Latihan soal
Ekspansi Fungsi Ke Bentuk Deret

Oleh:
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1. Tentukan deret dari: $\frac{\sin\sqrt{x}}{\sqrt{x}}$

Penyelesaian:

$$\frac{\sin\sqrt{x}}{\sqrt{x}} = \frac{1}{\sqrt{x}}\left\{\sqrt{x} - \frac{(\sqrt{x})^3}{3!} + \frac{(\sqrt{x})^5}{5!} - \frac{(\sqrt{x})^7}{7!} + \cdots\right\} = 1 - \frac{x}{3!} + \frac{x^2}{5!} - \frac{x^3}{7!} + \cdots$$

2. Tentukan deret dari: $\sinh x$

Penyelesaian:

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$= \frac{1}{2}\left\{\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{4^3}{4!} + \frac{x^5}{5!} + \cdots\right)\right.$$

$$\left. - \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{4^3}{4!} - \frac{x^5}{5!} + \cdots\right)\right\}$$

$$= \frac{1}{2}\left(2x + 2\frac{x^3}{3!} + 2\frac{x^5}{5!} + \cdots\right)$$

$$= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

3. Tentukan deret dari: $\frac{e^x}{1-x}$

Penyelesaian:

$$\frac{e^x}{1-x} = e^x(1-x)^{-1}$$

$$= \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{4^3}{4!} + \frac{x^5}{5!} + \cdots\right)\left(1 - x + \frac{2x^2}{2!} - \frac{2.3x^3}{3!} + \frac{2.3.4x^4}{4!} - \frac{2.3.4.5.\,x^5}{5!} + \cdots\right)$$

$$= \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{4^3}{4!} + \frac{x^5}{5!} + \cdots\right)(1 - x + x^2 - x^3 + x^4 - x^5 + \cdots)$$

4. Tentukan nilai dari: $\ln 3 + \frac{(\ln 3)^2}{2!} + \frac{(\ln 3)^3}{3!} + \frac{(\ln 3)^4}{4!} + \cdots$

Penyelesaian:

- $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{4^3}{4!} + \frac{x^5}{5!} + \cdots$
- $e^{\ln 3} = 1 + \ln 3 + \frac{(\ln 3)^2}{2!} + \frac{(\ln 3)^3}{3!} + \frac{(\ln 3)^4}{4!} + \cdots$
- $\ln 3 + \frac{(\ln 3)^2}{2!} + \frac{(\ln 3)^3}{3!} + \frac{(\ln 3)^4}{4!} + \cdots = e^{\ln 3} - 1 = 3 - 1 = 2$

5. Tentukan nilai dari : $\frac{\pi^2}{3!} - \frac{\pi^3}{5!} + \frac{\pi^4}{7!} + \cdots$

Penyelesaian:

- $1 - \frac{\sin \pi}{\pi} = 1 - \frac{1}{\pi}\left\{\pi - \frac{\pi^3}{3!} + \frac{\pi^5}{5!} - \frac{\pi^7}{7!} + \cdots\right\} = 1 - 1 + \frac{\pi^2}{3!} - \frac{\pi^4}{5!} + \frac{\pi^6}{7!} + \cdots = \frac{\pi^2}{3!} - \frac{\pi^4}{5!} + \frac{\pi^6}{7!} + \cdots$
- Sehingga: $\frac{\pi^2}{3!} - \frac{\pi^4}{5!} + \frac{\pi^6}{7!} + \cdots = 1 - \frac{\sin \pi}{\pi} = 1 - \frac{0}{\pi} = 1$

6. Tentukan nilai dari: $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{1-\cos^2 x}\right)$

Penyelesaian:

- $1-\cos^2 x=\sin^2 x=\left(x-\frac{x^3}{3!}+\cdots\right)^2=x^2-\frac{2x^4}{3!}+\frac{x^6}{3!3!}+\cdots=x^2-\frac{x^4}{3}+\frac{x^6}{3!3!}+\cdots$

- $\frac{1}{x^2}-\frac{1}{1-\cos^2 x}=\frac{1}{x^2}-\frac{1}{\sin^2 x}=\frac{\sin^2 x-x^2}{x^2\sin^2 x}=\frac{x^2-\frac{x^4}{3}+\frac{x^6}{3!3!}+\cdots-x^2}{x^2\left(x^2-\frac{x^4}{3}+\frac{x^6}{3!3!}+\cdots\right)}=\frac{-\frac{x^4}{3}+\frac{x^6}{3!3!}+\cdots}{x^4-\frac{x^6}{3}+\frac{x^8}{3!3!}+\cdots}=\frac{-\frac{1}{3}+\frac{x^2}{3!3!}+\cdots}{1-\frac{x^2}{3}+\frac{x^4}{3!3!}+\cdots}$

- $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{1-\cos^2 x}\right)=\lim_{x\to 0}\left(\frac{-\frac{1}{3}+\frac{x^2}{3!3!}+\cdots}{1-\frac{x^2}{3}+\frac{x^4}{3!3!}+\cdots}\right)=\frac{-\frac{1}{3}+\frac{0}{3!3!}+\cdots}{1-\frac{0}{3}+\frac{0}{3!3!}+\cdots}=-\frac{1}{3}$

7. Tentukan nilai dari: $\lim_{x \to 0} \left(\frac{\ln(1+x)}{x^2} - \frac{1}{x} \right)$

Penyelesaian:

- $\frac{\ln(1+x)}{x^2} - \frac{1}{x} = \frac{\ln(1+x) - x}{x^2} = \frac{x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots - x}{x^2} = \frac{-\frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots}{x^2} = -\frac{1}{2} + \frac{x}{3} - \frac{x^2}{4} + \cdots$
- $\lim_{x \to 0} \left(-\frac{1}{2} + \frac{x}{3} - \frac{x^2}{4} + \cdots \right) = -\frac{1}{2} + \frac{0}{3} - \frac{0}{4} + \cdots = -\frac{1}{2}$

8. Tentukan nilai dari: $\left.\frac{d^6\left(x^4e^{x^2}\right)}{dx^6}\right|_{x=0}$

Penyelesaian:

$$\left.\frac{d^6\left(x^4e^{x^2}\right)}{dx^6}\right|_{x=0} = \left.\frac{d^6}{dx^6}\left[x^4\left(1+x^2+\frac{x^4}{2}+\frac{x^6}{3!}+\cdots\right)\right]\right|_{x=0}$$

$$= \left.\frac{d^6}{dx^6}\left[x^4+x^6+\frac{x^8}{2}+\frac{x^{10}}{3!}+\cdots\right]\right|_{x=0}$$

$$= \left.\frac{d^5}{dx^5}\left[4x^3+6x^5+4x^7+\frac{10x^9}{3!}+\cdots\right]\right|_{x=0}$$

$$= \left.\frac{d^4}{dx^4}\left[4.3x^2+6.5x^4+4.7x^6+10.3x^8+\cdots\right]\right|_{x=0}$$

$$= \left.\frac{d^3}{dx^3}\left[4.3.2x+6.5.4x^3+4.7.6x^5+10.3.8x^7+\cdots\right]\right|_{x=0}$$

$$= \left.\frac{d^2}{dx^2}\left[4.3.2+6.5.4.3x^2+4.7.6.5x^4+10.3.8.7x^6+\cdots\right]\right|_{x=0}$$

$$= \left.\frac{d}{dx}\left[6.5.4.3.2x+4.7.6.5.4x^3+10.3.8.7.6x^5+\cdots\right]\right|_{x=0}$$

$$= \left.\left[6.5.4.3.2+4.7.6.5.4.3x^2+10.3.8.7.6.5x^4+\cdots\right]\right|_{x=0} = 6.5.4.3.2 = 6! = 720$$