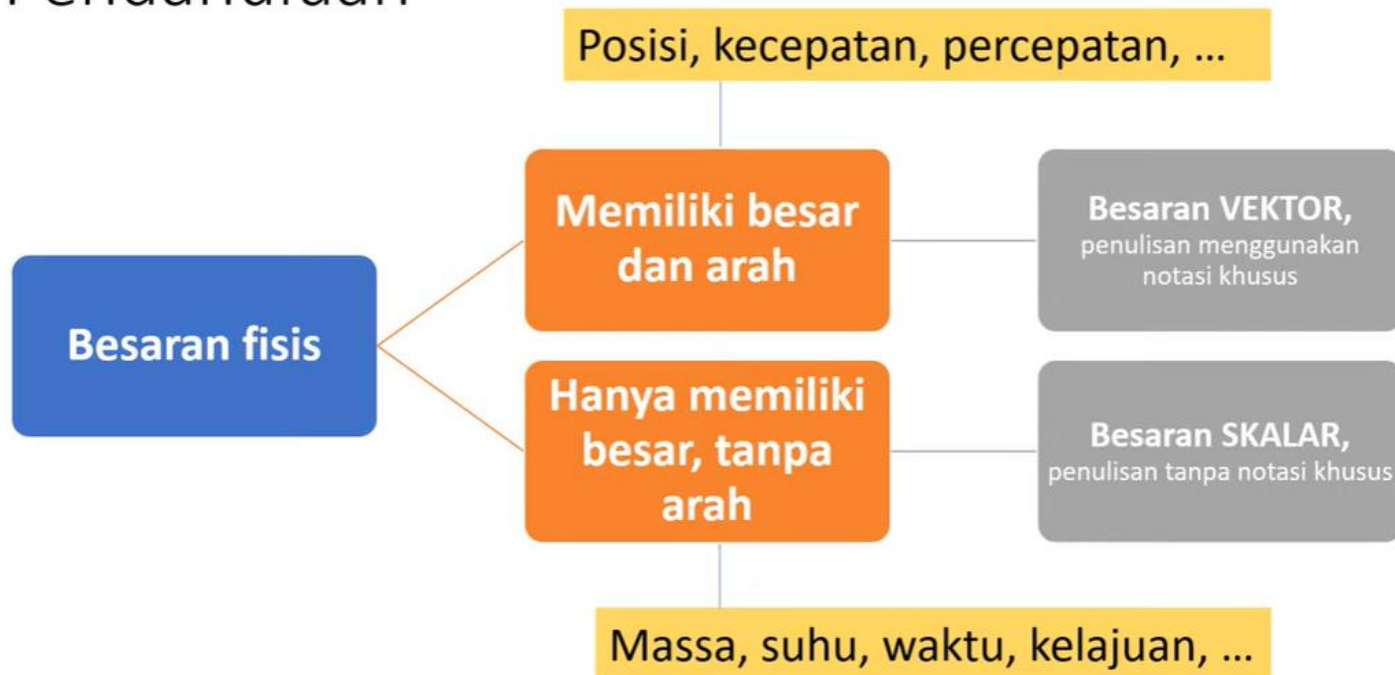


VEKTOR

Miftahul Djana, S.T., M.T

Pendahuluan

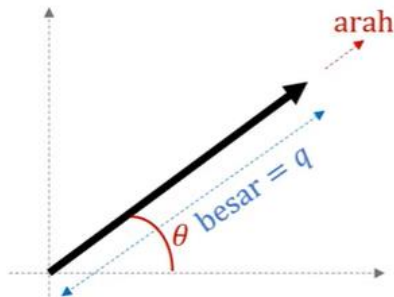


Representasi Vektor

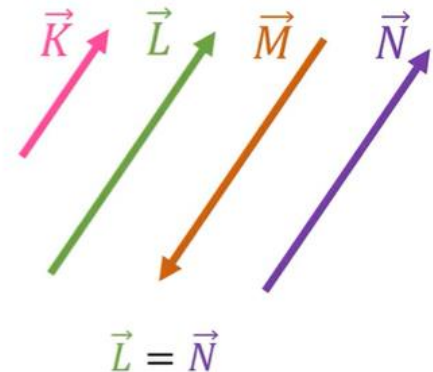
Penulisan besaran vektor menggunakan suatu notasi khusus.

Misal vektor Q ditulis dengan : $\vec{Q}$ atau $\mathbf{Q}$

Vektor direpresentasikan dalam sebuah anak panah yang menunjukkan besar dan arah tertentu.

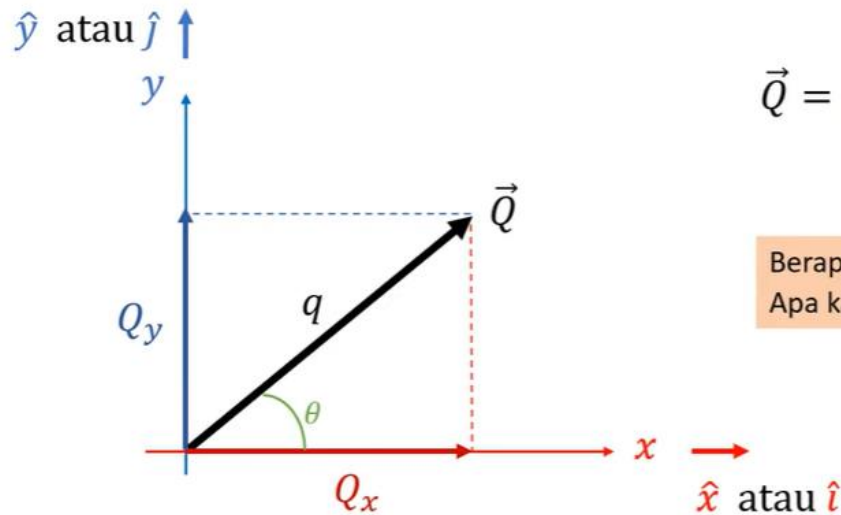


Vektor $\vec{Q}$,
besarnya q ,
arahnya θ terhadap sumbu datar



Komponen Vektor dan Vektor Satuan (2D)

Vektor dapat diuraikan menjadi komponen dalam setiap sumbu

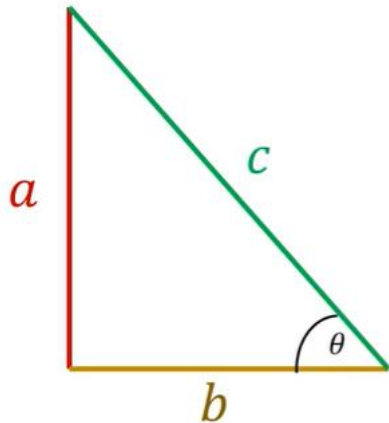


$$\vec{Q} = Q_x \hat{x} + Q_y \hat{y} = Q_x \hat{i} + Q_y \hat{j}$$

Berapa nilai Q_x dan Q_y ?
Apa kaitannya dengan besar vektor q ?



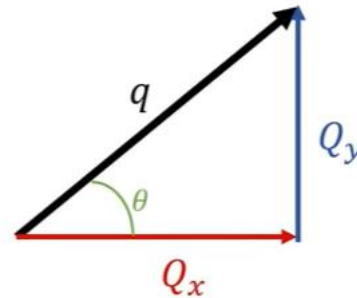
Recalling: Trigonometri



$$\sin \theta = \frac{a}{c}$$

$$\cos \theta = \frac{b}{c}$$

$$\tan \theta = \frac{a}{b}$$



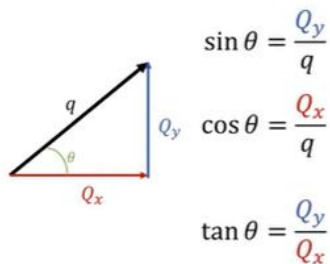
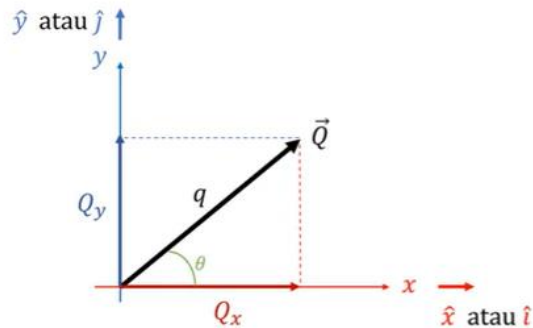
$$\sin \theta = \frac{Q_y}{q}$$

$$\cos \theta = \frac{Q_x}{q}$$

$$\tan \theta = \frac{Q_y}{Q_x}$$

Komponen Vektor dan Vektor Satuan (2D)

Vektor dapat diuraikan menjadi komponen dalam setiap sumbu



$$\vec{Q} = Q_x \hat{x} + Q_y \hat{y} = Q_x \hat{i} + Q_y \hat{j}$$

Berapa nilai Q_x dan Q_y ?

Apa kaitannya dengan besar vektor q ?

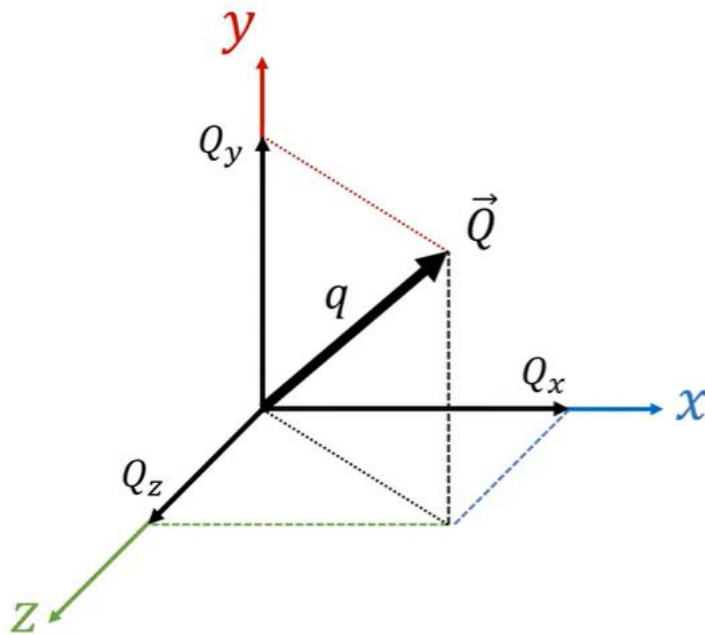


$$Q_y = q \sin \theta \quad \text{Komponen dlm arah sb-y}$$

$$Q_x = q \cos \theta \quad \text{Komponen dlm arah sb-x}$$

$$q^2 = Q_x^2 + Q_y^2$$

Vektor dalam 3D



$$\vec{Q} = Q_x \hat{x} + Q_y \hat{y} + Q_z \hat{z}$$

atau

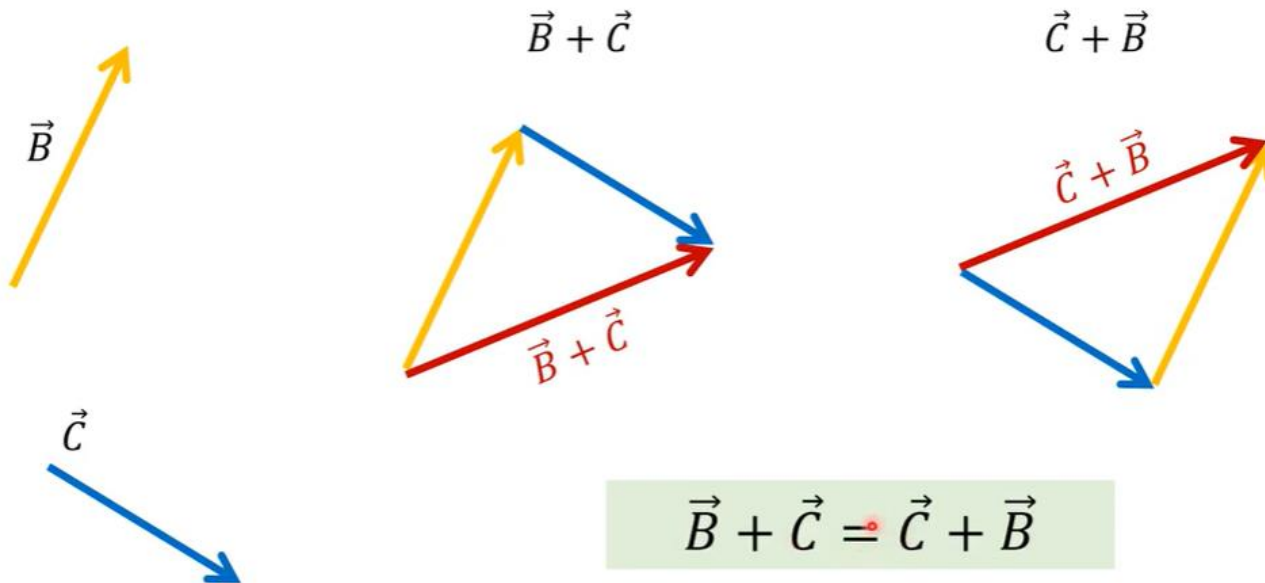
$$\vec{Q} = Q_x \hat{i} + Q_y \hat{j} + Q_z \hat{k}$$

$$q^2 = Q_x^2 + Q_y^2 + Q_z^2$$

Besar vektor

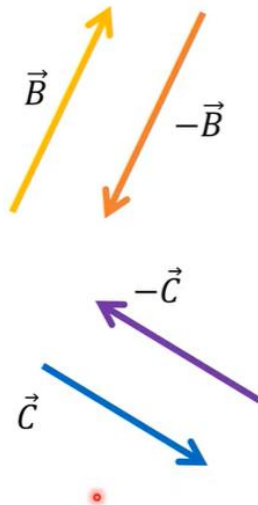
$$q = \sqrt{Q_x^2 + Q_y^2 + Q_z^2}$$

Penjumlahan Vektor: Grafis

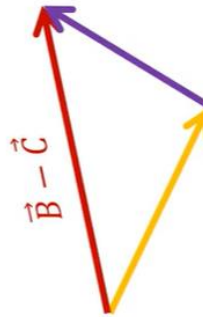


Penjumlahan Vektor: Grafis

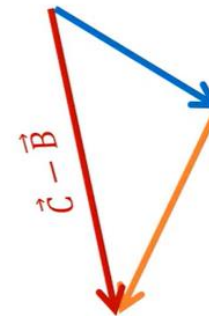
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$$\vec{B} - \vec{C} = \vec{B} + (-\vec{C})$$



$$\vec{C} - \vec{B} = \vec{C} + (-\vec{B})$$



$$\vec{B} - \vec{C} = -(\vec{C} - \vec{B})$$

Penjumlahan Vektor: Analitis

Penjumlahan hanya dapat dilakukan pada vektor yang memiliki arah yang sama

Misal: $\vec{V} = 2\hat{i}$, $\vec{W} = 7\hat{j}$. Tentukan: $\vec{V} + \vec{W}$

Jawaban yang sering muncul:

$$\begin{aligned}\vec{V} + \vec{W} &= 9 \\ \vec{V} + \vec{W} &= (\hat{i} + \hat{j}) \\ \vec{V} + \vec{W} &= 9\hat{i}\hat{j}\end{aligned}$$

$$\vec{V} + \vec{W} = 2\hat{i} + 7\hat{j}$$

Contoh:

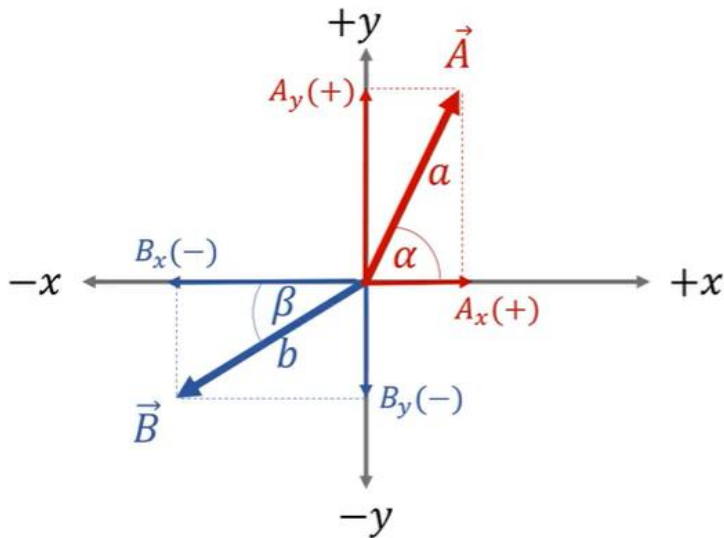
$\vec{G} = \hat{i} + 5\hat{j} - 6\hat{k}$, $\vec{H} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Tentukan: $\vec{G} + \vec{H}$

$$\vec{G} + \vec{H} = (\hat{i} + 5\hat{j} - 6\hat{k}) + (2\hat{i} - 3\hat{j} + 4\hat{k}) = (1\hat{i} + 2\hat{i}) + (5\hat{j} - 3\hat{j}) + (-6\hat{k} + 4\hat{k})$$

$$\therefore \vec{G} + \vec{H} = 3\hat{i} + 2\hat{j} - 2\hat{k}$$

Penjumlahan Vektor: Analitis

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$\vec{A} + \vec{B} \equiv$ Resultan vektor $\vec{A}$ dan $\vec{B} = \vec{R}$

$$\vec{R} = \vec{A} + \vec{B} = (A_x \hat{i} + A_y \hat{j}) + (B_x \hat{i} + B_y \hat{j})$$

Penjumlahan hanya dapat dilakukan pada vektor yang memiliki arah yang sama

Lakukan penjumlahan untuk tiap komponen arah

$$R_x = A_x + B_x = A \cos \alpha + (-B \cos \beta)$$

$$R_y = A_y + B_y = A \sin \alpha + (-B \sin \beta)$$

$$\therefore \vec{R} = R_x \hat{i} + R_y \hat{j}$$

Perkalian Vektor

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Perkalian vektor dengan skalar

Misal suatu vektor: $\vec{A} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ dikalikan dengan suatu konstanta c (c bilangan riil):

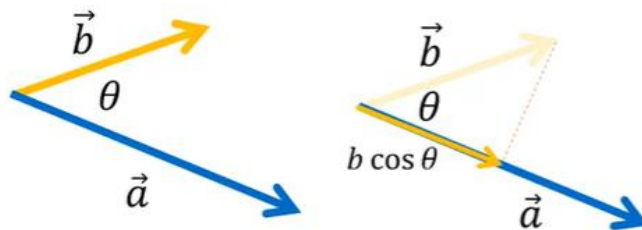
$$c\vec{A} = c(a_x\hat{i} + a_y\hat{j} + a_z\hat{k}) = ca_x\hat{i} + ca_y\hat{j} + ca_z\hat{k}$$

Contoh: $\vec{P} = 2\hat{i} + 10\hat{j} - 6\hat{k}$. Hitung $(-\frac{1}{2})\vec{P}$.

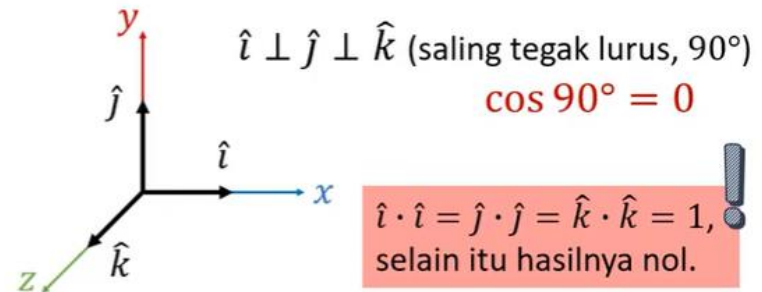
$$\left(-\frac{1}{2}\right)\vec{P} = \left(-\frac{1}{2}\right)(2\hat{i}) + \left(-\frac{1}{2}\right)(10\hat{j}) + \left(-\frac{1}{2}\right)(-6\hat{k}) = -\hat{i} - 5\hat{j} + 3\hat{k}$$

Perkalian Titik (*dot product*) “.”

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$$\vec{a} \cdot \vec{b} = ab \cos \theta$$



$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

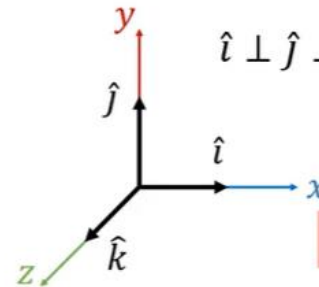
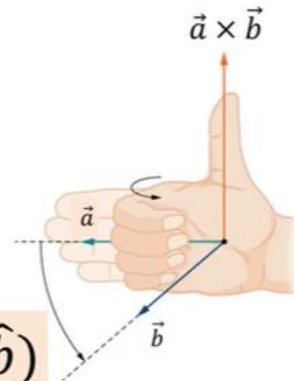
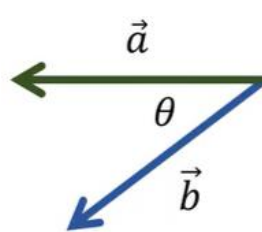
$$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$$

$$\vec{a} \cdot \vec{b} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \cdot (b_x \hat{i} + b_y \hat{j} + b_z \hat{k})$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

Perkalian Silang (*cross product*) “ $\times$ ”

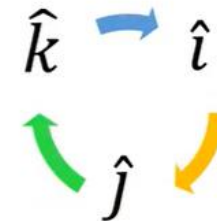
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$\hat{i} \perp \hat{j} \perp \hat{k}$ (saling tegak lurus, 90°)

$\sin 90^\circ = 1$

$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$!

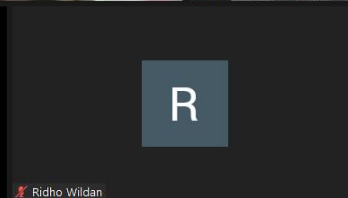


$$\vec{a} \times \vec{b} = ab \sin \theta (\hat{a} \times \hat{b})$$

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$$

$$\begin{aligned} \vec{a} \times \vec{b} &= (\cancel{a_x \hat{i}} \times b_x \hat{i}) + (a_x \hat{i} \times b_y \hat{j}) + (a_x \hat{i} \times b_z \hat{k}) \\ &\quad + (a_y \hat{j} \times b_x \hat{i}) + (\cancel{a_y \hat{j}} \times b_y \hat{j}) + (a_y \hat{j} \times b_z \hat{k}) \\ &\quad + (a_z \hat{k} \times b_x \hat{i}) + (a_z \hat{k} \times b_y \hat{j}) + (\cancel{a_z \hat{k}} \times b_z \hat{k}) \end{aligned}$$

 Qanita Wulandari	 Teknik Lingkungan	 Fhernando Okta Pratama	 Ananda Emelia Yusuf
 Thorif Adnan Nafis	 MAHARANI KENCANA PUSPANEGARA NASYAROEKA	 wendi Putra wiguna	 muhammad farhan setiadi
 Putri Aryanti Fadillah	 Elysia Azhara Hendra	 Dina Endang Ristanti	 DD DESNARIA MARCELIA SITANGGANG
 PSTL Ridho Wildan Ramdhani	 Aqhsal rizkianata	 Ridho Wildan	