No.

Date :

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Kelas : B

Metoda Penentuan Potensial

- Persamaan laplace

$\oint E \cdot dl = 0$, untuk E konservatif

$\oint E \cdot dl = \int \nabla \times E \cdot dA = 0$

$\nabla \times E = 0$

$\nabla E = \frac{\rho}{\varepsilon_0}$; $E = -\nabla V$

$\nabla E = -\nabla \cdot \nabla V$

$\frac{1}{\varepsilon_0} \rho = -\nabla^2 V$

$\nabla^2 V = -\frac{1}{\varepsilon_0} \rho$ → hukum poisson

$\nabla^2 V = 0$ → pers laplace

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{\partial^2 V}{\partial x^2} = 0 \ ; \ \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \ ; \ \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Pembahasan ini dibatasi pada persamaan laplace 1 dimensi, untuk menghitung potensial listrik dan rumus poisson $\nabla^2 V = \frac{1}{\varepsilon_0} \rho$

dalam persamaan laplace, $\rho = 0$, $\varepsilon_0 = c$ (konstan)

$\nabla^2 V = 0$, untuk satu dimensi maka : $\frac{\partial^2 V}{\partial x^2} = 0$

Karena V fungsi dari x saja, maka $\frac{d^2 V}{dx^2} = \frac{d}{dx}\left(\frac{dV}{dx}\right) = 0$

$\frac{dV_x}{dx} = k$ (konstan)

$dV_x = k\,dx$

$V_x = kx + b$, b (konstan)

$V(x) = kx + b$

Untuk koordinat bola maka persamaan laplace satu dimensi

$\nabla^2 V = 0$

$\frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{dV}{dr}\right) = 0$

$r^2 \frac{\partial V}{\partial r} = r^2 \frac{dV}{dr} = k$

$dV = \frac{1}{r^2} k\,dr$

$V(r) = -\frac{1}{r} k + c$

- Metoda Bayangan

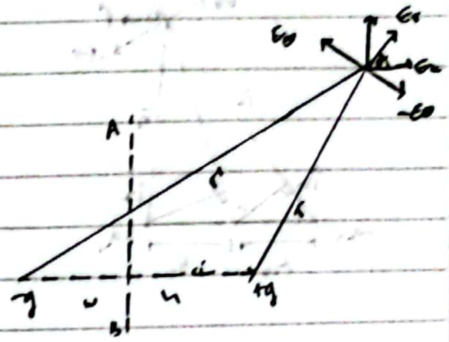
Muatan titik dengan batang konduktor

$$E_r = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r^2} - \frac{(r + 2h\cos\theta)}{(r^2 + 4h^2 + 4hr\cos\theta)^{3/2}}\right]$$

$$E_\theta = \frac{q}{4\pi\varepsilon_0}\left[\frac{2h\sin\theta}{(r^2 + 4h^2 + 4hr\cos\theta)^{3/2}}\right]$$

$$E_z = E_r\cos\theta - E_\theta\sin\theta$$

$$E_z = \frac{q}{4\pi\varepsilon_0}\left[\frac{\cos\theta}{r^2} - \frac{r\cos\theta + 2h}{r'^3}\right]$$



Untuk setiap titik pada bidang AB, $r = r' = a$, sehingga

$$E_z(a) = \frac{q}{4\pi\varepsilon_0}\left(\frac{\cos\theta}{a^2} - \frac{a\cos\theta + 2h}{a^3}\right)$$

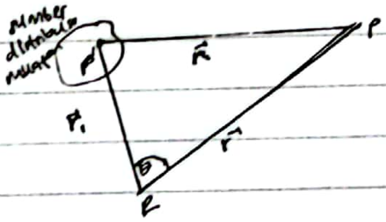
$$= \frac{-q}{4\pi\varepsilon_0}\left(\frac{2h}{a^3}\right) = \frac{-qh}{2\pi\varepsilon_0 a^3} = \frac{1}{\varepsilon_0}\sigma(a)$$

$$\sigma(a) = \frac{-qh}{2\pi a^3}$$

↳ Muatan induksi per satuan luas pada pelat konduktor AB.

- Expansi Multipol

- Monopol (kutub tunggal)



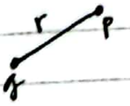
$$V_P = -\int_\infty^P \vec{E}\,dr \qquad (i)$$

$$V_R = -\int_\infty^R \vec{E}.dr \qquad (ii)$$

dari (i) dan (ii) $= V_P - V_R = -\int_\infty^P \vec{E}.dr + \int_\infty^R \vec{E}.dr$

$$V_{RP} = -\int_R^P \vec{E}\,dr = -\frac{1}{4\pi\varepsilon_0}\int_R^P \frac{\rho' d\tau}{r}$$

Jika muatan berupa titik



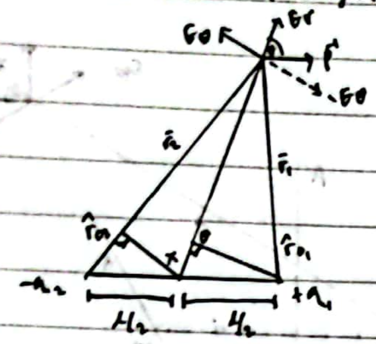
$$V_P = -\int_\infty^{P,r} \vec{E}.dr$$

$$V_P = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Jadi $V_{M(P)} \sim \frac{1}{r}$; $V_M(P)$ = potensial monopol di P.

- dipole (dua kutub) listrik.

Merupakan pasangan muatan listrik yang mempunyai muatan berlawanan $(\Sigma q_i = 0)$



$$V_p = V(r,\theta) = \frac{q_1}{4\pi\varepsilon_0 r_1} - \frac{q_2}{4\pi\varepsilon_0 r_2}$$

dimana $q_1 = q_2 = q$

$$V(r,\theta) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$r_1 = r - l/2 \cos\theta$$

$$r_2 = r + l/2 \cos\theta$$

$$V(r,\theta) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r - l/2\cos\theta} - \frac{1}{r + l/2\cos\theta}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{l\cos\theta}{r^2 - \frac{l^2}{4}\cos^2\theta}\right)$$; untuk $r \gg l$, maka:

$$V(r,\theta) = \frac{q\,l\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$V(r,\theta) = \frac{P\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\vec{P}\cdot\vec{r}_0}{4\pi\varepsilon_0 r^2}$$