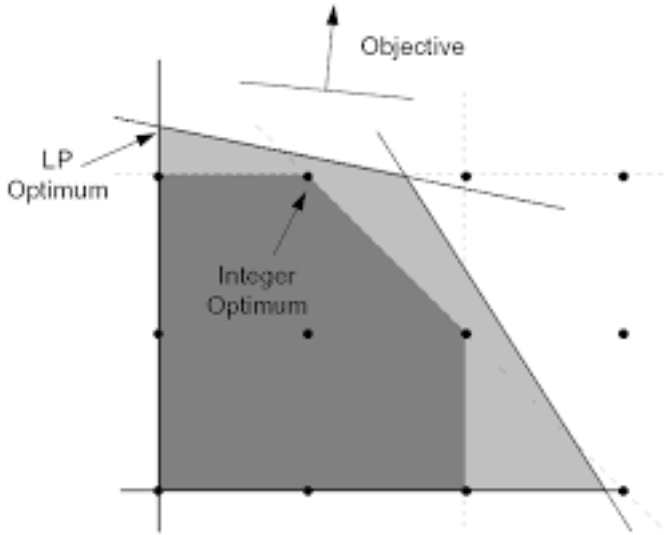
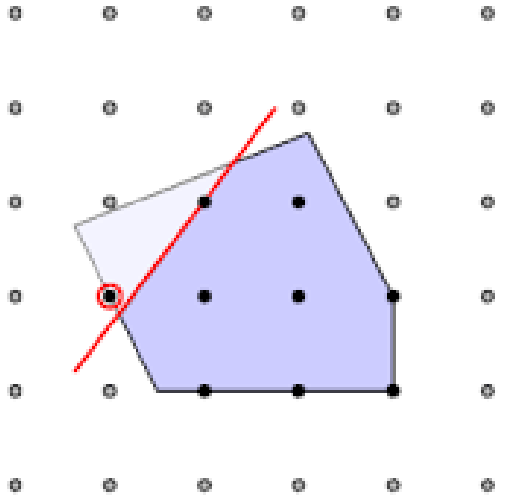
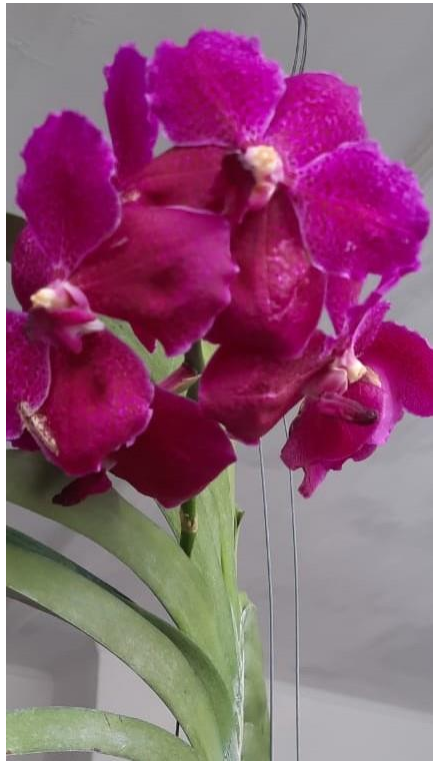




Perbedaan Program Linear dan Program Integer



Program Linear

Max /Min : $\boldsymbol{c^T x}$

Kendala $\boldsymbol{Ax} \leq \mathbf{b}$

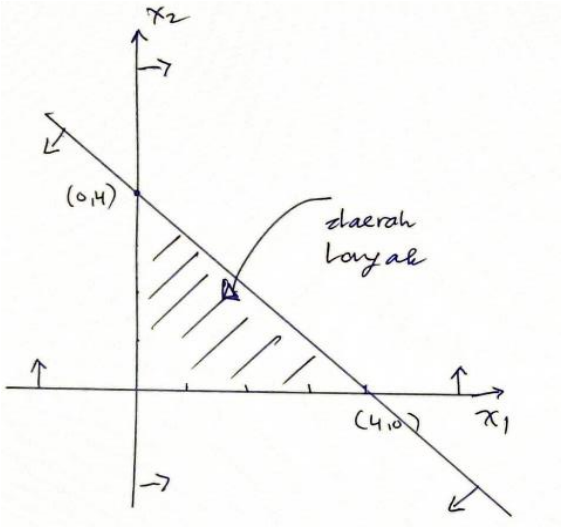
$\boldsymbol{x} \geq \boldsymbol{0}$

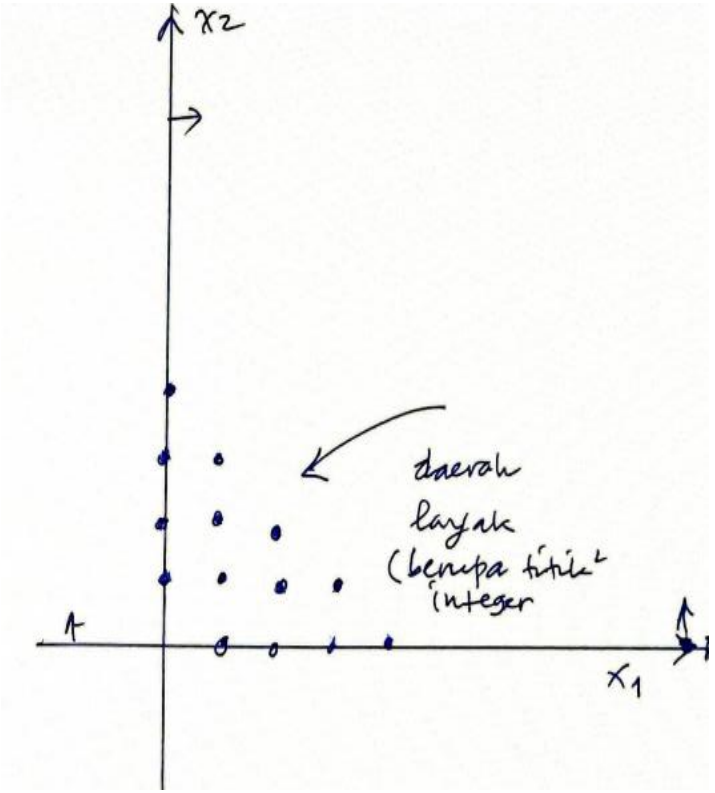
Contoh:

Max : x1 + x2

Kendala : $x_1 + x_2 \leq 4$

$x_1 \geq 0,\ x_2 \geq 0$





Program Integer

Max /Min : $\boldsymbol{c^T x}$

Kendala $\boldsymbol{Ax} \leq \mathbf{b}$

$\boldsymbol{x} \geq \boldsymbol{0}$

$\boldsymbol{x} \in \boldsymbol{Z}$

Contoh:

Max : x1 + x2

Kendala : $x_1 + x_2 \leq 4$

$x_1 \geq 0,\ x_2 \geq 0$

$x_1,\ x_2 \in \boldsymbol{Z}$

Linear programs: example

- We make reproductions of two paintings

- Painting 1 sells for $30, painting 2 sells for $20
- Painting 1 requires 4 units of blue, 1 green, 1 red
- Painting 2 requires 2 blue, 2 green, 1 red
- We have 16 units blue, 8 green, 5 red

maximize $3x + 2y$

subject to

$4x + 2y \leq 16$

$x + 2y \leq 8$

$x + y \leq 5$

$x \geq 0$

$y \geq 0$

Solving the linear program graphically

maximize $3x + 2y$

subject to

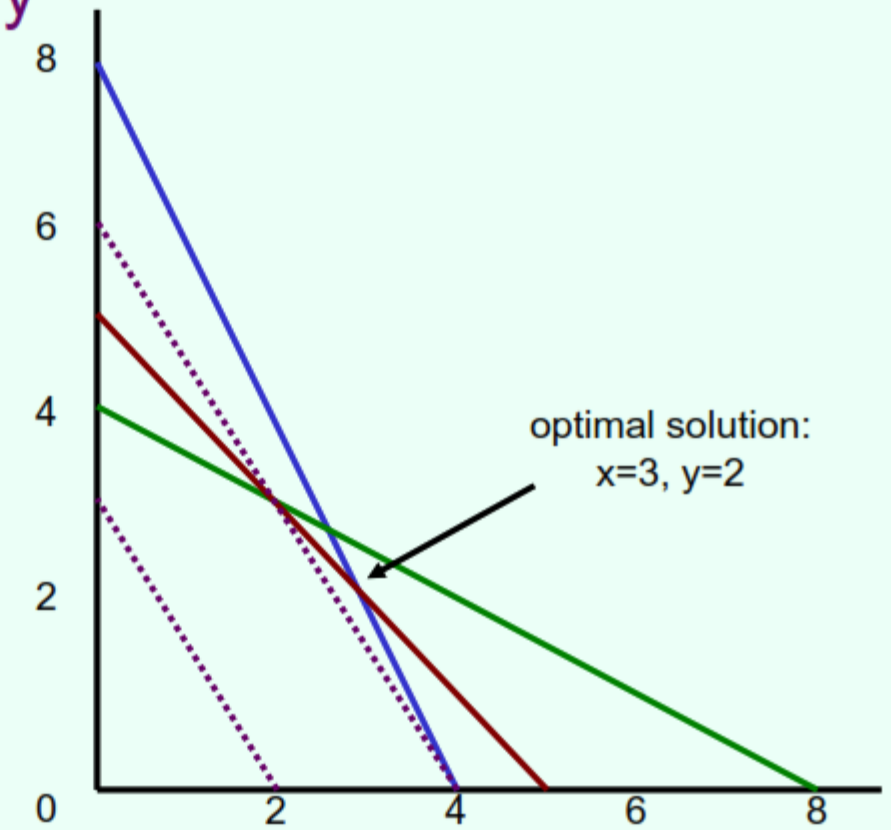
$4x + 2y \leq 16$

$x + 2y \leq 8$

$x + y \leq 5$

$x \geq 0$

$y \geq 0$



Modified LP

maximize $3x + 2y$

subject to

$4x + 2y \leq 15$

$x + 2y \leq 8$

$x + y \leq 5$

$x \geq 0$

$y \geq 0$

Optimal solution: x = 2.5, y = 2.5

Solution value = 7.5 + 5 = 12.5

Half paintings?

Integer (linear) program

maximize $3x + 2y$

subject to

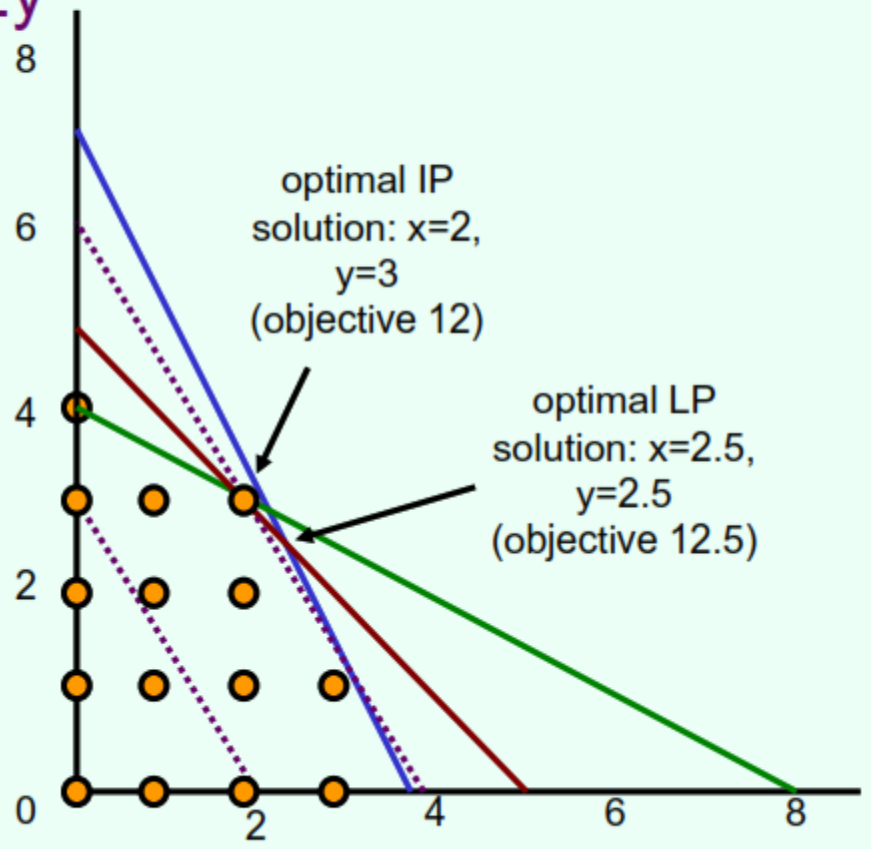
$4x + 2y \leq 15$

$x + 2y \leq 8$

$x + y \leq 5$

$x \geq 0$, integer

$y \geq 0$, integer



Mixed integer (linear) program

maximize $3x + 2y$

subject to

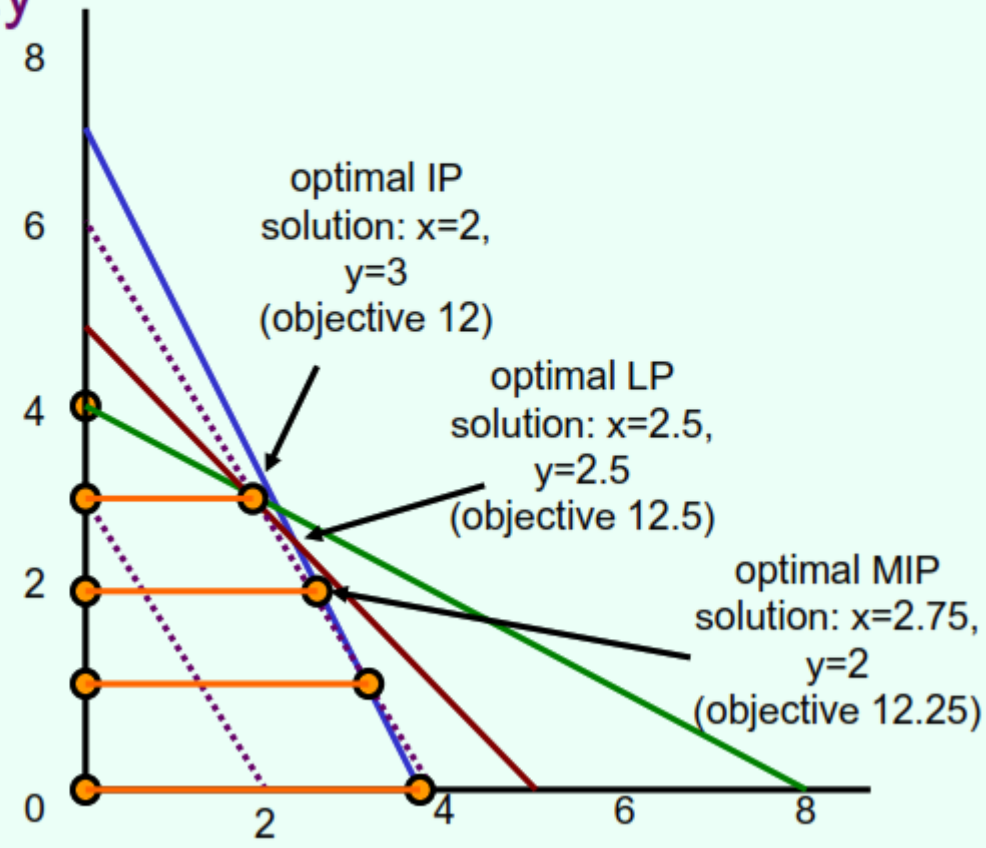
$4x + 2y \leq 15$

$x + 2y \leq 8$

$x + y \leq 5$

$x \geq 0$

$y \geq 0$, integer



Solving linear/integer programs

- Linear programs can be solved efficiently
  - Simplex, ellipsoid, interior point methods…
- (Mixed) integer programs are NP-hard to solve
  - Quite easy to model many standard NP-complete problems as integer programs (try it!)
  - Search type algorithms such as branch and bound
- Standard packages for solving these
  - GNU Linear Programming Kit, CPLEX, …
- LP relaxation of (M)IP: remove integrality constraints
  - Gives upper bound on MIP (~admissible heuristic)

Perbedaan mendasar antara solusi LP (Linear Programming) dan IP (Integer Programming) adalah:

1. Daerah solusi dari LP merupakan daerah convex, sedangkan daerah solusi dari IP berupa titik-titik bilangan bulat.

2. Pada IP, walaupun solusi layaknya banyaknya tak berhingga, tetapi solusi optimal hanya terdapat pada titik-titik ekstrim (yang jumlahnya berhingga) dan dapat dengan mudah diselesaikan dengan metode simpleks, sedangkan pada Program Integer, walaupun solusi layaknya berhingga, tetapi untuk menentukannya tidak mudah, sehingga banyak metode pendekatan/heuristic yang dikembangkan untuk menyelesaikan masalah IP.