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Prodi : Pendidikan Fisika (B)

Metoda Penentuan Potensial

3.1. Persamaan Laplace

$\oint \vec{E} \cdot d\vec{l} = 0$, untuk $\vec{E}$ konservatif.

$\oint \vec{E} \cdot d\vec{l} = \int \nabla \times \vec{E} \cdot d\vec{A} = 0$

$\nabla \times \vec{E} = 0$,

$\nabla E = \rho/\varepsilon_0$

$E = -\nabla V$ ✓

$\nabla \vec{E} = -\nabla \cdot \nabla V$ ✓

$\frac{1}{\varepsilon_0} \rho = -\nabla^2 V.$ ✓

$\nabla^2 V = \frac{-1}{\varepsilon_0} \rho$ Hukum Poisson

$\nabla^2 V = 0$ => Persamaan laplace

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{\partial^2 V}{\partial x^2} = 0; \quad \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0; \quad \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Pembahasan kali ini kita batasi pada Persamaan Laplace satu dimensi, untuk menghitung Potensial listrik dari rumus Poisson.

$\nabla^2 V = \frac{1}{\varepsilon_0} \rho$

Dalam Pers. laplace $\rho = 0$, ε_0 = konstan

$\nabla^2 V = 0$ untuk satu dimensi, maka

$\frac{\partial^2 V_x}{\partial x^2} = 0$ => karena V fungsi dari x saja, maka

$\frac{d^2 V_x}{dx^2} = \frac{d}{dx}\left(\frac{dV}{dx}\right) = 0$

$\frac{dV_x}{dx} = k$, k = konstan

$dV_x = k\,dx$

$V_x = kx + b$

$V(x) = kx + b$

b = bil. konstan

untuk koordinat bola, maka pers. laplace satu dimensi: $\nabla^2 V = 0$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = 0$$

$$r^2\frac{\partial V}{\partial r} = r^2\frac{dV}{dr} = k$$

$$dV = \frac{1}{r^2}\,k\,dr$$

$$V(r) = -\frac{1}{r}k + C$$

$V_a = 10\ V$

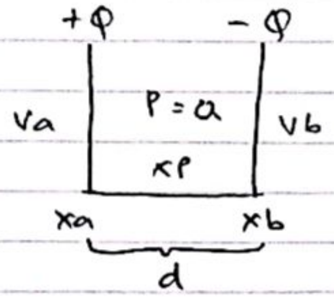
$V_b = 10\ V$

$x_a = 0$

$x_b = 10$

$x_P = 5$

Contoh : 0.10



$V(x_P) = ?$

$V(x) = k x_P + b$

$V_a = k(x_a) + b$

$V_b = k(x_b) + b$ −

$V_a - V_b = k(x_a - x_b)$

$x_a = 0$

$x_b = 10$

$$k = \frac{V_a - V_b}{x_a - x_b}$$

$$V_x = \left(\frac{V_a - V_b}{x_a - x_b}\right) x_a + b : \left[\frac{10-0}{10-0}\right)$$

$$b : V_a - \left(\frac{V_a - V_b}{x_a - x_b}\right) \cdot x_a{}^{2-1}$$

$$V_x = \left(\frac{V_a - V_b}{x_a - x_b}\right) x_a . x + \left(V_a - \left(\frac{V_a - V_b}{x_a - x_b}\right) . x_a\right)$$

$$= x_a\left[\left(\frac{V_a - V_b}{x_a - x_b}\right) . x + \left(\frac{V_a}{x_a} - \left(\frac{V_a - V_b}{x_a - x_b}\right)\right)\right]$$

$$V_{xP} = x_a\left[\left(\frac{V_a - V_b}{x_a - x_b}\right) , x_a + \left(\frac{V_a}{x_a} - \left(\frac{V_a - V_b}{x_a - x_b}\right)\right]$$

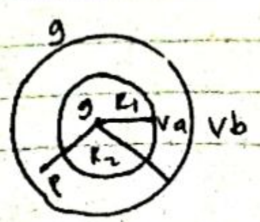
solusi

$$V_x = \left[\frac{(V_a - V_b)}{(x_a - x_b)}\,x + \left(V_a - \left(\frac{x_a - V_b}{x_a - x_b}\right) x_a\right]\right.$$

$V(P) = ?$

$$V(r) = \frac{-1}{r}k + c$$

$$V_a = -\frac{1}{R_1}k + C$$



$$V_a = -\frac{1}{R_1} k + C$$

$$V_b = -\frac{1}{R_2} k + C$$

$$V_a - V_b = k\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

$$V_a - V_b = k\left(\frac{R_1 - R_2}{R_2 R_1}\right)$$

$$k = \frac{R_2 R_1 (V_a - V_b)}{(R_1 - R_2)}$$

$$V_a = -\frac{1}{R_1}\left[\frac{R_2 R_1 (V_a - V_b)}{(R_1 - R_2)}\right] + C$$

$$C = V_a + \frac{1}{R_1}\left[\frac{R_2 R_1 (V_a - V_b)}{(R_1 - R_2)}\right]$$

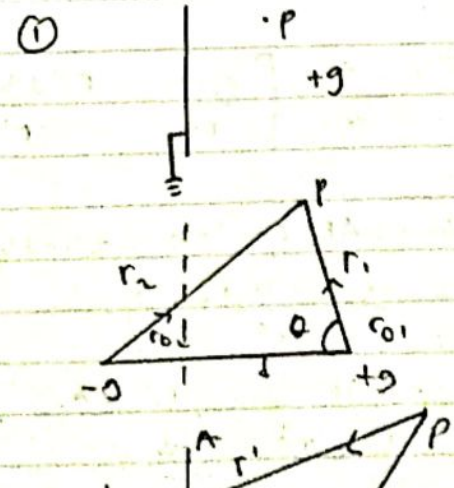
$$V(r_p) = -\frac{1}{r_p}\,\frac{R_2 R_1 (V_a - V_b)}{(R_1 - R_2)} + \left[V_a + \frac{1}{R_1}\left(\frac{R_2 R_1 (V_a - V_b)}{R_1 - R_2}\right)\right]$$

$$V(r_p) = \frac{R_2 R_1 (V_a - V_b)}{R_1 - R_2}\left(-\frac{1}{r_p} + \frac{1}{R_1}\right) + V_a$$

3.2 Metoda Bayangan dengan

a). Muatan titik batas konduktor

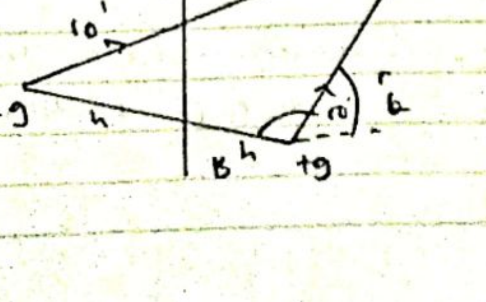
①



$V_P = ?$ $\quad E_r = -\frac{\partial V}{\partial r}$; $E_\theta = \frac{\partial V}{\partial r}$

$$V_P = \frac{1}{4\pi\varepsilon_0}\,\frac{+q}{r_1} + \frac{-q}{4\pi\varepsilon_0 r_2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$



Penjumlahan biasa

$$V_P = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r'}\right)$$

$$V_P = \frac{q}{4\pi\varepsilon_0}\left(\frac{r' - r}{r \cdot r'}\right)$$

$$V_P = V(r,\theta)$$

Perhatikan gambar

$$V(r,\theta) = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{(r^2+4h^2+4hr\cos\theta)^{1/2}}\right]$$

$$E_r = -\frac{\partial V}{\partial r} =$$

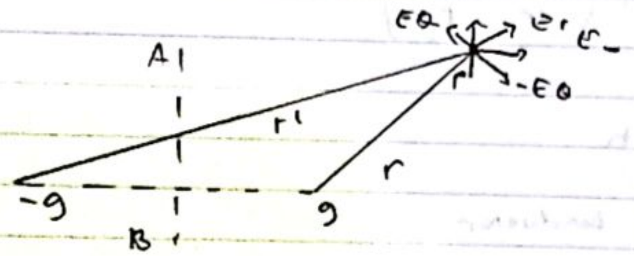
$$= \frac{\partial}{\partial r}\left(\frac{1}{r} - \frac{1}{(r^2+4h^2+4hr\cos\theta)^{\frac{1}{2}}}\right)$$

$$= -\frac{1}{r^2} + \frac{\frac{1}{2}(r^2+4h^2+4hr\cos\theta)^{-1/2}(2r+4h\cos\theta)}{(r^2+4h^2+4hr\cos\theta)}$$

$$= -\frac{1}{r^2} + \frac{(r+2h\cos\theta)}{(r^2+4h^2+4hr\cos\theta)^{3/2}}$$

$$E_r = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r^2} - \frac{(r+2h\cos\theta)}{(r^2+4h^2+4hr\cos\theta)^{3/2}}\right]$$

$$E_\theta = \frac{q}{4\pi\varepsilon_0 r}\left[\frac{2h\sin\theta}{(r^2+4h^2+4hr\cos\theta)^{3/2}}\right]$$



$$E_z = E_r\cos\theta - E_\theta\sin\theta$$

$$E_z = \frac{q}{4\pi\varepsilon_0}\left[\frac{\cos\theta}{r^2} - \frac{r\cos\theta+2h}{r'^3}\right]$$

untuk setiap titik pada bidang AB, $r = r' = a$, sehingga

$$E_z(a) = \frac{q}{4\pi\varepsilon_0}\left(\frac{\cos\theta}{a^2} - \frac{a\cos\theta+2h}{a^3}\right)$$

$$= \frac{-q}{4\pi\varepsilon_0}\left(\frac{2h}{a^3}\right) = \frac{-qh}{2\pi\varepsilon_0 a^3} = \frac{1}{\varepsilon_0}\sigma(A)$$

$$\sigma(A) = \frac{-qh}{2\pi a^3}$$

$\sigma(A)$ = muatan induksi luas pada pelat konduktor AB.