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BAB III
Metoda Penentuan Potensial

3.1 Persamaan Laplace

$\oint \vec{E}.d\vec{\ell} = 0$. untuk $\vec{E}$ konservatif

$$\oint \vec{E}.d\vec{\ell} = \int \nabla \times \vec{E} . d\vec{A} = 0.$$

$$\nabla \times \vec{E} = 0,$$

$$\nabla . E = \frac{\rho}{\varepsilon_0}$$

$$E = -\nabla V.$$

$$\nabla . \vec{E} = -\nabla . \nabla V$$

$$\frac{1}{\varepsilon_0}\rho = -\nabla^2 V.$$

$$\boxed{\nabla^2 V = -\frac{1}{\varepsilon_0}\rho}$$ Hukum Poisson.

$\nabla^2 V = 0 \Rightarrow$ persamaan Laplace.

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{\partial^2 V}{\partial x^2} = 0 ; \quad \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 ;$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$

Pembahasan kali ini dibatasi pada persamaan Laplace satu dimensi, untung menghitung potensial listrik. Dari rumus Poisson.

$$\nabla^2 V = \frac{1}{\varepsilon_0}\rho$$

Dalam pers. Laplace $\rho = 0$, ε_0 = konstan.

$\boxed{\nabla^2 V = 0}$; untuk satu dimensi maka

$\frac{\partial^2 V_x}{\partial x^2} = 0 \Rightarrow$ karena V fungsi dari x saja, maka

$$\frac{d^2 V_x}{dx^2} = \frac{d}{dx}\left(\frac{dV}{dx}\right) = 0.$$

$\frac{dV_x}{dx} = K$, K = konstan.

$$dV_x = K\,dx$$

$$\begin{cases} V_x = Kx + b. \\ V(x) = Kx + b. \end{cases}$$; b = bilangan konstanta.

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Untuk koordinat bola, maka pers. Laplace satu dimensi :

$$\nabla^2 V = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

$$r^2 \frac{\partial V}{\partial r} = r^2 \frac{dV}{dr} = k.$$

$$dV = \frac{1}{r^2} k\, dr.$$

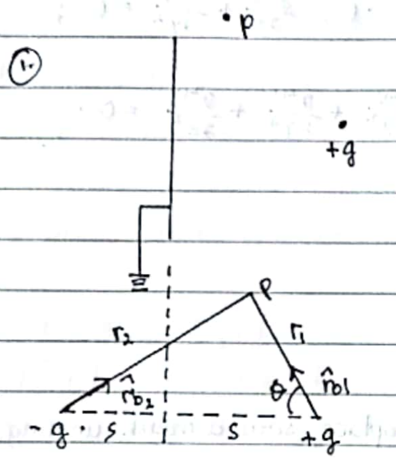
$$V_{(r)} = -\frac{1}{r}k + C$$

$V_a = 10V$
$V_b = 20V$
$x_a = 0$
$x_b = 10$
$x_p = 5.$

3.2 Metoda Bayangan dengan

a.) Muatan titik oleh batang konduktor.

1.



$V_p = ?..;\ E_r = -\frac{\partial V}{\partial r}\ ;\ E_\theta = -\frac{1}{r}\frac{\partial V}{\partial r}$

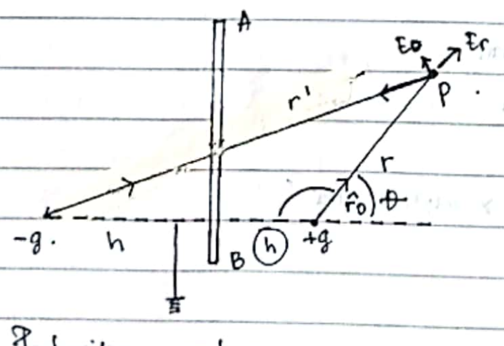
$$V_p = \frac{1}{4\pi\varepsilon_0}\frac{+q}{r_1} + \frac{-q}{4\pi\varepsilon_0 r_2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{r_2 - r_1}{r_1 \cdot r_2}\right)$$

$\vec{E}_0 = -\frac{\partial V}{\partial \rho}$

$E_\theta = -\frac{1}{\rho}\frac{\partial V}{\partial \theta}.$

2. Penjumlahan biasa.



$$V_p = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r'}\right)$$

$$V_p = \frac{q}{4\pi\varepsilon_0}\left(\frac{r' - r}{r \cdot r'}\right).$$

$$V_p = V(r, \theta)$$

Perhatikan gambar.

$$V(r,\theta) = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{(r^2 + 4h^2 + 4hr\cos\theta)^{\frac{1}{2}}}\right]$$

$$E_r = -\frac{\partial V}{\partial r} =$$

$$\hookrightarrow \frac{\partial}{\partial r}\left(\frac{1}{r} - \frac{1}{(r^2+4h^2+4hr\cos\theta)^{\frac{1}{2}}}\right)$$

$$= -\frac{1}{r^2} + \frac{\frac{1}{2}(r^2 + 4h^2 + 4hr\cos\theta)^{-\frac{1}{2}}(2r + 4h\cos\theta)}{(r^2 + 4h^2 + 4hr\cos\theta)}$$

$$= -\frac{1}{r^2} + \frac{(r + 2h\cos\theta)}{(r^2 + 4h^2 + 4hr\cos\theta)^{3/2}}$$

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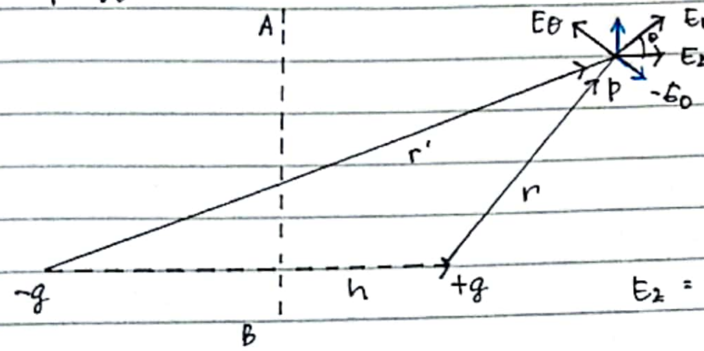
$$\therefore E_r = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r^2} + \frac{(r+2h\cos\theta)}{(r^2+4h^2+4hr\cos\theta)^{3/2}}\right]$$

$$\therefore E_\theta = \frac{q}{4\pi\varepsilon_0 r}\left[\frac{2h\sin\theta}{(r^2+4h^2+4hr\cos\theta)^{3/2}}\right] . \quad = \frac{q}{4\pi\varepsilon_0}\left[\frac{2h\sin\theta}{4r^2}\right]$$

$$\vec{E}_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta}$$



$$E_z = E_r\cos\theta - E_\theta\sin\theta$$

$$E_z = \frac{q}{4\pi\varepsilon_0}\left[\frac{\cos\theta}{r^2} - \frac{r\cos\theta + 2h}{r'^3}\right]$$

Untuk setiap titik pada bidang AB, $r = r' = a$, sehingga

$$E_z(a) = \frac{q}{4\pi\varepsilon_0}\left(\frac{\cos\theta}{a^2} - \frac{a\cos\theta + 2h}{a^3}\right)$$

$$= -\frac{q}{4\pi\varepsilon_0}\left(\frac{2h}{a^3}\right) = \frac{-qh}{2\pi\varepsilon_0 a^3} = \frac{1}{\varepsilon_0}\sigma_{(a)}$$

$$\sigma_{(a)} = \frac{-qh}{2\pi a^3}$$ $\sigma_{(a)}$ = muatan induksi persatuan luas pada pelat konduktor AB.

Muatan induksi total pada pelat AB :

$$\int_0^\infty \sigma\, dA = \int_0^\infty \sigma\,(2\pi x\, dx) = -hq\int_0^\infty \frac{x}{a^3}\,dx$$

$$= -hq\int_0^\infty \frac{a}{a^3}\,da$$

$$= \boxed{-q}$$ ⇒ ini bukti sebagai muatan bayangan.