi\varepsilon_0}\left[\frac{-(a-d\cos\theta)}{(a^2+d^2-2ad\cos\theta)^{\frac{3}{2}}} + \frac{a/d - a^2/d\cos\theta}{\left(a^2 + a^4/d^2 - 2a\left(a^2/d\right)\cos\theta\right)^{\frac{1}{2}}}\right]$.

$$E_r = -\frac{a^2/d}{a^3/d^3}\frac{\left(1-a/d\cos\theta\right)}{(d^2+a^2-2ad\cos\theta)^{3/2}} + \frac{a/d\left(a-a^3/d\cos\theta\right)}{\left(a^2+a^4/d^2-2a\left(a^2/d\right)\cos\theta\right)^{3/2}}$$

$$E_r = -\frac{q_1}{4\pi\varepsilon_0}\left[\frac{-(a-d\cos\theta)+d^2/a\left(1-a/d\cos\theta\right)}{\left(a^2+d^2-2ad\cos\theta\right)^{3/2}}\right]$$

Dengan Cara yang sama.

$E_\theta = 0$, terjadi pergeseran listrik D

$D = \varepsilon_0 E$, $-D = \sigma_F = \sigma$

$\sigma = \varepsilon_0 E_r$

$$\sigma = -\frac{q_1}{4\pi a}\left(\frac{d^2-a}{(a^2+d^2-2ad\cos\theta)^{3/2}}\right)$$

σ = muatan induksi persatuan luas pada bola konduktor

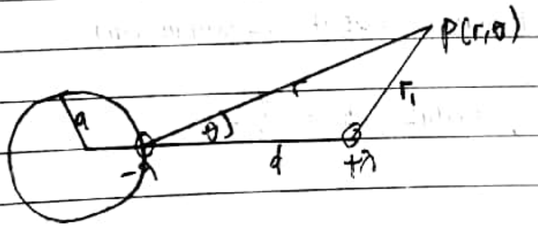
F = gaya antara bola dengan q_1.

$$F = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{(d-b)^2}.$$

$$F = -\frac{1}{4\pi\varepsilon_0}\frac{a q_1^2}{a(dd)^2}.$$

c) . Muatan garis dengan silinder bermuatan

Ada 2 muatan garis saling sejajar terpisah sejarak d.



dimana $V=0$

V permukaan $= V = 0$.

$$V_P = \frac{+\lambda}{4\pi\varepsilon_0}\ln r' - \frac{\lambda}{2\pi\varepsilon_0}\ln r + C.$$

agar $V=0$ maka

$$C = \frac{-\lambda}{4\pi\varepsilon_0}\ln\left(\frac{r_1}{r}\right)^2,$$ V semua permukaan dimana

$$\left(\frac{r_1'}{r}\right)^2 = \frac{r^2+d^2-2rd\cos\theta}{r^2} = m^2 = \text{konstanta dg } 0 < m < \infty$$

$r^2+d^2-2rd\cos\theta = m^2r^2$ untuk $x = r\cos\theta$, $y = r\sin\theta$.

maka :

$r^2+d^2-2rd\cos\theta = m^2r^2$.

diperoleh : $\left(x + \frac{a}{m^2-1}\right)^2 + y^2 = \frac{m^2d^2}{(m^2-1)^2} \rightarrow \left(-\frac{d}{m^2-1} ; 0\right)$.

Jari-jari silinder $a = \frac{md}{m^2-1}$, $x = \frac{-d}{m^2-1}$, $y = 0$.

untuk $m > 1$ letak sumbu silinder dengan harga $V=0$ adalah pd jarak $\frac{a}{m^2-1}$ disebelah kiri titik O.

→ Jarak sumbu silinder hingga $+\lambda$ adalah

$$p = d + \frac{a}{m^2-1} = \frac{m^2d}{m^2-1} = m\,a.$$

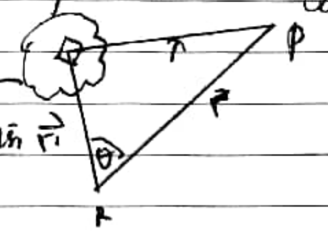
dan jarak

$$x = \frac{a}{m^2-1} = \frac{a}{m} = \frac{a^2}{r}$$

$$x = \frac{a}{p}.$$

3.3. Ekspansi Multipol

1) Monopol (kutub tunggal).



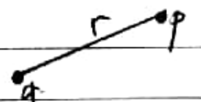
$$V_p = -\int_\infty^p \vec{E}\, dr \quad ...(1)$$

$$V_R = -\int_\infty^R \vec{E}\, dr \quad ...(2).$$

$$V_p - V_R = -\int_\infty^p E\,dr + \int_\infty^R \vec{E}\,dr$$

$$V_{Rp} = -\int_R^p \vec{E}\,dr = \frac{1}{4\pi\varepsilon_0}\int_R^p \frac{\rho' d\tau}{r}$$

Jika muatan tsb tanpa titik



$$V_p = -\int_\infty^{p\,r} E\,dr$$

$$V_p = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}.$$

Jadi, $V_{m(p)} \sim \frac{1}{r}$; V_{mp} = potensial monopol di p.

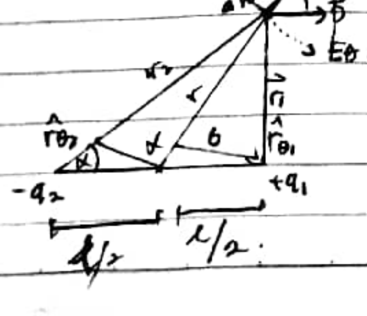
2). Dipole (dua kutub) listrik adalah pasangan muatan listrik yang mempunyai muatan berlawanan $(\Sigma q_i = 0)$. Ada dipol, maka dipol tersebut akan timbul momen dipol. Besarnya $(\vec{p})$ adalah muatan kali jarak.

$\vec{p} = q \cdot \vec{e} \rightarrow$ arahnya dari − ke +.

ciri² dipol listrik

1). Jumlah muatan = 0
2) Jarak relatif dekat
3) Jumlah momen dipol ≠ 0
4) gaya tidak muatan titik.

Besar potensial dipol listrik.



$V_p = 0$, dimana $q_1 = q_2 = q$.

$$V(r,\theta) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

$r_1 = r - \frac{\ell}{2}\cos\theta$, $r_2 = r + \frac{\ell}{2}\cos\theta$.

$$V(r,\theta) = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r^2 - \ell^2/4\cos^2\theta} - \frac{1}{r + \ell/2\cos\theta}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{\ell\cos}{r^2 - \ell^2/4\cos^2\theta}\right)$$

•) untuk $r \gg \ell$ maka

$$V(r,\theta) = \frac{q\,\ell\cos\theta}{4\pi\varepsilon_0 r^2}.$$

$$V(r,\theta) = \vec{p}\frac{\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\vec{p}\,\hat{r}}{4\pi\varepsilon_0 r^2}.$$

•) $E(r,\theta) = -\nabla V$.

$$E(r,\theta) = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$

$$E(r,\theta) = \frac{1}{4\pi\varepsilon_0 r^3}\left((2p\cos\theta)\,\hat{r} + \vec{p}\sin\theta\,\hat{\theta}\right)$$

Jadi, $E(r,\theta) = \frac{1}{4\pi\varepsilon_0 r^3}\left[3(\vec{p}\,\hat{r})\,\hat{r} - \vec{p}\right]$.