Chapter Fifteen

Market Risk

INTRODUCTION

market risk
Risk related to the uncertainty of an FI's earnings on its trading portfolio caused by changes in market conditions.

Market risk can be defined as the risk related to the uncertainty of an FI's earnings on its trading portfolio caused by changes, and particularly extreme changes, in market conditions such as the price of an asset, interest rates, market volatility, and market liquidity.[1] Thus, risks such as interest rate risk (discussed in Chapters 8 and 9), credit risk (including credit risk from sovereign debt exposure, discussed in Chapters 10, 11, and 14), liquidity risk (discussed in Chapter 12), and foreign exchange risk (discussed in Chapter 13) affect market risk. However, market risk emphasizes the risks to FIs that actively trade assets and liabilities (and derivatives) rather than hold them for longer-term investment, funding, or hedging purposes.

Indeed, market risk was at the heart of much of the losses associated with the financial crisis. Signs of significant problems in the U.S. economy first arose in late 2006 and the first half of 2007 when home prices plummeted and defaults by subprime mortgage borrowers began to affect the mortgage lending industry as a whole, as well as other parts of the economy, noticeably. As mortgage borrowers defaulted on their mortgages, financial institutions that held and actively traded these mortgages and mortgage-backed securities started announcing huge losses on them. Losses from the falling value of subprime mortgages and securities backed by these mortgages reached more than $1 trillion worldwide through mid-2009. Investment banks and securities firms were major traders of mortgage-backed securities. As mortgage borrowers defaulted on their mortgages, investment banks were particularly hard hit with huge losses on the mortgages and securities backing them.

A prime example of the losses incurred is that of Bear Stearns. In the summer of 2007, two Bear Stearns hedge funds suffered heavy market risk related losses on investments in the subprime mortgage market. The two funds filed for bankruptcy in the fall of 2007. Bear Stearns' market value was hurt badly from these losses. The losses became so great that in March 2008 J.P. Morgan Chase and the Federal Reserve stepped in to rescue the then fifth largest investment bank in the United States before it failed or was sold piecemeal to various financial institutions. The market risk meltdown continued through the summer and fall of 2008. On Monday, September 15, Lehman Brothers (the 158-year-old investment bank) filed for bankruptcy, Merrill Lynch was bought by Bank of America, AIG (one of the world's

[1] Market risk used by FI managers and regulators is not synonymous with systematic market risk analyzed by investors in securities markets. Systematic (market) risk reflects the co-movement of a security with the market portfolio (reflected by the security's beta), although beta is used to measure the market risk of equities, as noted below.

largest insurance companies) met with federal regulators to raise desperately needed cash, and Washington Mutual (the largest savings institution in the United States) was acquired by J.P. Morgan Chase. As news spread that Lehman Brothers would not survive, financial institutions moved to disentangle trades made with Lehman. The Dow fell more than 500 points, the largest drop in over seven years. Also by Wednesday, tension mounted around the world. Stock markets saw huge swings in value as investors tried to sort out who might survive (markets from Russia to Europe were forced to suspend trading as stock prices plunged). By mid-September, financial markets froze and banks stopped lending to each other at anything but exorbitantly high rates. Market risk was the root cause of much of this market failure and substantial losses incurred by financial institutions.

Conceptually, an FI's trading portfolio can be differentiated from its investment portfolio on the basis of time horizon and liquidity. The trading portfolio contains assets, liabilities, and derivative contracts that can be quickly bought or sold on organized financial markets (such as long and short positions in bonds, commodities, foreign exchange, equity securities, interest rate swaps, and options). Further, with the increasing securitization of bank loans (e.g., mortgages), more and more assets have become liquid and tradable (e.g., mortgage-backed securities). Additionally, many large syndicated loans are often partly sold off (participations in loans, see Chapter 25). The lead bank usually retains a percentage (normally 15 to 30 percent). These syndicated loans can be viewed as held for sale and thus part of the trading book. The investment portfolio (or, in the case of banks, the so-called banking book) contains assets and liabilities that are relatively illiquid and held for longer holding periods (such as consumer and commercial loans, retail deposits, and branches). Table 15–1 shows a hypothetical breakdown between banking book and trading book assets and liabilities. Note that capital produces a cushion against losses on either the banking or trading books—see Chapter 20.

Income from trading activities is increasingly replacing income from traditional FI activities of deposit taking and lending. The resulting earnings uncertainty, or market risk, can be measured over periods as short as a day or as long as a year. While bank regulators have normally viewed tradable assets as those being held for horizons of less than one year, private FIs take an even shorter-term view. In particular, FIs are concerned about the fluctuation in value—or value at risk (VAR)—of their trading account assets and liabilities for periods as short as one day—especially if such fluctuations pose a threat to their solvency. Moreover, market risk can be defined in absolute terms as a *dollar* exposure amount or as a

TABLE 15–1
The Investment (Banking) Book and Trading Book of a Commercial Bank

	Assets	**Liabilities**
Banking Book	Cash Loans Premises and equipment Other illiquid assets	Deposits Other illiquid borrowed funds Capital
Trading Book	Bonds (long) Commodities (long) FX (long) Equities (long) Mortgage-backed securities (long)	Bonds (short) Commodities (short) FX (short) Equities (short)
	Derivatives* (long)	Derivatives* (short)

* Derivatives are off-balance-sheet (as discussed in Chapter 6).

relative amount against some benchmark. For example, Bank of America's 2011 Annual Report (p. 114) states, "To evaluate risk in our trading activities, we focus on the actual and potential volatility of individual positions as well as portfolios. VAR is the key statistic used to measure market risk. In order to manage day-to-day risks, VAR is subject to trading limits both for our overall trading portfolio and within individual businesses." In recent years, market risk of FIs has raised considerable concern among regulators as well. So important is market risk in determining the viability of an FI that since 1998, U.S. regulators have included market risk in determining the required level of capital an FI must hold.

Further, part of the Wall Street Reform and Consumer Protection Act, passed in 2010 in response to the financial crisis, is the Volker Rule (to be implemented by banks by July 2014 at the earliest). The Volcker Rule prohibits U.S. depository institutions (DIs) from engaging in proprietary trading (i.e., any transaction to purchase or sell as a principal for the trading account of the bank) and from investing in hedge funds or private equity funds. However, a DI may organize and offer a hedge fund or private equity fund if it does not have an ownership interest in the fund except for a seed investment that is limited to no more than 3 percent of total ownership interest of the fund within one year after the date of establishment of the fund. Additionally, the DI's overall investment in hedge funds or private equity funds may not exceed 3 percent of the DI's tier 1 capital. The rule was named after former Federal Reserve Chairman Paul Volcker, who had been outspoken in his claims that such activities played a major part in the financial crisis. The Volker Rule is intended to restrict speculative trades made by depository institutions with their own money and, thus, is intended to reduce market risk at depository institutions. However, some have said the new rules are anti–bank specialness. This argument stems from the fact that the new rules on FIs' trading portfolios virtually force FIs to hold a matched maturity book. This limits the traditional specialness in bank maturity intermediation—that is, borrow in the short-term funds market to lend in the long-term market.

Table 15–2 summarizes several benefits of measuring market risk, including providing management with information on the extent of market risk exposure, market risk limits, resource allocation, and performance evaluation, as well as providing regulators with information on how to protect banks and the financial system against failure due to extreme market risk. The sections that follow concentrate on absolute dollar measures of market risk. We look at three major approaches that are being used to measure market risk: RiskMetrics, historic or back simulation, and Monte Carlo simulation. The link between market risk and required capital levels is also discussed in the chapter.

CALCULATING MARKET RISK EXPOSURE

Large commercial banks, investment banks, insurance companies, and mutual funds have all developed market risk models. In the development of these models, four major approaches have been followed:

- RiskMetrics (or the variance/covariance approach).
- Historic or back simulation.
- Monte Carlo simulation.
- Expected shortfall.

TABLE 15–2
Benefits of Market Risk Measurement (MRM)

1. *Management information.* MRM provides senior management with information on the risk exposure taken by FI traders. Management can then compare this risk exposure to the FI's capital resources.
2. *Setting limits.* MRM considers the market risk of traders' portfolios, which will lead to the establishment of economically logical position limits per trader in each area of trading.
3. *Resource allocation.* MRM involves the comparison of returns to market risks in different areas of trading, which may allow for the identification of areas with the greatest potential return per unit of risk into which more capital and resources can be directed.
4. *Performance evaluation.* MRM considers the return-risk ratio of traders, which may allow a more rational bonus (compensation) system to be put in place. That is, those traders with the highest returns may simply be the ones who have taken the largest risks. It is not clear that they should receive higher compensation than traders with lower returns and lower risk exposures.
5. *Regulation.* With the Bank for International Settlements (BIS) and Federal Reserve currently regulating market risk through capital requirements (discussed later in this chapter), private sector benchmarks are important, since it is possible that regulators will overprice some risks. MRM conducted by the FI can be used to point to potential misallocations of resources as a result of prudential regulation. As a result, in certain cases regulators are allowing banks to use their own (internal) models to calculate their capital requirements.

The first three models offer different methods used to calculate value at risk. We consider RiskMetrics first and then compare it with other internal model approaches, such as historic or back simulation. The expected shortfall model (also called the conditional value at risk) is an alternative to the traditional value at risk measure that is more sensitive to the shape of the loss tail of the probability distribution of returns. Starting in January 2013, regulators have replaced value at risk with the expected shortfall measure as the main measure of market risk.

THE RISKMETRICS MODEL

The ultimate objective of market risk measurement models can best be seen from the following question from an FI manager: "I am X% sure that the FI will not lose more than $VAR in the next T days." In a nutshell, the FI manager wants a single *dollar* number that tells him the FI's market risk exposure over the next days—especially if those days turn out to be extremely "bad" days.

This can be nontrivial, given the extent of a large or even mid-sized FI's trading business. When JPM developed its RiskMetrics model in 1994 it had 14 active trading locations with 120 independent units trading fixed-income securities, foreign exchange, commodities, derivatives, emerging-market securities, and proprietary assets.[2] In 2011, J.P. Morgan Chase operated worldwide and held a trading portfolio worth over more than $444 billion. This scale and variety of activities is typical of the major money center banks, large overseas banks (e.g., Deutsche Bank and Barclays), and major insurance companies and investment banks.

[2] J.P. Morgan (JPM) first developed RiskMetrics in 1994. In 1998 the Corporate Risk Management Department that operated RiskMetrics was spun off from J.P. Morgan and became known as RiskMetrics Group. The company went public in January 2008 and was subsequently acquired, in June 2010, by MSCI. The material presented in this chapter is an overview of the RiskMetrics model. The details, additional discussion, and examples are found in "Return to RiskMetrics: The Evolution of a Standard," April 2001, available at the J.P. Morgan Chase website, **www.jpmorganchase.com**, or **www.riskmetrics.com**.

Here, we will concentrate on measuring the market risk exposure of a major FI on a daily basis using the RiskMetrics approach. As will be discussed later, measuring the risk exposure for periods longer than a day (e.g., five days) is under certain assumptions a simple transformation of the daily risk exposure number. Essentially, the FI is concerned with how to preserve equity if market conditions move adversely tomorrow; that is:

$$\text{Market risk} = \text{Estimated potential loss under adverse circumstances}$$

daily earnings at risk (DEAR)
Market risk exposure over the next 24 hours.

More specifically, the market risk is measured in terms of the FI's **daily earnings at risk (DEAR)** and has three components:

$$\begin{matrix}\text{Daily earnings}\\ \text{at risk}\end{matrix} = \begin{pmatrix}\text{Dollar market}\\ \text{value of}\\ \text{the position}\end{pmatrix} \times \begin{pmatrix}\text{Price}\\ \text{sensitivity of}\\ \text{the position}\end{pmatrix} \times \begin{pmatrix}\text{Potential}\\ \text{adverse move}\\ \text{in yield}\end{pmatrix}$$

Since price sensitivity multiplied by adverse yield move measures the degree of price volatility of an asset, we can also write this equation as:

$$\begin{matrix}\text{Daily earnings}\\ \text{at risk}\end{matrix} = \begin{pmatrix}\text{Dollar market}\\ \text{value of}\\ \text{the position}\end{pmatrix} \times \begin{pmatrix}\text{Price}\\ \text{volatility}\end{pmatrix} \tag{1}$$

How price sensitivity and an adverse yield move will be measured depends on the FI and its choice of a price-sensitivity model as well as its view of what exactly is a potentially adverse price (yield) move.

We concentrate on how the RiskMetrics model calculates daily earnings at risk in three trading areas—fixed income, foreign exchange (FX), and equities—and then on how it estimates the aggregate risk of the entire trading portfolio to meet an FI manager's objective of a single aggregate dollar exposure measure across the whole bank on a given day.

The Market Risk of Fixed-Income Securities

Suppose an FI has a $1 million market value position in zero-coupon bonds of seven years to maturity with a face value of $1,631,483. Today's yield on these bonds is 7.243 percent per year.[3] These bonds are held as part of the trading portfolio. Thus,

$$\text{Dollar market value of position} = \$1 \text{ million}$$

The FI manager wants to know the potential exposure the FI faces should interest rates move against the FI as the result of an adverse or reasonably bad market

[3] The face value of the bonds is $1,631,483—that is, $1,631,483/(1.07243)^7 = $1,000,000 market value. In the original model, prices were determined using a discrete rate of return, R_j. In the April 2001 document "Return to RiskMetrics: The Evolution of a Standard," prices are determined using a continuously compounded return, e^{-rt}. The change was implemented because continuous compounding has properties that facilitate mathematical treatment. For example, the logarithmic return on a zero-coupon bond equals the difference of interest rates multiplied by the maturity of the bond. That is:

$$\ln\left(\frac{e^{-\bar{r}t}}{e^{-rt}}\right) = -(\bar{r} - r)t$$

where $\bar{r}$ is the expected return.

move the next day. How much the FI will lose depends on the bond's price volatility. From the duration model in Chapter 9 we know that:

$$\begin{aligned}\text{Daily price volatility} &= (\text{Price sensitivity to a small change in yield}) \\ &\quad \times (\text{Adverse daily yield move}) \\ &= (MD) \times (\text{Adverse daily yield move})\end{aligned} \tag{2}$$

The modified duration (MD) of this bond is:[4]

$$MD = \frac{D}{1+R} = \frac{7}{(1.07243)} = 6.527$$

given that the yield on the bond is $R = 7.243$ percent. To estimate price volatility, multiply the bond's MD by the expected adverse daily yield move.

EXAMPLE 15–1
Daily Earnings at Risk on Fixed-Income Securities

Suppose we define bad yield changes such that there is only a 1 percent chance that the yield changes will exceed this amount in either direction—or, since we are concerned only with bad outcomes, and we are long in bonds, that there is 1 chance in 100 (or a 1 percent chance) that the next day's yield increase (or shock) will exceed this given adverse move.

If we assume that yield changes are normally distributed,[5] we can fit a normal distribution to the histogram of recent past changes in seven-year zero-coupon interest rates (yields) to get an estimate of the size of this adverse rate move. From statistics, we know that (the middle) 98 percent of the area under the normal distribution is to be found within $\pm$ 2.33 standard deviations (σ) from the mean—that is, 2.33σ—and 2 percent of the area under the normal distribution is found beyond $\pm 2.33\sigma$ (1 percent under each tail, -2.33σ and $+2.33\sigma$, respectively).[6] Suppose that during the last year the mean change in daily yields on seven-year zero-coupon bonds was 0 percent,[7] while the standard deviation was 10 basis points (or 0.001). Thus, 2.33σ is 23.3 basis points (bp).[8] In other words, over the last year, daily yields on seven-year, zero-coupon bonds have fluctuated (either positively or negatively) by more than 23.3 bp 2 percent of the time. Adverse moves in yields are those that decrease the value of the security (i.e., the yield increases). These occurred 1 percent of the time, or 1 in 100 days. This is shown in Figure 15–1.

[4] Assuming annual compounding for simplicity.

[5] In reality, many asset return distributions—such as exchange rates and interest rates—have "fat tails." Thus, the normal distribution will tend to underestimate extreme outcomes. This is a major criticism of the RiskMetrics modeling approach and a major reason for regulators' move to the use of expected shortfall from the traditional value at risk measure of market risk. Further, the original CreditMetrics calculation of DEAR incorporated a 5 percent chance that the next day's yield increase will exceed this given adverse move. The use of 1 percent to measure adverse moves produces a more conservative estimate of an FI's value at risk.

[6] For 95 percent of the area under the normal distribution (2.5 percent under each tail), we use $\pm$ 1.96, and for 90 percent of the area (5 percent under each tail) we use $\pm$ 1.65. CreditMetrics originally used the 90 percent confidence level.

[7] If the mean were nonzero (e.g., -1 basis point), this could be added to the 23.3 bp (i.e., 22.3 bp) to project the yield shock.

[8] RiskMetrics weights more recent observations more highly than past observations (this is called *exponential weighting).* This allows more recent news to be more heavily reflected in the calculation of σ. Regular σ calculations put an equal weight on all past observations.

We can now calculate the potential daily price volatility on seven-year discount bonds using equation (2) as:

$$\begin{aligned}\text{Price volatility} &= (MD) \times (\text{Potential adverse move in yield})\\ &= (6.527) \times (0.00233)\\ &= 0.01521 \text{ or } 1.521\%\end{aligned}$$

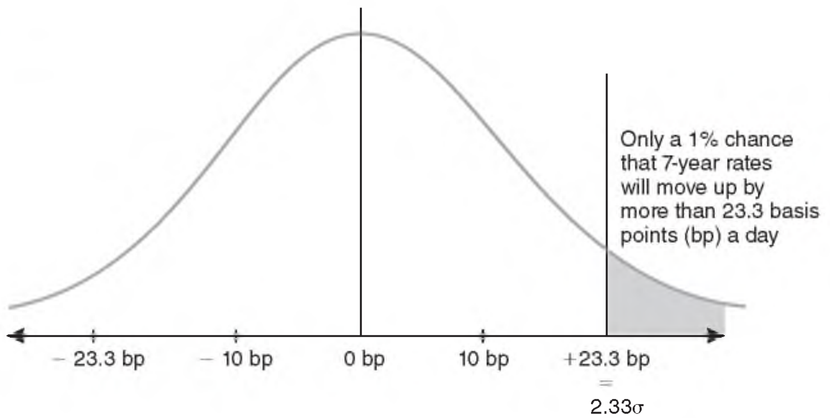
Given this price volatility and the initial market value of the seven-year bond portfolio, then equation (1) can be used to calculate the daily earnings at risk as:[9]

$$\begin{aligned}\text{Daily earnings at risk} &= (\text{Dollar market value of position}) \times (\text{Price volatility})\\ &= (\$1{,}000{,}000) \times (0.01521)\\ &= \$15{,}210\end{aligned}$$

That is, the potential daily loss in earnings on the $1 million position is $15,210 if the 1 bad day in 100 occurs tomorrow.

We can extend this analysis to calculate the potential loss over 2, 3, . . . *N* days. If we assume that yield shocks are independent and daily volatility is approximately constant,[10] and that the FI is locked in to holding this asset for *N* number

FIGURE 15–1
Adverse Rate Move, Seven-Year Rates



[9] Since we are calculating loss, we drop the minus sign here.

[10] The assumptions that daily volatility is constant and that there is no autocorrelation in yield shocks are strong assumptions. Much recent literature suggests that shocks are autocorrelated in many asset markets over relatively long horizons. To understand why we take the square root of *N,* consider a five-day holding period. The σ_5^2, or five-day variance of asset returns, will equal the current one-day variance, σ_1^2, times 5 under the assumptions of constant daily variance and no autocorrelation in shocks, or:

$$\sigma_5^2 = \sigma_1^2 \times 5$$

The standard deviation of this equation is:

$$\sigma_5 = \sigma_1 \times \sqrt{5}$$

or since DEAR is measured in the same dimensions as a standard deviation (σ), in the terminology of RiskMetrics, the five-day value at risk is:

$$\text{5-day } VAR = DEAR \times \sqrt{5}$$

of days, then the N-day market value at risk (VAR) is related to daily earnings at risk (DEAR) by:

$$N\text{-day } VAR = DEAR \times \sqrt{N} \tag{3}$$

That is, the earnings the FI has at risk, should interest rate yields move against the FI, are a function of the value or earnings at risk for one day (DEAR) and the (square root of the) number of days that the FI is forced to hold the securities because of an illiquid market. Specifically, DEAR assumes that the FI can sell all the bonds tomorrow, even at the new lower price. In reality, it may take many days for the FI to unload its position. This relative illiquidity of a market exposes the FI to magnified losses (measured by the square root of N).[11] If N is five days, then:

$$5\text{-day } VAR = \$15{,}210 \times \sqrt{5} = \$34{,}011$$

If N is 10 days, then:[12]

$$10\text{-day } VAR = \$15{,}210 \times \sqrt{10} = \$48{,}098$$

In the preceding calculations, we estimated price sensitivity using modified duration. However, the RiskMetrics model generally prefers using the present value of cash flow changes as the price-sensitivity weights over modified durations. Essentially, each cash flow is discounted by the appropriate zero-coupon rate to generate the daily earnings at risk measure. If we used the direct cash flow calculation in this case, the loss would be $15,209.63.[13] The estimates in this case are very close.

Foreign Exchange

Large FIs also actively trade in foreign exchange (FX). Remember that:

$$DEAR = (\text{Dollar value of position}) \times (\text{Price volatility})$$

EXAMPLE 15–2
Daily Earnings at Risk of Foreign Exchange Contracts

Suppose the FI had a €800,000 trading position in spot euros at the close of business on a particular day. The FI wants to calculate the daily earnings at risk from this position (i.e., the risk exposure on this position should the next day be a bad day in the FX markets with respect to the value of the euro against the dollar).

The first step is to calculate the dollar value of the position:

$$\text{Dollar equivalent value of position} = (\text{FX position}) \times (\$ \text{ per unit of foreign currency})$$

Suppose for simplicity that the exchange rate is €0.8000/$1 or $1.25/€ at the daily close; then:

$$\begin{aligned}\text{Dollar value of position} &= €800{,}000 \times \$1.25/€ \\ &= \$\ 1 \text{ million}\end{aligned}$$

[11] In practice, a number of FIs calculate N internally by dividing the position held in a security by the median daily volume of trading of that security over recent days. Thus, if trading volume is low because of a "one-way market," in that most people are seeking to sell rather than buy, then N can rise substantially; that is, N = ($ position in security/median daily $ volume of trading).

[12] Under the BIS market risk capital requirements, a 10-day holding period ($N = 10$) is assumed to measure exposure.

[13] The initial market value of the seven-year zero was $1,000,000, or $\$1{,}631{,}483/(1.07243)^7$. The (loss) effect on each $1 (market value) invested in the bond of a rise in rates by 1 bp from 7.243 percent to 7.253 percent is 0.00065277. However, the adverse rate move is 23.3 bp. Thus:

$$DEAR = (\$1 \text{ million}) \times (0.00065277) \times (23.3) = \$15{,}210$$

Suppose that, looking back at the €/$ exchange rate over the past year, we find that the volatility, or standard deviation (σ), of daily percentage changes in the spot exchange rate was 56.5 bp. However, suppose that the FI is interested in adverse moves—that is, bad moves that will not occur more than 1 percent of the time, or 1 day in every 100. Statistically speaking, if changes in exchange rates are historically "normally" distributed, the exchange rate must change in the adverse direction by 2.33σ (2.33 × 56.5 bp) for this change to be viewed as likely to occur only 1 day in every 100 days:[14]

$$\text{FX volatility} = 2.33 \times 56.5 \text{ bp} = 131.645 \text{ bp}$$

In other words, during the last year, the euro declined in value against the dollar by 131.645 bp 1 percent of the time. As a result:

$$\begin{aligned} DEAR &= (\text{Dollar value of position}) \times (\text{FX volatility}) \\ &= (\$1 \text{ million}) \times (0.0131645) \\ &= \$13,164 \end{aligned}$$

This is the potential daily earnings exposure to adverse euro to dollar exchange rate changes for the FI from the €1.4 million spot currency holdings.

Equities

Many large FIs also take positions in equities. As is well known from the Capital Asset Pricing Model (CAPM), there are two types of risk to an equity position in an individual stock *i*:

$$\begin{aligned} \text{Total risk} &= \text{Systematic risk} + \text{Unsystematic risk} \\ (\sigma_{it}^2) &= (\beta_i^2 \sigma_{mt}^2) + (\sigma_{eit}^2) \end{aligned} \quad \textbf{(4)}$$

beta
Systematic (undiversifiable) risk reflecting the co-movement of the returns on a specific stock with returns on the market portfolio.

Systematic risk reflects the co-movement of that stock with the market portfolio reflected by the stock's **beta** (β_i) and the volatility of the market portfolio (σ_{mt}), while unsystematic risk is specific to the firm itself (σ_{eit}).

In a very well diversified portfolio, unsystematic risk (σ^2_{eit}) can be largely diversified away (i.e., will equal zero), leaving behind systematic (undiversifiable) market risk ($\beta^2_i \sigma^2_{mt}$). If the FI's trading portfolio follows (replicates) the returns on the stock market index, the β of that portfolio will be 1, since the movement of returns on the FI's portfolio will be one to one with the market,[15] and the standard deviation of the portfolio, σ_{it}, will be equal to the standard deviation of the stock market index, σ_{mt}.

[14] Technically, 98 percent of the area under a normal distribution lies between +/− 2.33σ from the mean. This means that 1 percent of the time, daily exchange rate changes will increase by more than 2.33σ, and 1 percent of the time, will decrease by 2.33σ. This case concerns only adverse moves in the exchange rate of euros to dollars (i.e., a depreciation of 2.33σ).

[15] If $\beta \neq 1$, as in the case of most individual stocks, $DEAR$ = dollar value of position × β_i × 2.33σ_m, where β_i is the systematic risk of the ith stock.

EXAMPLE 15–3
Daily Earnings at Risk on Equities

Suppose the FI holds a $1 million trading position in stocks that reflect a U.S. stock market index (e.g., the Wilshire 5000). Then $\beta = 1$ and the DEAR for equities is:

$$\begin{aligned} DEAR &= (\text{Dollar market value of position}) \times (\text{Stock market return volatility}) \\ &= (\$1{,}000{,}000) \times (2.33\ \sigma_m) \end{aligned}$$

If, over the last year, the σ_m of the daily returns on the stock market index was 200 bp, then $2.33\sigma_m = 466$ bp (i.e., the adverse change or decline in the daily return on the stock market exceeded 466 bp only 1 percent of the time). In this case:

$$\begin{aligned} DEAR &= (\$1{,}000{,}000) \times (0.0466) \\ &= \$46{,}600 \end{aligned}$$

That is, the FI stands to lose at least $46,600 in earnings if adverse stock market returns materialize tomorrow.[16]

In less well diversified portfolios or portfolios of individual stocks, the effect of unsystematic risk σ_{eit} on the value of the trading position would need to be added. Moreover, if the CAPM does not offer a good explanation of asset pricing compared with, say, multi-index arbitrage pricing theory (APT), a degree of error will be built into the DEAR calculation.[17]

Portfolio Aggregation

The preceding sections analyzed the daily earnings at risk of individual trading positions. The examples considered a seven-year, zero-coupon, fixed-income security ($1 million market value); a position in spot euros ($1 million market value); and a position in the U.S. stock market index ($1 million market value). The individual DEARs were:

1. Seven-year, zero-coupon bonds = $15,210
2. Euro spot = $13,164
3. U.S. equities = $46,600

However, senior management wants to know the aggregate risk of the entire trading position. To calculate this, we *cannot* simply sum the three DEARs—$15,210 + $13,164 + $46,600 = $74,974—because that ignores any degree of offsetting covariance or correlation among the fixed-income, FX, and equity trading positions. In particular, some of these asset shocks (adverse moves) may be negatively correlated. As is well known from modern portfolio theory, anything but perfect positive correlations among asset shocks will reduce the degree of portfolio risk.

[16] If we consider a single equity security with a beta (β) = 1.25 (i.e., one that is more sensitive than the market, such that as market returns increase [decrease] by 1 percent, the security's return increases [decreases] by 1.25 percent), then with a $1 million investment and the same (assumed) volatility (σ) of 2 percent, the FI would stand to lose at least $58,250 in daily earnings if adverse stock returns materialize (i.e., $DEAR = \$1{,}000{,}000 \times 1.25 \times 2.33 \times 0.02 = \$58{,}250$).

[17] As noted in the introduction, derivatives are also used for trading purposes. In the calculation of its DEAR, a derivative has to be converted into a position in the underlying asset (e.g., bond, FX, or equity).

EXAMPLE 15–4

Calculation of the DEAR of a Portfolio

Table 15–3 shows a hypothetical correlation matrix between daily seven-year, zero-coupon bond yield changes, €/$ spot exchange rate changes, and changes in daily returns on a U.S. stock market index (Wilshire 5000). From Table 15–3, the correlation between the seven-year, zero-coupon bonds and €/$ exchange rates, $\rho_{z,€}$, is negative (−0.2), while U.S. stock return changes with, respectively, seven-year, zero coupon yields, $\rho_{z,U.S.}$, (0.4) and €/$ shocks, $\rho_{U.S.,€}$, (0.1) are positively correlated.

Using this correlation matrix along with the individual asset DEARs, we can calculate the risk or standard deviation of the whole (three-asset) trading portfolio as:[18]

$$DEAR\text{ portfolio} = \begin{bmatrix} [(DEAR_z)^2 + (DEAR_€)^2 + (DEAR_{U.S.})^2 \\ + (2 \times \rho_{z,€} \times DEAR_z \times DEAR_€) \\ + (2 \times \rho_{z,U.S.} \times DEAR_z \times DEAR_{U.S.}) \\ + (2 \times \rho_{U.S.,€} \times DEAR_{U.S.} \times DEAR_€)] \end{bmatrix}^{1/2} \tag{5}$$

This is a direct application of modern portfolio theory (MPT) since DEARs are directly similar to standard deviations. Substituting into equation (5) the calculated individual DEARs, we get:

$$DEAR\text{ portfolio} = \begin{bmatrix} [(15{,}210)^2 + (13{,}164)^2 + (46{,}600)^2 + 2(-0.2)(15{,}210)(13{,}164) \\ + 2(0.4)(15{,}210)(46{,}600) + 2(0.1)(13{,}164)(46{,}600)] \end{bmatrix}^{1/2}$$

$$= \$56{,}443$$

The equation indicates that considering the risk of each trading position as well as the correlation structure among those positions' returns results in a lower measure of portfolio trading risk ($56,443) than when risks of the underlying trading positions (the sum of which was $74,974) are added. A quick check will reveal that had we assumed that all three assets were perfectly positively correlated (i.e., $\rho_{ij} = 1$), DEAR for the portfolio would have been $74,974 (i.e., equal to the sum of the three DEARs). Clearly, even in abnormal market conditions, assuming that asset returns are perfectly correlated will exaggerate the degree of actual trading risk exposure.

Table 15–4 shows the type of spreadsheet used by FIs to calculate DEAR. As you can see, in this example, positions are taken in 13 different country (currency) bonds in eight different maturity buckets.[19] There is also a column for FX risk (and, if necessary, equity risk) in these different country markets, although in this example, the FI has no FX risk exposure (all the cells are empty).

[18] This is a standard relationship from modern portfolio theory in which the standard deviation or risk of a portfolio of three assets is equal to the square root of the sum of the variances of returns on each of the three assets individually plus two times the covariances among each pair of these assets. With three assets there are three covariances. Here we use the fact that a correlation coefficient times the standard deviations on each pair of assets equals the covariance between each pair of assets. Note that DEAR is measured in dollars and has the same dimensions as a standard deviation. We discussed modern portfolio theory in more detail in Chapter 11.

[19] Bonds held with different maturity dates (e.g., six years) are split into two and allocated to the nearest two of the eight maturity buckets (here, five years and seven years) using three criteria: (1) The sum of the current market *value* of the two resulting cash flows must be identical to the market value of the original cash flow; (2) the market *risk* of the portfolio of two cash flows must be identical to the overall market risk of the original cash flow; and (3) the two cash flows have the same *sign* as the original cash flow. See J.P. Morgan, "RiskMetrics—Technical Document," November 1994, and "Return to RiskMetrics: The Evolution of a Standard," April 2001, **www.msci.com**.

TABLE 15–3 **Correlations (ρ_{ij}) among Assets**

	7-Year Zero	€/$1	U.S. Stock Index
7-year zero	—	−0.2	.4
€/$1		—	.1
U.S. stock index			—

TABLE 15–4 **Portfolio DEAR Spreadsheet**

	Interest Rate Risk Notional Amounts (US$ millions equivalents)								Interest	FX Risk		Total	
	1 Month	1 Year	2 Years	3 Years	4 Years	5 Years	7 Years	10 Years	DEAR ($000s)	Spot FX	FX DEAR	Portfolio Effect	Total DEAR
Australia										AUD			
Brazil										BRL			
Canada										CAD			
Denmark	19			−30				11	48	DKK			48
European Union	−19			30				−11	27	EUR			27
Hong Kong										HKD			
Japan										YEN			
Mexico										MXN			
Singapore										SGD			
Sweden										SEK			
Switzerland										CHF			
United Kingdom										GBP			
United States						10		10	76	USD			76
Total						10		10	151				151
						Portfolio effect			(62)				(62)
RISK	DATA	PRINT	CLOSE			Total DEAR ($000s)			89				89

In the example in Table 15–4, while the FI is holding offsetting long and short positions in both Danish bonds and Eurobonds, it is still exposed to trading risks of $48,000 and $27,000, respectively (see the column Interest DEAR). This happens because the European Union yield curve is more volatile than the Danish and shocks at different maturity buckets are not equal. The DEAR figure for a U.S. bond position of long $20 million is $76,000. Adding these three positions yields a DEAR of $151,000. However, this ignores the fact that Danish, European Union, and U.S. yield shocks are not perfectly correlated. Allowing for diversification effects (the portfolio effect) results in a total DEAR of only $89,000. This would be the number reported to the FI's senior management. Most financial institutions establish limits for value at risk, daily earnings at risk, position limits, and dollar trading loss limits for their trading portfolios. Actual activity compared with these limits is then monitored daily. Should a risk exposure level exceed approved limit levels, management must provide a strategy for bringing risk levels within approved limits. Table 15–5 reports the average, minimum, and maximum daily earnings at risk for several large U.S. commercial banks in 2005 and 2011. Note the increase in market risk for all of these FIs over this period. For example, Citigroup was exposed to an average DEAR of $109 million in 2005 and $153 million in 2011. Currently, the number of markets covered by Citigroup's traders and the number of correlations among those markets require the daily production and updating of over 250,000 volatility estimates (σ) and correlations (ρ). These data are updated daily.

TABLE 15–5
Daily Earnings at Risk for Large U.S. Commercial Banks, 2005 and 2011* (in millions of dollars)

Source: Year 2011 and 2005 10-K reports for the respective companies.

Name	Average DEAR for the year	Minimum DEAR during the year	Maximum DEAR during the year
2011:			
Bank of America	$167	$ 75	$319
Citigroup	153	104	205
J.P. Morgan Chase	101	67	147
KeyCorp	2	1	2
Wells Fargo	29	19	42
Sun Trust	5	3	7
2005:			
Bank of America	$ 62	$ 38	$ 92
Citigroup	109	78	157
J.P. Morgan Chase	86	53	130
KeyCorp	2	1	5
Wells Fargo	18	11	24
Sun Trust	4	2	6

* The figures are based on these banks' internal models, i.e., they may be based on methodologies other than RiskMetrics.

Concept Questions

1. What is the ultimate objective of market risk measurement models?
2. Refer to Example 15–1. What is the DEAR for this bond if σ is 15 bp?
3. Refer to Example 15–4. What is the DEAR of the portfolio if the returns on the three assets are independent of each other?

HISTORIC (BACK SIMULATION) APPROACH

A major criticism of RiskMetrics is the need to assume a symmetric (normal) distribution for all asset returns.[20] Clearly, for some assets, such as options and short-term securities (bonds), this is highly questionable. For example, the most an investor can lose if he or she buys a call option on an equity is the call premium.

[20] Another criticism is that VAR models like RiskMetrics ignore the (risk in the) payments of accrued interest on an FI's debt securities. Thus, VAR models will underestimate the true probability of default and the appropriate level of capital to be held against this risk. Also, because of the distributional assumptions, while RiskMetrics produces reasonable estimates of downside risk for FIs with highly diversified portfolios, FIs with small, undiversified portfolios will significantly underestimate their true risk exposure using RiskMetrics. Further, a number of authors have argued that many asset distributions have "fat tails" and that RiskMetrics, by assuming the normal distribution, underestimates the risk of extreme losses. One alternative approach to dealing with the "fat-tail" problem is extreme value theory. Simply put, one can view an asset distribution as being explained by two distributions. For example, a normal distribution may explain returns up to the 95 percent threshold, but for losses beyond that threshold another distribution, such as the generalized Pareto distribution, may provide a better explanation of loss outcomes such as the 99 percent level and beyond. In short, the normal distribution is likely to underestimate the importance and size of observations in the tail of the distribution, which is, after all, what value at risk models are meant to be measuring. Finally, VAR models by definition concern themselves with risk rather than return. It should be noted that minimizing risk may be highly costly in terms of the return the FI gives up. Indeed, there may be many more return–risk combinations preferable to that achieved at the minimum risk point in the trading portfolio. Recent upgrades to RiskMetrics (see the RiskMetrics Web site at **www.msci.com**) allow management to incorporate a return dimension to VAR analysis so that management can evaluate how trading portfolio returns differ as VAR changes.

However, the investor's potential upside returns are unlimited. In a statistical sense, the returns on call options are nonnormal since they exhibit a positive skew.[21]

Because of these and other considerations discussed herein, many FIs that have developed market risk models have employed a historic or back simulation approach. The advantages of this approach are that (1) it is simple, (2) it does not require that asset returns be normally distributed, and (3) it does not require that the correlations or standard deviations of asset returns be calculated.

The essential idea is to take the current market portfolio of assets (FX, bonds, equities, etc.) and revalue them on the basis of the actual prices (returns) that existed on those assets yesterday, the day before that, and so on. Frequently, the FI will calculate the market or value risk of its current portfolio on the basis of prices (returns) that existed for those assets on each of the last 500 days. It will then calculate the 1 percent worst case—the portfolio value that has the 5th lowest value out of 500. That is, on only 5 days out of 500, or 1 percent of the time, would the value of the portfolio fall below this number based on recent historic experience of exchange rate changes, equity price changes, interest rate changes, and so on.

Consider the following simple example in Table 15–6, where a U.S. FI is trading two currencies: the Japanese yen and the Swiss franc. At the close of trading on December 1, 2015, it has a long position in Japanese yen of 500 million and a long position in Swiss francs of 20 million. It wants to assess its VAR. That is, if tomorrow is that 1 bad day in 100 (the 1 percent worst case), how much does it stand to lose on its total foreign currency position? As shown in Table 15–6, six steps are required to calculate the VAR of its currency portfolio. It should be noted that the same methodological approach would be followed to calculate the VAR of any asset, liability, or derivative (bonds, options, etc.) as long as market prices were available on those assets over a sufficiently long historic time period.

- *Step 1: Measure exposures.* Convert today's foreign currency positions into dollar equivalents using today's exchange rates. Thus, an evaluation of the FX position of the FI on December 1, 2015, indicates that it has a long position of $5,000,000 (¥500,000,000/(¥100/$1)) in yen and $18,181,818 (SF20,000,000/(SF1.1/$1)) in Swiss francs.
- *Step 2: Measure sensitivity.* Measure the sensitivity of each FX position by calculating its delta, where delta measures the change in the dollar value of each FX position if the yen or the Swiss franc depreciates (declines in value) by 1 percent against the dollar.[22] As can be seen from Table 15–6, line 6, the delta for the Japanese yen position is −$49,505 (or, (¥500,000,000/(¥101/$1) − ¥500,000,000/(¥100/$1)), and for the Swiss franc position, it is −$180,018 (or (SF20,000,000/(SF1.111/$1) − SF20,000,000/(SF1.1/$1)).
- *Step 3: Measure risk.* Look at the actual percentage changes in exchange rates, ¥/$ and SF/$, on each of the past 500 days. Thus, on November 30, 2015, the yen declined in value against the dollar over the day by 0.5 percent while the Swiss franc declined in value against the dollar by 0.2 percent. (It might be noted that if the currencies were to appreciate in value against the dollar, the sign against the number in row 7 of Table 15–6 would be negative; that is, it

[21] For a normal distribution, its skew (which is the third moment of a distribution) is zero.

[22] That is, in the case of FX, delta measures the dollar change in FX holdings for a 1 percent change in the foreign exchange rate. In the case of equities, it would measure the change in the value of those securities for a 1 percent change in price, while for bonds, it measures the change in value for a 1 percent change in the yield on the bond (note that delta measures sensitivity of a bond's value to a change in yield, not price).

TABLE 15–6 Hypothetical Example of the Historic, or Back Simulation, Approach Using Two Currencies, as of December 1, 2015

	Yen	Swiss Franc
Step 1. Measure exposures		
1. Closing position on December 1, 2015	¥500,000,000	SF20,000,000
2. Exchange rate on December 1, 2015	¥100/$1	SF1.1/$1
3. U.S. $ equivalent position on December 1, 2015	$5,000,000	$18,181,818
Step 2. Measure sensitivity		
4. 1.01 × current exchange rate	¥101/$1	SF1.111/$1
5. Revalued position in $s	$4,950,495	$18,001,800
6. Delta of position ($s) (measure of sensitivity to a 1% adverse change in exchange rate, or row 5 minus row 3)	−$49,505	−$180,018
Step 3. Measure risk of December 1, 2015, closing position using exchange rates that existed on each of the last 500 days		
November 30, 2015	**Yen**	**Swiss Franc**
7. Change in exchange rate (%) on November 30, 2015	0.5%	0.2%
8. Risk (delta × change in exchange rate)	−$24,752.5	−$36,003.6
9. Sum of risks = −$60,756.1		
Step 4. Repeat step 3 for each of the remaining 499 days		
November 29, 2015		
⋮		
April 15, 2014		
⋮		
November 30, 2013		
⋮		
Step 5. Rank days by risk from worst to best		
Date	**Risk ($)**	
1. May 6, 2014	−$119,096	
2. Jan 27, 2015	−$116,703	
3. Dec 1, 2013	−$104,366	
4. Sept 14, 2013	−100,248	
5. Aug 8, 2014	−97,210	
⋮	⋮	
25. Nov 30, 2015	−$60,756.1	
⋮	⋮	
499. April 8, 2015	+$112,260	
500. July 28, 2014	+$121,803	
Step 6. VAR (5th worst day out of last 500)		
VAR = −97,210 (August 8, 2014)		

takes fewer units of foreign currency to buy a dollar than it did the day before). As can be seen in row 8, combining the delta and the actual percentage change in each FX rate means a total loss of $60,756.1 if the FI had held the current ¥500,000,000 and SF20,000,000 positions on that day (November 30, 2015).

- *Step 4: Repeat step 3.* Step 4 repeats the same exercise for the yen and Swiss franc positions but uses actual exchange rate changes on November 29, 2015; November 28, 2015; and so on. That is, we calculate the FX losses and/or gains on each of the past 500 trading days, excluding weekends and holidays, when the FX market is closed. This amounts to going back in time over two years. For each of these days the actual change in exchange rates is calculated (row 7) and multiplied by the deltas of each position (the numbers in row 6 of Table 15–6). These two numbers are summed to attain total risk measures for each of the past 500 days.
- *Step 5: Rank days by risk from worst to best.* These risk measures can then be ranked from worst to best. Clearly the worst-case loss would have occurred on this position on May 6, 2014, with a total loss of $119,096. While this worst case scenario is of interest to FI managers, we are interested in the 1 percent worst case, that is, a loss that does not occur more than 5 days out of the 500 days (5 ÷ 500 = 1 percent). As can be seen, in our example, the 5th worst loss out of 500 occurred on August 8, 2014. This loss amounted to $97,210.
- *Step 6: VAR.* If it is assumed that the recent past distribution of exchange rates is an accurate reflection of the likely distribution of FX rate changes in the future — that exchange rate changes have a stationary distribution—then the $97,210 can be viewed as the FX value at risk (VAR) exposure of the FI on December 1, 2015. That is, if tomorrow (in our case, December 2, 2015) is a bad day in the FX markets, and given the FI's position of long yen 500 million and long Swiss francs 20 million, the FI can expect to lose $97,210 (or more) with a 1 percent probability. This VAR measure can then be updated every day as the FX position changes and the delta changes. For example, given the nature of FX trading, the positions held on December 5, 2015, could be very different from those held on December 1, 2015.[23]

The Historic (Back Simulation) Model versus RiskMetrics

One obvious benefit of the historic, or back simulation, approach is that we do not need to calculate standard deviations and correlations (or assume normal distributions for asset returns) to calculate the portfolio risk figures in row 9 of Table 15–6.[24] A second advantage is that it directly provides a worst-case scenario number, in our example, a loss of $119,096—see step 5. RiskMetrics, since it assumes asset returns are normally distributed (that returns can go to plus and minus infinity), provides no such worst-case scenario number.[25]

The disadvantage of the back simulation approach is the degree of confidence we have in the 1 percent VAR number based on 500 observations. Statistically speaking, 500 observations are not very many, so there will be a very wide confidence

[23] As in RiskMetrics, an adjustment can be made for illiquidity of the market, in this case, by assuming the FI is locked into longer holding periods. For example, if it is estimated that it will take five days for the FI to sell its FX position, then the FI will be interested in the weekly (i.e., five trading days) changes in FX rates in the past. One immediate problem is that with 500 past trading days, only 100 weekly periods would be available, which reduces the statistical power of the VAR estimate (see below).

[24] The reason is that the historic, or back simulation, approach uses actual exchange rates on each day that explicitly include correlations or comovements with other exchange rates and asset returns on that day.

[25] The 1 percent number in RiskMetrics tells us that we will lose more than this amount on 1 day out of every 100. It does not tell us the maximum amount we can lose. As noted in the text, theoretically, with a normal distribution, this could be an infinite amount.

band (or standard error) around the estimated number ($97,210 in our example). One possible solution to the problem is to go back in time more than 500 days and estimate the 1 percent VAR based on 1,000 past daily observations (the 10th worst case) or even 10,000 past observations (the 100th worst case). The problem is that as one goes back farther in time, past observations may become decreasingly relevant in predicting VAR in the future. For example, 10,000 observations may require the FI to analyze FX data going back 40 years. Over this period we have moved through many very different FX regimes: from relatively fixed exchange rates in the 1950–70 period, to relatively floating exchange rates in the 1970s, to more managed floating rates in the 1980s and 1990s, to the abolition of exchange rates and the introduction of the euro in January 2002, to large fluctuations in exchange rates during the financial crisis of 2008–2009. Clearly, exchange rate behavior and risk in a fixed–exchange rate regime will have little relevance to an FX trader or market risk manager operating and analyzing risk in a floating–exchange rate regime.

This seems to confront the market risk manager with a difficult modeling problem. There are, however, at least two approaches to this problem. The first is to weight past observations in the back simulation unequally, giving a higher weight to the more recent past observations. The second is to use a Monte Carlo simulation approach, which generates additional observations that are consistent with recent historic experience. The latter approach, in effect, amounts to simulating or creating artificial trading days and FX rate changes.

The Monte Carlo Simulation Approach

To overcome the problems imposed by a limited number of actual observations, we can generate additional observations (in our example, FX changes). Normally, the simulation or generation of these additional observations is structured using a Monte Carlo simulation approach so that returns or rates generated reflect the probability with which they have occurred in recent historic time periods. The first step is to calculate the historic variance–covariance matrix (Σ) of FX changes. This matrix is then decomposed into two symmetric matrices, A and A'.[26] This allows the FI to generate scenarios for the FX position by multiplying the A' matrix, which reflects the historic volatilities and correlations among FX rates, by a random number vector z:[27] 10,000 random values of z are drawn for each FX exchange rate.[28] This simulation approach results in realistic FX scenarios being generated as historic volatilities and correlations among FX rates are multiplied by the randomly drawn values of z. The VAR of the current position is then calculated as in Table 15–6, except that in the Monte Carlo approach, the VAR is the 100th worst simulated loss out of 10,000.

Monte Carlo simulation is, therefore, a tool for considering portfolio valuation under all possible combinations of factors that determine a security's value. The model generates random market values drawn from the multivariate normal distributions representing each variable. The Industry Perspectives box outlines the process Citigroup follows in estimating VAR using the Monte Carlo simulation approach and gives more detail on its 2011 VARs.

[26] The only difference between A and A' is that the numbers in the rows of A become the numbers in the columns of A'. The technical term for this procedure is the Cholesky decomposition, where $\Sigma = AA'$.

[27] Where z is assumed to be normally distributed with a mean of zero and a standard deviation of 1 or $z \sim N(0, 1)$.

[28] Technically, let y be an FX scenario; then $y = A'z$. For each FX rate, 10,000 values of z are randomly generated to produce 10,000 values of y. The y values are then used to revalue the FX position and calculate gains and losses.

Industry Perspectives Citigroup—Value at Risk

Value at risk (VAR) estimates, at a 99 percent confidence level, the potential decline in the value of a position or a portfolio under normal market conditions. VAR statistics can be materially different across firms due to differences in portfolio composition, differences in VAR methodologies, and differences in model parameters. Citi believes VAR statistics can be used more effectively as indicators of trends in risk taking within a firm, rather than as a basis for inferring differences in risk taking across firms.

Citi uses Monte Carlo simulation, which it believes is conservatively calibrated to incorporate the greater of short-term (most recent month) and long-term (three years) market volatility. The Monte Carlo simulation involves approximately 300,000 market factors, making use of 180,000 time series, with market factors updated daily and model parameters updated weekly.

The conservative features of the VAR calibration contribute approximately 20 percent add-on to what would be a VAR estimated under the assumption of stable and perfectly normally distributed markets. Under normal and stable market conditions, Citi would thus expect the number of days where trading losses exceed its VAR to be less than two or three exceptions per year. Periods of unstable market conditions could increase the number of these exceptions. During the last four quarters, there was one back-testing exception where trading losses exceeded the VAR estimate at the Citigroup level (back-testing is the process in which the daily VAR of a portfolio is compared to the actual daily change in the market value of transactions). This occurred on August 8, 2011, after the U.S. government rating was downgraded by S&P.

The accompanying table summarizes VAR for Citi-wide trading portfolios at and during 2011 and 2010, including quarterly averages. Historically, Citi included only the hedges associated with the CVA (credit valuation adjustment) of its derivative transactions in its VAR calculations and disclosures (these hedges were, and continue to be, included within the relevant risk type e.g., interest rate, foreign exchange, equity). However, Citi now includes both the hedges associated with the CVA of its derivatives and the CVA on the derivative counterparty exposure (included in the line "Incremental Impact of Derivative CVA"). The inclusion of the CVA on derivative counterparty exposure reduces Citi's total trading VAR; Citi believes this calculation and presentation reflect a more complete and accurate view of its mark-to-market risk profile as it incorporates both the CVA underlying derivative transactions and related hedges.

In Millions of Dollars	Dec. 31, 2011	2011 Average	Dec. 31, 2010	2010 Average
Interest rate	**$250**	**$246**	$235	$234
Foreign exchange	**51**	**61**	52	61
Equity	**36**	**46**	56	59
Commodity	**16**	**22**	19	23
Covariance adjustment[1]	**(118)**	**(162)**	(171)	(172)
Total Trading VAR—all market risk factors, including general and specific risk (excluding derivative CVA)	**$235**	**$213**	$191	$205
Specific risk-only component[2]	**$ 14**	**$ 22**	$ 8	$ 18
Total—general market factors only	**$221**	**$191**	$183	$187
Incremental impact of derivative CVA	**$ (52)**	**$ (24)**	$ (5)	N/A
Total Trading and CVA VAR	**$183**	**$189**	$186	N/A

(1) Covariance adjustment (also known as diversification benefit) equals the difference between the total VAR and the sum of the VARs tied to each individual risk type. The benefit reflects the fact that the risks within each and across risk types are not perfectly correlated and, consequently, the total VAR on a given day will be lower than the sum of the VARs relating to each individual risk type. The determination of the primary drivers of changes to the covariance adjustment is made by an examination of the impact of both model parameter and position changes.

(2) The specific risk-only component represents the level of equity and fixed income issuer-specific risk embedded in VAR.

N/A Not available

Source: *Citigroup 2011 10-k Report,* March 2012, p. 98.

EXAMPLE 15–5
Calculating Value at Risk Using Monte Carlo Simulation

Consider an FI with a long position in a one-year, zero-coupon €1,000,000 bond. The current one year interest rate on the Eurobond is 10 percent. So, the present value of the one-year, €1m notional Eurobond is €909,091. The current $/€ exchange rate is 0.65 (i.e., the €/$ exchange rate is 1.538461). Thus, the FI has a long position of $590,909 in the Eurobond. The FI wants to evaluate the value at risk for this bond based on changes in interest rates and FX rates over the next 10 days.

The two underlying bond characteristics to be simulated are the $/€ exchange rate and the one year Eurobond price for changes in one year interest rates. Historical daily volatilities of the $/€ exchange rate and the bond price are such that $\sigma_{FX} = 0.0042$ and $\sigma_B = 0.0008$. The historic correlation between the two is $\rho_{FX,B} = -0.17$. To generate one thousand scenarios for values of the two underlying assets in 10 days, Monte Carlo analysis first generates one thousand pairs of standard normal variates whose correlation is $\rho_{FX,B} = -0.17$. Label each pair z_{FX} and z_B. Histograms for the results are shown in Figure 15–2. Note that the distributions are essentially the same.

Next, Monte Carlo simulation creates the actual scenarios for the variables, FX and B. That is, for each pair z_{FX} and z_B future values are created by applying

$$P_{FX} = 0.65e^{0.0042 \times \sqrt{5} \times z_{FX}} \quad \textbf{(6)}$$

and

$$P_B = €909{,}091e^{0.0008 \times \sqrt{5} \times z_B} \quad \textbf{(7)}$$

To express the bond price in dollars (accounting for both the exchange rate and interest rate risk for the bond), it is necessary to multiply the simulated bond price by the exchange rate in each scenario. Figures 15–3 and 15–4 show the distributions of future values, P_{FX} and P_B, respectively, obtained by one thousand simulations. Note that the distributions are no longer normal, and for the bond price, the distribution shows a marked asymmetry. This is due to the transformation made from normal to lognormal variates by applying Equations (6) and (7). Table 15–7 lists the first ten scenarios generated from Monte Carlo analysis. The process would be repeated until the 10,000 random observations are generated. Then with the observations rank ordered from worst (biggest loss) to best (biggest gain), the VAR is the 100th worst estimate out of 10,000.

FIGURE 15–2
Frequency Distribution for Z_{FX} and Z_B (1000 trials)

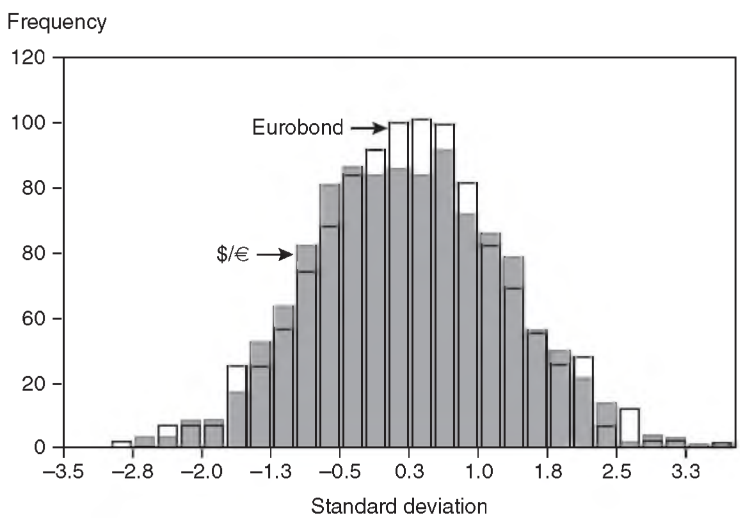


FIGURE 15–3
Frequency Distribution for Eurobond Price (1000 trials)

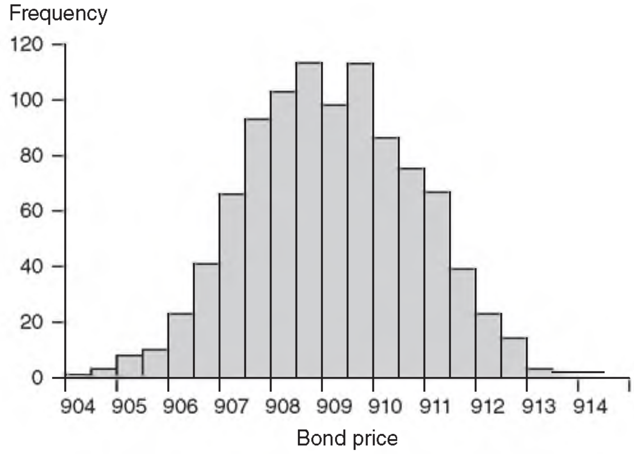


FIGURE 15–4
Frequency Distribution for $/€ Exchange Rate (1000 trials)

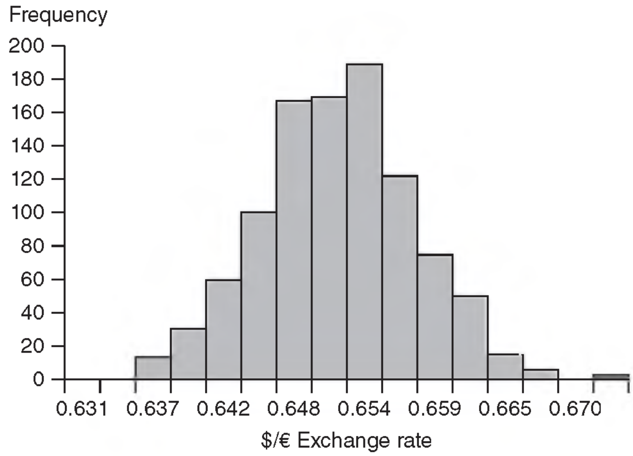


TABLE 15–7
Monte Carlo Scenarios (1000 trials)

$/€	PV of Cash Flow (in €s)	PV of Cash Flow (in $s)
0.6500	€906,663	$589,350
0.6540	907,898	593,742
0.6606	911,214	601,935
0.6513	908,004	591,399
0.6707	910,074	610,430
0.6444	908,478	585,460
0.6569	908,860	597,053
0.6559	906,797	594,789
0.6530	906,931	592,267
0.6625	920,768	603,348

Concept Questions

1. What are the advantages of the historic, or back simulation, approach over RiskMetrics to measure market risk?
2. What are the steps involved with the historic, or back simulation, approach to measuring market risk?
3. What is the Monte Carlo simulation approach to measuring market risk?

Expected Shortfall

As mentioned earlier, a criticism of VAR is that it tells the FI manager the level of possible losses that might occur with a given confidence level—that is, the 99th percentile—assuming a normally shaped return distribution. Expected shortfall (ES), also referred to as conditional VAR and expected tail loss, tells us the average of the losses in the tail of the distribution beyond the 99th percentile—that is, if 1 in every 100 days there is a loss, ES tells us the average of those 1 in 100 day losses. For example, in Table 15–6, the FI's 99 percent confidence level VAR is $97,210. Thus, if tomorrow is a bad day, there is a 1 percent probability that the FI's losses will exceed $97,210 assuming a normal probability distribution. However, many return distributions have "fat tails." Consider Figure 15–5. The VAR of the probability distribution is $97,210—that is, assuming a normal probability distribution, there is a 1 in 100 chance that the FI will lose $97,210. However, clearly the probability distribution is not normal, but has a fat-tail loss. Thus, the average of the 1 in 100 day losses will be larger than $97,210.

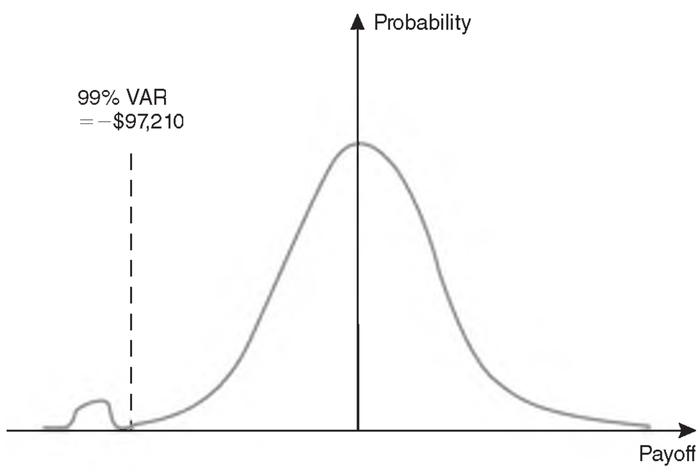
VAR corresponds to a specific point of loss on the probability distribution. It does not provide information about the potential size of the loss that exceeds it—that is, VAR completely ignores the patterns and the severity of the losses in the extreme tail. Thus, VAR gives only partial information about the extent of possible losses, particularly when probability distributions are non-normal. The drawbacks of VAR became painfully evident during the financial crisis as asset returns plummeted into the fat-tail region of non-normally shaped distributions. FI managers and regulators were forced to recognize that VAR projections of possible losses far underestimated actual losses on extreme bad days.

ES is a measure of market risk that estimates the expected value of losses beyond a given confidence level—that is, it is the average of VARs beyond a given confidence level. Specifically, for a confidence level *c*, ES can be solved using the following formula for a continuous probability distribution:

$$ES(c) = \frac{1}{1-c}\int_{c}^{1} \mathrm{Var}(u)\,du$$

That is, for a confidence level of, say, 99 percent (i.e., *c*), we measure the area under the probability distribution from the 99th to 100th percentile.

FIGURE 15–5
Probability Distribution of Returns for a Security



For a discrete distribution,

$$ES = -E(\Delta V \mid \Delta V < -\text{Var})$$

That is, for a confidence level of, say, 99 percent, we sum the weighted value of any observation in the discrete probability distribution from the 99th to 100th percentile.

In Figure 15–5, VAR tells the FI manager the loss at a particular point, *c*, on the probability distribution (i.e., 99th percentile). It, however, fails to incorporate information regarding the shape of the probability distribution below that particular point. Expected shortfall is the average VAR to the left of the 99 percent confidence level. Thus, VAR is identical for both probability distributions. However, ES, which incorporates points to the left of VAR, is larger when the probability distribution exhibits fat-tail losses. Accordingly, ES provides more information about possible market risk losses than VAR. For situations in which probability distributions exhibit fat-tail losses, VAR may look relatively small, but ES may be very large.

EXAMPLE 15–6
Simple Example of VAR versus ES

Consider the following discrete probability distribution of payoffs for two securities, A and B, held in the trading portfolio of an FI:

Probability	A	Probability	B
50.00%	$100 m	50.00%	$100 m
49.00	80 m	49.00	92 m
1.00	−920 m	0.25	−920 m
		0.75	−1,704 m

The FI wants to estimate which of the two securities will add more market risk to its trading portfolio according to both the VAR and ES measures.

$$\text{Expected return on security A} = 0.50(\$100\text{m}) + 0.49(\$80\text{m}) + 0.01(-\$920\text{m}) = \$80\text{m}$$

$$\text{Expected return on security B} = 0.50(\$100\text{m}) + 0.49(\$92\text{m}) + 0.0025(-\$920\text{m}) + 0.0075(-\$1{,}704\text{m}) = \$80\text{m}$$

For a 99 percent confidence level,

$$VAR_A = VAR_B = -\$920\text{ m}$$

Yet, for a 99 percent confidence level,

$$ES_A = -\$920\text{ m, while } ES_B = 0.25(-\$920\text{ m}) + 0.75(-\$1{,}704\text{ m}) = -\$1{,}508\text{ m}$$

Thus, while the VAR is identical for both securities, the ES finds that security B has the potential to subject the FI to much greater losses than security A. Specifically, if tomorrow is a bad day, VAR finds that there is a 1 percent probability that the FI's losses will exceed $920 million on either security. However, if tomorrow is a bad day, ES finds that there is a 1 percent probability that the FI's losses will exceed $920 million if security A is in its trading portfolio, but losses will exceed $1,508 million if security B is in its trading portfolio.

For continuous probability distributions ES uses a scaling factor based on a fat-tailed student's t distribution.[29] Thus, while the scaling factors for VAR are 2.33 for a 1 percent confidence level (and 1.65 for a 5 percent confidence level), ES scales up the risk factor to account for fat tails in the probability distribution, using 2.665 for a 1 percent confidence level (and 2.063 for a 5 percent confidence level).

EXAMPLE 15–7
Estimating VAR and ES of Trading Portfolio Securities

An FI has €1 million in its trading portfolio on the close of business on a particular day. The FI wants to calculate the one-day VAR and ES from this position. The first step is to calculate the dollar value position:

Suppose the current exchange rate of euros for dollars is €0.7983/$, or dollars for euros is $1.2527, at the daily close. So,

$$\text{Dollar value of position} = €1 \text{ million} \times 1.2527 = \$1{,}252{,}700$$

Suppose also that looking back at the daily percentage changes in the €/$ exchange rate over the past year, we find that the volatility, or standard deviation (σ), of daily percentage changes in the spot exchange rate was 44.3 bp. However, the FI is interested in adverse moves—bad moves that will not occur more than 1 percent of the time, or 1 day in every 100.

Value at Risk

Using VAR, which assumes that changes in exchange rates are normally distributed, the exchange rate must change in the adverse direction by 2.33σ (2.33×44.3 bp) for this change to be viewed as likely to occur only 1 day in every 100 days:

$$\text{FX volatility} = 2.33 \times 44.3 \text{ bp} = 103.219 \text{ bp}$$

In other words, using VAR during the last year the euro declined in value against the dollar by 103.219 bp 1 percent of the time. As a result, the one-day VAR is:

$$\text{VAR} = \$1{,}252{,}700 \times 0.0103219 = \$12{,}930$$

Expected Shortfall

Using ES, which assumes that changes in exchange rates are normally distributed but with fat tails, the exchange rate must change in the adverse direction by 2.665σ (2.665×44.3 bp) for this change to be viewed as likely to occur only 1 day in every 100 days:

$$\text{FX volatility} = 2.665 \times 44.3 \text{ bp} = 118.0595 \text{ bp}$$

In other words, using ES during the last year the euro declined in value against the dollar by 118.0595 bp 1 percent of the time. As a result, the one-day ES is:

$$\text{ES} = \$1{,}252{,}700 \times 0.01180595 = \$14{,}798$$

The potential loss exposure to adverse euro to dollar exchange rate changes for the FI from the €1 million spot currency holdings are higher using the ES measure of market risk. ES estimates potential losses that are $1,868 higher than VAR. This is because VAR focuses on the location of the extreme tail of the probability distribution. ES also considers the shape of the probability distribution once VAR is exceeded.

[29] Specifically,

$$\text{ES} = \text{Scale}^{ES} \times \sigma \times X$$

where,

$$\text{Scale}^{ES} = -\frac{N^{pdf}\left(N^{-1}(1-c)\right)}{1-c}$$

Concept Questions

1. What is the difference between VAR and ES?
2. Why is ES superior to VAR as a measure of market risk?

REGULATORY MODELS: THE BIS STANDARDIZED FRAMEWORK

www.bis.org

www.federalreserve.gov

The development of internal market risk models by FIs such as J.P. Morgan Chase was partly in response to proposals by the Bank for International Settlements (BIS) in 1993 to measure and regulate the market risk exposures of banks by imposing capital requirements on their trading portfolios.[30] As noted in Chapter 7, the BIS is an organization encompassing the largest central banks in the world. After refining these proposals over a number of years, most recently in 2013, the BIS (including the Federal Reserve) decided on a final approach to measuring market risk and the capital reserves necessary for an FI to hold to withstand and survive market risk losses. These required levels of capital held to protect against market risk exposure are in addition to the minimum level of capital banks are required to hold for credit risk purposes (see Chapter 20). Since January 1998 banks in the countries that are members of the BIS can calculate their market risk exposures in one of two ways. The first is to use a simple standardized framework (to be discussed below). The second, with regulatory approval, is to use their own internal models, which are similar to the models described above. However, if an internal model is approved for use in calculating capital requirements for the FI, it is subject to regulatory audit and certain constraints. Before looking at these constraints, we examine the BIS standardized framework. Additional details of this model can be found at the BIS website, **www.bis.org**.[31]

The financial crisis exposed a number of shortcomings in the way market risk was being measured in accordance with Basel II rules. Although the crisis largely exposed problems with the large-bank internal models approach to measuring market risk, the BIS also identified shortcomings with the standardized approach. These included a lack of risk sensitivity, a very limited recognition of hedging and diversification benefits, and an inability to sufficiently capture risks associated with more complex instruments. To address shortcomings of the standardized approach to measuring market risk, Basel III proposes a "partial risk factor" approach as a revised standardized approach. Basel III also introduces a "fuller risk factor" approach as an alternative to the revised partial risk factor standardized approach.

Partial Risk Factor Approach

The partial risk factor approach applies risk weights to the market values of trading portfolio securities, with enhancements to prudently reflect hedging of and diversification across securities. Particularly, the partial risk factor approach requires the following process be followed by FIs to determine capital requirements:

1. *Assign instruments to asset "buckets."* Instruments are placed in one of 20 asset buckets across each of five risk classes according to their risk similarity. The five

[30] BIS, Basel Committee on Banking Supervision, "The Supervisory Treatment of Market Risks," Basel, Switzerland, April 1993; "The New Basel Capital Accord: Third Consultative Paper," Basel, Switzerland, April 2003; and Revisions to Basel II Market Risk Framework, Basel, Switzerland, January 2009.

[31] Specifically, Basel 2.5 and Basel 3 changes are discussed in "Fundamental Review of the Trading Book," BIS Basel Committee on Banking Supervision, May 2012.

risk classes include FX, interest rates, equities, credit (including securitizations), and commodities.

2. *Calculate each bucket's risk measure.* A risk measure is calculated for each bucket using a regulator-specified formula based on ES estimates. The market values of the assets in each bucket are then multiplied by the risk weight.

3. *Aggregate the buckets.* The risk measures of the individual asset buckets are aggregated to obtain the capital requirement for the trading portfolio. The formula used to aggregate is:

$$\text{Capital} = \sqrt{\sum_{b=1}^{B} K_b^2 + \sum_{b=1}^{B}\sum_{c \neq b} \gamma_{bc} S_b S_c}$$

where $S_b = \Sigma_{i \neq b} RW_i MV_i$, and γ_{bc} is correlation parameter between buckets b and c, defined by regulators. The first term in this formula aggregates risk across buckets without considering cross-bucket diversification (the "sum of squares"). The second term adjusts for the "same direction" correlation between the asset types in b and c (i.e., long/long or short/short), γ_{bc}.

Fuller Risk Factor Approach

The fuller risk factor approach maps each trading portfolio security to a set of risk factors and associated shocks that explain the variation in the security values. The set of risk factors and shocks to the risk factors are established by regulators. The risk factors are organized in a hierarchy. Those risk factors listed at the top of the hierarchy affect the largest number of securities. Risk factors listed further down in the hierarchy are more specific in nature. Thus, changes in these risk factors would impact a smaller number of instruments. Finally, risk factors listed at the bottom of the hierarchy are nonhedgeable risk factors (i.e., risk that cannot easily be hedged in period of financial stress). Table 15–8 provides an illustration of the

TABLE 15–8 Hierarchy of Hedgeable Risk Factors under the Standardized Model Fuller Risk Factor Approach

Source: "Fundamental Review of the Trading Book," BIS Basel Committee on Banking Supervision, May 2012.

Level	FX Risk	Interest Rate Risk	Equity Risk	Credit Risk	Commodity Risk
I	Exchange rate of domestic currency to worldwide currency basket	Worldwide interest rate index	Worldwide equity index	Worldwide credit spread index	Commodity price index
II	Exchange rate of worldwide currency basket to respective foreign currency	Level of money market/swap rate curve in respective currency	Equity index by broad industry category	Credit spread index by industry category	Price index for commodity type
III		Slope of money market/swap rate curve in respective currency	Price of individual equity	Credit spread for individual issuer	Price index for physical type of commodity
IV		Money market/swap rate between vertex points in respective currency (residual)			

order of risk factors proposed by the BIS for Basel III. FIs then apply BIS empirically estimated standard deviations of shocks to these underlying risk factors. The capital charge is then determined by converting the risk position to an expected shortfall (ES) similar to that described in the previous section. The fuller risk factor approach requires the following process be followed by FIs to determine capital requirements:

1. *Assign each instrument to applicable risk factors.* The BIS defines a set of risk factors and associated shocks that explain the variation in the value of an FI's trading portfolio securities. Using a BIS-provided description of the mapping of securities to each risk factor, FIs determine which risk factors influence the value of their trading portfolio securities.

2. *Determine the size of the net risk position in each risk factor.* Once the FI determines the risk factors that apply to each of its trading portfolio securities, it uses a pricing model to determine the size of the risk positions from each security with respect to the applicable risk factors. The size of the risk positions is based on the sensitivity of the instruments to the prescribed risk factors. The FI then aggregates all negative and positive gross risk positions to determine the net risk position. For nonhedgeable risk factors, the gross risk position would equal the net risk position.

3. *Aggregate overall risk position across risk factors.* To compute the overall capital requirement for each risk factor class, the net risk positions determined in step 2 are aggregated. Regulators specify the distribution of the risk factors (i.e., the standard deviations to apply against each of the risk factors). One option offered by the BIS is to assume that all risk factors of the same risk factor class are independently distributed.[32] Thus, the overall portfolio standard deviation is calculated using a sum of squares multiplied by a scalar that approximates the average across the loss tail of the portfolio distribution (i.e., the ES). The ES scalar factor implemented by regulators in Basel III is four. Thus, the overall capital requirement is four times the overall portfolio standard deviation.

The following example is the BIS illustration of the fuller risk factor approach of the standardized model.[33]

EXAMPLE 15–8
Calculating Market Risk Capital Requirement Using the Fuller Risk Factor Approach

In its trading portfolio, an FI holds 1,000 Daimler shares at a share price of €101 and has sold 500 Volkswagen shares under a forward contract that matures in one year. The current share price for Volkswagen is €20. To calculate the market risk capital charge on these securities, the FI proceeds as follows.

Step 1. Assign each instrument to applicable risk factors
From Table 15–8, hedgeable risk factors for these equities include level I movements in global equity markets (worldwide equity index), level II movements in sectoral equity indices (equity index by broad industry category), and level III movements in the prices of individual equity. Daimler and Volkswagen have the same hedgeable risk factors at levels I and II (i.e., global and industry specific equity indices). However, movements in the prices of the two firms are unique. Thus, they do not have the same risk factor at level III, and as a result, they are

[32] The BIS stated that to recognize diversification it would be necessary to impose a distribution on the risk factors. However, specifying a distribution of risk factors, with appropriate pairwise correlations between risk factors, is likely to be a burdensome and complex task for regulators and would also complicate FIs' calculations considerably. Thus, the BIS decided that the computationally simplest approach is to treat all risk factors of the same risk factor class as independently distributed.

[33] See "Fundamental Review of the Trading Book," BIS Basel Committee on Banking Supervision, May 2012.

mapped to different individual equity risk factors.[34] There is also a nonhedgeable risk factor for the Volkswagen equity price to capture basis risk from the forward contract.

Step 2. Determine the size of the net risk position in each risk factor
For each risk factor, the FI determines a net risk position, calculated as the sum of gross risk positions for all instruments that are subject to that risk factor.[35] Table 15–9 shows the gross and net positions for Daimler and Volkswagen equities for the equity risk factor. The size of the gross position in Daimler for the three applicable risk factors is €101,000 (1,000 shares × €101) and for the short position in Volkswagen is −€10,000 (500 shares × €20). Note again that the two securities do not have the same risk factor at level III. Thus, they are mapped to different individual equity risk factors. Further, to capture basis risk from the forward contract, there is a nonhedgeable risk factor for the Volkswagen equity price, −€10,000. The net risk position of the two securities for each risk factor, listed in the last column of Table 15–9, is the sum of the gross risk factors for the securities at each level—that is, €91,000 for levels I and II, €101,000 and −€10,000, respectively, for level III, and −€10,000 for nonhedgeable risk.

Step 3. Aggregate overall risk position across risk factors
The net risk positions is then converted into a capital charge by multiplying by regulator specified standard deviations (i.e., shift risk factors). Table 15–10 shows the calculations of the capital charge for market risk. The net risk positions (listed in column 3 for each risk level) are multiplied by the standard deviations assigned for each level (column 4) to produce the standard deviations of the net risk position (column 5). For example, the standard deviation of the net risk position for the level I worldwide equity index is equal to the net risk (€91,000) times the regulator set shift risk factor (5 percent) to give the standard deviation associated with level I risk factor (€4,550). The square of the standard deviation (the variance) is then listed in column 6 (i.e., €20,702,500 for level I). Summing the squared standard deviations gives the portfolio variance (€164,289,400) and taking the square root of this gives the portfolio standard deviation (€12,818). Finally, this portfolio standard deviation is multiplied by a scalar (currently set at 4) to achieve the overall expected shortfall for the portfolio.

TABLE 15–9 Calculation of Gross and Net Risk Position

Level	Equity Risk	Daimler Gross Risk Position	Volkswagen Gross Risk Position	Total Size of Net Risk Position
I	Worldwide equity index	€101,000	−€10,000	€91,000
II	Industry equity index	€101,000	−€10,000	€91,000
III	Daimler share price	€101,000	—	€101,000
	Volkswagen share price	—	−€10,000	−€10,000
N-h*	Volkswagen share price	—	−€10,000	−€10,000

* Nonhedgeable risk factor.

[34] However, these risk factors can be hedged with other positions that shared this risk factor, such as Daimler equity options.

[35] For equities, this is equivalent to assuming that equity betas are homogeneous and equal to one. For FX, the size of the gross risk position is the market value of the instrument converted to the reporting currency of the FI. For linear interest rate risk and credit risk–related instruments, the size of the gross risk position is determined by applying a small shift to the respective risk factor and determining the value change of the instrument in relation to the shift applied.

TABLE 15–10 Calculation of Market Risk Capital Charge

Level	Equity Risk: Portfolio	Net Risk Position (EUR)	Standard Deviation (i.e., shift of risk factor)	Standard Deviation of Net Risk Position	Square the Standard Deviation of the Net Risk Position (i.e, variance)
I	Worldwide equity index	€91,000	5%	€4,550	€20,702,500
II	Industry equity index	91,000	7%	6,370	40,576,900
III	Daimler share price	101,000	10%	10,100	102,010,000
	Volkswagen share price	−10,000	−10%	1,000	1,000,000
N-h*	Volkswagen share price	−10,000	1%	100	10,000
Portfolio	Sum the squared standard deviations (portfolio variance)				€164,289,400
Portfolio	Take the square root (portfolio standard deviation)				€12,818
Portfolio	Multiply by scalar to obtain expected shortfall				€51,270

* Nonhedgeable risk factor.
Notes: ES scalar factor decided by regulators = 4 × standard deviation. Correlation (ρ) between stocks = 0 is assumed by the model.

THE BIS REGULATIONS AND LARGE-BANK INTERNAL MODELS

As discussed above, the BIS capital requirement for market risk exposure introduced in January 1998 allows large banks (subject to regulatory permission) to use their own internal models to calculate market risk instead of the standardized framework. The initial market risk capital requirements were included as part of what became known as Basel I capital rules. However, details of the capital calculations have been refined and revised over the years. Today, FIs' internal models are governed by Basel 2.5 (implemented in 2012) and Basel III (being phased in between 2013 and 2019) versions of the rules for adequate capital at FIs. (We examine the initiatives taken by the BIS and the major central banks, e.g., the Federal Reserve, in controlling bank risk exposure through capital requirements in greater detail in Chapter 20.)

During the financial crisis, losses due to market risk were significantly higher than the minimum market risk capital requirements under BIS Basel I and Basel II rules. As a result, in July 2009 the BIS announced Basel 2.5, a final version of revised rules for market risk capital requirements. Specifically, in addition to the risk capital charge already in place (steps 1 and 2 listed below), an incremental capital charge is assessed which includes a "stressed value at risk" capital requirement taking into account a one-year observation period of significant financial stress relevant to the FI's portfolio (step 3 listed below). The introduction of stressed VAR in Basel 2.5 is intended to reduce the cyclicality of the VAR measure and alleviate the problem of market stress periods dropping out of the data period used to calculate VAR after some time. Basel 2.5 requires the following process be followed by large FIs using internal models to calculate the market risk capital charge.

1. In calculating DEAR, the FI must define an adverse change in rates as being in the 99th percentile (multiply σ by 2.33).
2. The FI must assume the minimum holding period to be 10 days (this means that daily DEAR would have to be multiplied by $\sqrt{10}$).
3. The FI must add to this a "stressed VAR" that is intended to replicate a VAR calculation that would be generated on the FI's trading portfolio if the relevant

market factors were experiencing a period of stress. The stressed VAR is based on the 10-day, 99th percentile VAR of the trading portfolio, with model inputs incorporating historical data from a one-year period of significant financial stress. The period used must be approved by the supervisor and regularly reviewed. For example, a 12-month period relating to significant losses during the financial crisis would adequately reflect a period of such stress.

The FI must consider its proposed capital charge or requirement as the sum of:

1. The higher of the previous day's VAR (value at risk or $DEAR \times \sqrt{10}$) and the average daily VAR over the previous 60 business days times a multiplication factor with a minimum value of 3, i.e., capital charge $= DEAR \times \sqrt{10} \times 3$ (in general, the multiplication factor makes required capital significantly higher than VAR produced from private models), plus
2. The higher of its latest available stressed VAR and an average of the stressed VAR over the preceding 60 business days times a multiplication factor with a minimum value of 3 and a maximum of 4.

From this,

$$\text{Capital charge for market risk} = (\text{VAR} \times \sqrt{10} \times 3) + (\text{Stressed VAR} \times \sqrt{10} \times 3)$$

For example, suppose an FI's portfolio VAR over the previous 60 days was \$10 million and stressed VAR over the previous 60 days was \$25 million using the 1 percent worst case (or 99th percentile). The minimum capital charge would be:[36]

$$\begin{aligned}\text{Capital charge} &= (\$10 \text{ million} \times \sqrt{10} \times 3) + (\$25 \text{ million} \times \sqrt{10} \times 3)\\ &= \$332.04 \text{ million}\end{aligned}$$

Basel III proposes to replace VAR models with those based on extreme value theory and expected shortfall (ES). As discussed earlier, the ES measure analyzes the size and likelihood of losses above the 99th percentile in a crisis period for a traded asset and thus measures "tail risk" more precisely. Thus, ES is a risk measure that considers a more comprehensive set of potential outcomes than VAR. The BIS change to ES highlights the importance of maintaining sufficient regulatory capital not only in stable market conditions, but also in periods of significant financial stress. Indeed, it is precisely during periods of stress that capital is vital for absorbing losses and safeguarding the stability of the banking system. Accordingly, the committee intends to move to a framework that is calibrated to a period of significant financial stress.

Two methods of identifying the stress period and calculating capital requirements under the internal models are the direct method and the indirect method. The direct method is based on the approach used in the Basel 2.5 stressed VAR. The FI would search the entire historical period and identify the period that produces

[36] The idea of a minimum multiplication factor of 3 is to create a scheme that is "incentive compatible." Specifically, if FIs using internal models constantly underestimate the amount of capital they need to meet their market risk exposures, regulators can punish those FIs by raising the multiplication factor to as high as 4. Such a response may effectively put the FI out of the trading business. The degree to which the multiplication factor is raised above 3 depends on the number of days an FI's model underestimates its market risk over the preceding year. For example, an underestimation error that occurs on more than 10 days out of the past 250 days will result in the multiplication factor's being raised to 4.

TABLE 15–11
Ratio of Market Risk to Total Risk-Based Capital for Bank Holding Companies Using Internal Models

Source: Federal Reserve Board, FR Y-9C Reports, 2011.

Name	Market Risk to Total Risk-Based Capital (%)
Bank of America	2.34%
Citigroup	0.81
J.P. Morgan Chase	4.42
HSBC North America	1.48
KeyCorp	0.67
Suntrust	0.45
Wells Fargo	0.80
Bank of New York Mellon	1.13
PNC Financial	0.81
US Bancorp	0.10

the highest ES result when all risk factors are included. However, Basel III would require the FI to determine the stressed period on the basis of a reduced set of risk factors. Once the FI has identified the stressed period, it must then determine the ES for the full set of risk factors for the stress period. The indirect method identifies the relevant historical period of stress by using a reduced set of risk factors. However, instead of calculating the full ES model to that period, the FI calculates a loss based on the reduced set of risk factors. This loss is then scaled using the ratio of the full ES model using current market data to the full ES model using the reduced set of risk factors using current market data.

Finally, it should be noted that the market risk framework discussed earlier is based on an assumption that an FI's trading book positions are liquid—that is, that FIs can exit or hedge the trading book positions over a 10-day horizon. The financial crisis proved this to be false. Thus, under the new liquidity risk measures the 10-day liquidity metric as used in the VAR calculations (i.e., VAR $\times \sqrt{10}$) are replaced with liquidity horizons based on a set of quantitative and qualitative criteria that allow for changes in market liquidity conditions. Specifically, FIs' exposures would be assigned to one of five liquidity horizon categories, ranging from 10 days to one year based on the time required to exit or hedge a risk position in a stressed market environment. Further, capital add-ons are included for jumps in liquidity premia. These add-ons would apply only to instruments that could become particularly illiquid to the extent that the market risk measures, even with extended liquidity horizons, would not sufficiently capture the risk to FI solvency from large fluctuations in liquidity premia on these securities.

Table 15–11 lists the market risk to the total risk-based capital for several large U.S. bank holding companies in 2011. Notice how small the market risk capital is relative to the total risk-based capital for these banks. Only J.P. Morgan Chase has a ratio greater than 4 percent. The average ratio of market risk capital to total risk-based capital required for the 10 bank holding companies is only 1.30 percent. Moreover, very few banks, other than the very largest (above), report market risk exposures at all.

Concept Questions

1. What is the BIS standardized framework for measuring market risk?
2. What is the effect of using the 99th percentile (1 percent worst case) rather than the 95th percentile (5 percent worst case) on the measured size of an FI's market risk exposures?

Summary

In this chapter we analyzed the importance of measuring an FI's market risk exposure. This risk is likely to continue to grow in importance as more and more loans and previously illiquid assets become marketable and as the traditional franchises of commercial banks, insurance companies, and investment banks shrink. Given the risks involved, both private FI management and regulators are investing increasing resources in models to measure and track market risk exposures. We analyzed in detail four approaches FIs have used to measure market risk: RiskMetrics, the historic (or back simulation) approach, the Monte Carlo simulation approach, and the expected shortfall (ES) approach. The four approaches were also compared in terms of simplicity and accuracy. Market risk is also of concern to regulators. Beginning in January 1998, banks in the United States have had to hold a capital requirement against the risk of their trading positions. The novel feature of the regulation of market risk is that the Federal Reserve and other central banks (subject to regulatory approval) have given large FIs the option to calculate capital requirements based on their own internal models rather than the regulatory model.

Questions and Problems

1. What is meant by *market risk?*
2. Why is the measurement of market risk important to the manager of a financial institution?
3. What is meant by *daily earnings at risk* (DEAR)? What are the three measurable components? What is the price volatility component?
4. Follow Bank has a $1 million position in a five-year, zero-coupon bond with a face value of $1,402,552. The bond is trading at a yield to maturity of 7.00 percent. The historical mean change in daily yields is 0.0 percent and the standard deviation is 12 basis points.
   a. What is the modified duration of the bond?
   b. What is the maximum adverse daily yield move given that we desire no more than a 1 percent chance that yield changes will be greater than this maximum?
   c. What is the price volatility of this bond?
   d. What is the daily earnings at risk for this bond?
5. How can DEAR be adjusted to account for potential losses over multiple days? What would be the VAR for the bond in problem 4 for a 10-day period? What statistical assumption is needed for this calculation? Could this treatment be critical?
6. The DEAR for a bank is $8,500. What is the VAR for a 10-day period? A 20-day period? Why is the VAR for a 20-day period not twice as much as that for a 10-day period?
7. The mean change in the daily yields of a 15-year, zero-coupon bond has been five basis points (bp) over the past year with a standard deviation of 15 bp. Use these data and assume that the yield changes are normally distributed.
   a. What is the highest yield change expected if a 99 percent confidence limit is required; that is, adverse moves will not occur more than 1 day in 100?
   b. What is the highest yield change expected if a 95 percent confidence limit is required; adverse moves will not occur more than 1 day in 20?
8. In what sense is duration a measure of market risk?

9. Bank Alpha has an inventory of AAA-rated, 15–year zero-coupon bonds with a face value of $400 million. The bonds currently are yielding 9.5 percent in the over-the-counter market.
   a. What is the modified duration of these bonds?
   b. What is the price volatility if the potential adverse move in yields is 25 basis points?
   c. What is the DEAR?
   d. If the price volatility is based on a 99 percent confidence limit and a mean historical change in daily yields of 0.0 percent, what is the implied standard deviation of daily yield changes?
10. Bank Beta has an inventory of AAA-rated, 10-year zero-coupon bonds with a face value of $100 million. The modified duration of these bonds is 12.5 years, the DEAR is $2,150,000, and the potential adverse move in yields is 35 basis points. What is the market value of the bonds, the yield on the bonds, and the duration of the bonds?
11. Bank Two has a portfolio of bonds with a market value of $200 million. The bonds have an estimated price volatility of 0.95 percent. What are the DEAR and the 10-day VAR for these bonds?
12. Suppose that an FI has a €1.6 million long trading position in spot euros at the close of business on a particular day. Looking back at the daily percentage changes in the exchange rate of the €/$ for the past year, the volatility or standard deviation (σ) of daily percentage changes in the €/$ spot exchange rate was 62.5 basis points (bp). Calculate the FI's daily earnings at risk from this position (i.e., adverse moves in the FX markets with respect to the value of the euro against the dollar will not occur more than 1 percent of the time, or 1 day in every 100 days) if the spot exchange rate is €0.80/$1, or $1.25/€, at the daily close.
13. Bank of Southern Vermont has determined that its inventory of 20 million euros (€) and 25 million British pounds (£) is subject to market risk. The spot exchange rates are $0.40/€ and $1.28/£, respectively. The σ's of the spot exchange rates of the € and £, based on the daily changes of spot rates over the past six months, are 65 bp and 45 bp, respectively. Determine the bank's 10-day VAR for both currencies. Use adverse rate changes in the 99th percentile.
14. Bank of Bentley has determined that its inventory of yen (¥) and Swiss franc (SF) denominated securities is subject to market risk. The spot exchange rates are ¥80.00/$ and SF0.9600/$, respectively. The σ's of the spot exchange rates of the ¥ and SF, based on the daily changes of spot rates over the past six months, are 75 bp and 55 bp, respectively. Using adverse rate changes in the 99th percentile, the 10-day VARs for the two currencies, ¥ and SF, are $350,000 and $500,000, respectively. Calculate the yen and Swiss franc-denominated value positions for Bank of Bentley.
15. Suppose that an FI holds a $15 million trading position in stocks that reflect the U.S. stock market index (e.g., the S&P 500). Over the last year, the σ_m of the daily returns on the stock market index was 156 bp. Calculate the VAR for this portfolio of stocks using a 99 percent confidence limit.
16. Bank of Alaska's stock portfolio has a market value of $10 million. The beta of the portfolio approximates the market portfolio, whose standard deviation (σ_m) has been estimated at 1.5 percent. What is the five-day VAR of this portfolio using adverse rate changes in the 99th percentile?

17. Jeff Resnick, vice president of operations at Choice Bank, is estimating the aggregate DEAR of the bank's portfolio of assets consisting of loans (L), foreign currencies (FX), and common stock (EQ). The individual DEARs are $300,700; $274,000; and $126,700, respectively. If the correlation coefficients (ρ_{ij}) between L and FX, L and EQ, and FX and EQ are 0.3, 0.7, and 0.0, respectively, what is the DEAR of the aggregate portfolio?
18. Calculate the DEAR for the following portfolio with the correlation coefficients and then with perfect positive correlation between various asset groups.

Assets	Estimated DEAR	($\rho_{S,FX}$)	($\rho_{S,B}$)	($\rho_{FX,B}$)
Stocks (*S*)	$300,000	−0.10	0.75	0.20
Foreign Exchange (*FX*)	200,000			
Bonds (*B*)	250,000			

What is the amount of risk reduction resulting from the lack of perfect positive correlation between the various asset groups?

19. What are the advantages of using the back simulation approach to estimate market risk? Explain how this approach would be implemented.
20. Export Bank has a trading position in Japanese yen and Swiss francs. At the close of business on February 4, the bank had ¥300 million and SF10 million. The exchange rates for the most recent six days are given below.

Exchange Rates per U.S. Dollar at the Close of Business

	2/4	2/3	2/2	2/1	1/29	1/28
Japanese yen	80.13	80.84	80.14	83.05	84.35	84.32
Swiss francs	0.9540	0.9575	0.9533	0.9617	0.9557	0.9523

a. What is the foreign exchange (FX) position in dollar equivalents using the FX rates on February 4?
b. What is the definition of delta as it relates to the FX position?
c. What is the sensitivity of each FX position; that is, what is the value of delta for each currency on February 4?
d. What is the daily percentage change in exchange rates for each currency over the five-day period?
e. What is the total risk faced by the bank on each day? What is the worst-case day? What is the best-case day?
f. Assume that you have data for the 500 trading days preceding February 4. Explain how you would identify the worst-case scenario with a 99 percent degree of confidence.
g. Explain how the 1 percent value at risk (VAR) position would be interpreted for business on February 5.
h. How would the simulation change at the end of the day on February 5? What variables and/or processes in the analysis may change? What variables and/or processes will not change?

21. Export Bank has a trading position in euros and Australian dollars. At the close of business on October 20, the bank had €20 million and A$30 million. The exchange rates for the most recent six days are given below:

Exchange Rates per U.S. Dollar at the Close of Business

	10/20	10/19	10/18	10/17	10/16	10/15
Euros	0.8000	0.7970	o.7775	0.7875	0.7950	0.8115
Australian $s	0.9700	0.9550	0.9800	0.9655	0.9505	0.9460

a. What is the foreign exchange (FX) position in dollar equivalents using the FX rates on October 20?
b. What is the sensitivity of each FX position; that is, what is the value of delta for each currency on October 20?
c. What is the daily percentage change in exchange rates for each currency over the five-day period?
d. What is the total risk faced by the bank on each day? What is the worst-case day? What is the best-case day?

22. What is the primary disadvantage of the back simulation approach in measuring market risk? What effect does the inclusion of more observation days have as a remedy for this disadvantage? What other remedies can be used to deal with the disadvantage?
23. How is Monte Carlo simulation useful in addressing the disadvantages of back simulation? What is the primary statistical assumption underlying its use?
24. What is the difference between VAR and expected shortfall (ES) as measure of market risk?
25. Consider the following discrete probability distribution of payoffs for two securities, A and B, held in the trading portfolio of an FI:

Probability	A	Probability	B
50.00%	$80m	50.00%	$80m
49.00	60m	49.00	68m
1.00	−740m	0.40	−740m
		0.60	−1,393m

Which of the two securities will add more market risk to the FI's trading portfolio according to the VAR and ES measures?

26. Consider the following discrete probability distribution of payoffs for two securities, A and B, held in the trading portfolio of an FI:

Probability	A	Probability	B
55.00%	$120m	55.00%	$120m
44.00	95m	44.00	100m
1.00	−1,100m	0.30	−1,100m
		0.70	−1,414m

Which of the two securities will add more market risk to the FI's trading portfolio according to the VAR and ES measures?

27. An FI has £5 million in its trading portfolio on the close of business on a particular day. The current exchange rate of pounds for dollars is £0.6400/$, or dollars for pounds is $1.5625, at the daily close. The volatility, or standard deviation (σ), of daily percentage changes in the spot £/$ exchange rate over the past year was 58.5 bp. The FI is interested in adverse moves–bad moves that will not occur more than 1 percent of the time, or 1 day in every 100. Calculate the one-day VAR and ES from this position.
28. An FI has ¥500 million in its trading portfolio on the close of business on a particular day. The current exchange rate of yen for dollars is ¥80.00/$, or dollars for yen is $0.0125, at the daily close. The volatility, or standard deviation (σ), of daily percentage changes in the spot ¥/$ exchange rate over the past year was 121.6 bp. The FI is interested in adverse moves–bad moves that will not occur more than 1 percent of the time, or 1 day in every 100. Calculate the one-day VAR and ES from this position.
29. Bank of Hawaii's stock portfolio has a market value of $250 million. The beta of the portfolio approximates the market portfolio, whose standard deviation (σ_m) has been estimated at 2.25 percent. What are the five-day VAR and ES of this portfolio using adverse rate changes in the 99th percentile?
30. Despite the fact that market risk capital requirements have been imposed on FIs since the 1990s, huge losses in value were recorded from losses incurred in FIs' trading portfolios. Why did this happen? What changes to capital requirements did regulators propose to prevent such losses from reoccurring?
31. In its trading portfolio, an FI holds 10,000 ExxonMobil (XOM) shares at a share price of $86.50 and has sold 5,000 General Electric (GE) shares under a forward contract that matures in one year. The current share price for GE is $20.50. The shift risk factor (i.e., standard deviation) for level 1 risk factor is 4 percent, for level II risk factor is 6 percent, for level III long positions is 9 percent, for level III short positions is −9 percent, and for nonhedgeable risk is 1 percent. Using the risk factors listed in Table 15–8, calculate the market risk capital charge on these securities.
32. In its trading portfolio, a U.S. FI is long £20 million worth of pound FX forward contracts and has sold €40 million of euro FX forward contracts that mature in one year. The current exchange rate of dollars for pounds is $1.5625 and the exchange rate of euros for pounds is $1.25 at the daily close. The shift risk factor (i.e., standard deviation) for level 1 risk factor is 5 percent, for level II risk factor for pounds is 8 percent, and for level II risk factors for euros is 12 percent. Using the risk factors listed in Table 15–8, calculate the market risk capital charge on these securities.
33. Suppose an FI's portfolio VAR for the previous 60 days was $3 million and stressed VAR for the previous 60 days was $8 million using the 1 percent worst case (or 99th percentile). Calculate the minimum capital charge for market risk for this FI.

Integrated Mini Case

CALCULATING DEAR ON AN FI'S TRADING PORTFOLIO

An FI wants to obtain the DEAR on its trading portfolio. The portfolio consists of the following securities.

Fixed-income securities:

(i) The FI has a $1 million position in a six-year, zero-coupon bond with a face value of $1,543,302. The bond is trading at a yield to maturity of 7.50 percent. The historical mean change in daily yields is 0.0 percent, and the standard deviation is 22 basis points.

(ii) The FI also holds a 12-year, zero-coupon bond with a face value of $1,000,000. The bond is trading at a yield to maturity of 6.75 percent. The price volatility if the potential adverse move in yields is 65 basis points.

Foreign exchange contracts:

The FI has a €2.0 million long trading position in spot euros at the close of business on a particular day. The exchange rate is €0.80/$1, or $1.25/€, at the daily close. Looking back at the daily percentage changes in the exchange rate of the euro to dollars for the past year, the FI finds that the volatility or standard deviation (σ) of the spot exchange rate was 55.5 basis points (bp).

Equities:

The FI holds a $2.5 million trading position in stocks that reflect the U.S. stock market index (e.g., the S&P 500). The $\beta = 1$. Over the last year, the standard deviation of the stock market index was 175 basis points. Correlations (ρ_{ij}) among assets are as follows:

	Six-Year, Zero-Coupon	12-Year, Zero-Coupon	€/$	U.S. Stock Index
Six-year, zero-coupon	–	0.75	−0.2	0.40
12-year, zero-coupon	–	–	−0.3	0.45
€/$	–	–	–	0.25
U.S. stock index	–	–	–	–

Calculate the DEAR of this trading portfolio.