11 Power System Security

11.1 INTRODUCTION

Up until now we have been mainly concerned with minimizing the cost of operating a power system. An overriding factor in the operation of a power system is the desire to maintain system security. System security involves practices designed to keep the system operating when components fail. For example, a generating unit may have to be taken off-line because of auxiliary equipment failure. By maintaining proper amounts of spinning reserve, the remaining units on the system can make up the deficit without too low a frequency drop or need to shed any load. Similarly, a transmission line may be damaged by a storm and taken out by automatic relaying. If, in committing and dispatching generation, proper regard for transmission flows is maintained, the remaining transmission lines can take the increased loading and still remain within limit.

Because the specific times at which initiating events that cause components to fail are unpredictable, the system must be operated at all times in such a way that the system will not be left in a dangerous condition should any credible initiating event occur. Since power system equipment is designed to be operated within certain limits, most pieces of equipment are protected by automatic devices that can cause equipment to be switched out of the system if these limits are violated. If any event occurs on a system that leaves it operating with limits violated, the event may be followed by a series of further actions that switch other equipment out of service. If this process of cascading failures continues, the entire system or large parts of it may completely collapse. This is usually referred to as a *system blackout.*

An example of the type of event sequence that can cause a blackout might start with a single line being opened due to an insulation failure; the remaining transmission circuits in the system will take up the flow that was flowing on the now-opened line. If one of the remaining lines is now too heavily loaded, it may open due to relay action, thereby causing even more load on the remaining lines. This type of process is often termed a *cascading outage.* Most power systems are operated such that any single initial failure event will not leave other components heavily overloaded, specifically to avoid cascading failures.

Most large power systems install equipment to allow operations personnel to monitor and operate the system in a reliable manner. This chapter will deal with the techniques and equipment used in these systems. We will lump these under the commonly used title *system security*.

Systems security can be broken down into three major functions that are carried out in an operations control center:

1. System monitoring.
2. Contingency analysis.
3. Security-constrained optimal power flow.

System monitoring provides the operators of the power system with pertinent up-to-date information on the conditions on the power system. Generally speaking, it is the most important function of the three. From the time that utilities went beyond systems of one unit supplying a group of loads, effective operation of the system required that critical quantities be measured and the values of the measurements be transmitted to a central location. Such systems of measurement and data transmission, called *telemetry systems*, have evolved to schemes that can monitor voltages, currents, power flows, and the status of circuit breakers, and switches in every substation in a power system transmission network. In addition, other critical information such as frequency, generator unit outputs and transformer tap positions can also be telemetered. With so much information telemetered simultaneously, no human operator could hope to check all of it in a reasonable time frame. For this reason, digital computers are usually installed in operations control centers to gather the telemetered data, process them, and place them in a data base from which operators can display information on large display monitors. More importantly, the computer can check incoming information against prestored limits and alarm the operators in the event of an overload or out-of-limit voltage.

State estimation is often used in such systems to combine telemetered system data with system models to produce the best estimate (in a statistical sense) of the current power system conditions or "state." We will discuss some of the highlights of these techniques in Chapter 12.

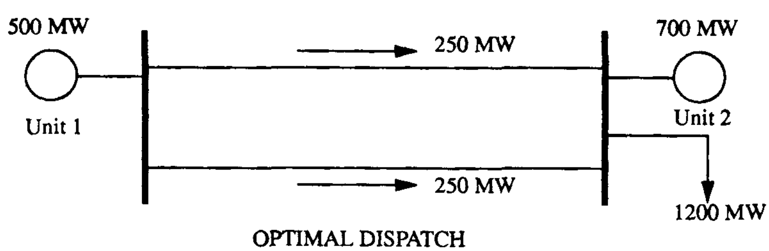
Such systems are usually combined with supervisory control systems that allow operators to control circuit breakers and disconnect switches and transformer taps remotely. Together, these systems are often referred to as *SCADA systems*, standing for supervisory control and data acquisition system. The SCADA system allows a few operators to monitor the generation and high-voltage transmission systems and to take action to correct overlords or out-of-limit voltages.

The second major security function is contingency analysis. The results of this type of analysis allow systems to be operated defensively. Many of the problems that occur on a power system can cause serious trouble within such a quick time period that the operator could not take action fast enough. This is often the case with cascading failures. Because of this aspect of systems operation, modern operations computers are equipped with contingency analysis programs that model possible systems troubles before they arise. These programs are based on a model of the power system and are used to study outage events and alarm the operators to any potential overlords or out-of-limit voltages. For example, the simplest form of contingency analysis can be put together with a standard power-flow program such as described in Chapter 4, together with procedures to set up the power-flow data for each outage to be studied by the power-flow program. Several variations of this type of contingency analysis scheme involve fast solution methods, automatic contingency event selection, and automatic initializing of the contingency power flows using actual system data and state estimation procedures.

The third major security function is security-constrained optimal power flow. In this function, a contingency analysis is combined with an optimal power flow which seeks to make changes to the optimal dispatch of generation, as well as other adjustments, so that when a security analysis is run, no contingencies result in violations. To show how this can be done, we shall divide the power system into four operating states.

- **Optimal dispatch:** this is the state that the power system is in prior to any contingency. It is optimal with respect to economic operation, but it may not be secure.
- **Post contingency:** is the state of the power system after a contingency has occurred. We shall assume here that this condition has a security violation (line or transformer beyond its flow limit, or a bus voltage outside the limit).
- **Secure dispatch:** is the state of the system with no contingency outages, but with corrections to the operating parameters to account for security violations.
- **Secure post-contingency:** is the state of the system when the contingency is applied to the base-operating condition—with corrections.

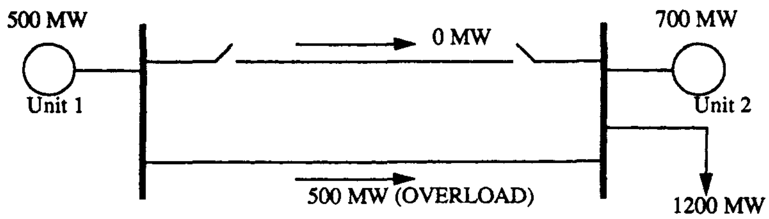
We shall illustrate the above with an example. Suppose the trivial power system consisting of two generators, a load, and a double circuit line, is to be operated with both generators supplying the load as shown below (ignore losses):



OPTIMAL DISPATCH

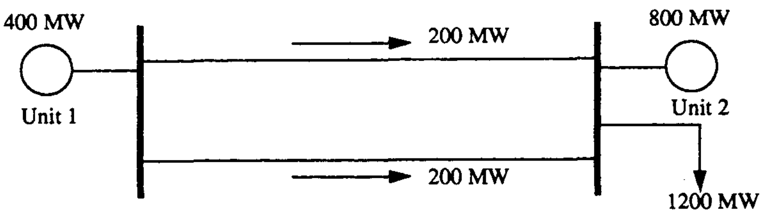
We assume that the system as shown is in economic dispatch, that is the 500 MW from unit 1 and the 700 MW from unit 2 is the optimum dispatch. Further, we assert that each circuit of the double circuit line can carry a maximum of 400 MW, so that there is no loading problem in the base-operating condition.

Now, we shall postulate that one of the two circuits making up the transmission line has been opened because of a failure. This results in



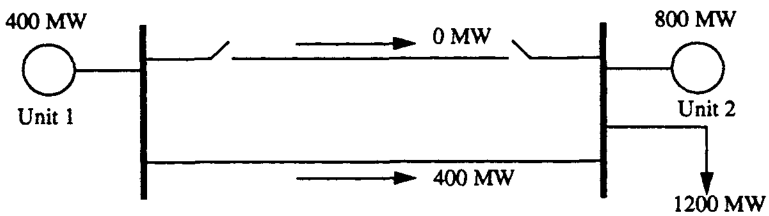
POST CONTINGENCY STATE

Now there is an overload on the remaining circuit. We shall assume for this example that we do not want this condition to arise and that we will correct the condition by lowering the generation on unit 1 to 400 MW. The secure dispatch is



SECURE DISPATCH

Now, if the same contingency analysis is done, the post-contingency condition is



SECURE POST CONTINGENCY STATE

By adjusting the generation on unit 1 and unit 2, we have prevented the post-contingency operating state from having an overload. This is the essence of what is called "security corrections." Programs which can make control adjustments to the base or pre-contingency operation to prevent violations in the post-contingency conditions are called "security-constrained optimal power flows" or SCOPF. These programs can take account of many contingencies and calculate adjustments to generator MW, generator voltages, transformer taps, interchange, etc. We shall show how the SCOPF is formed in Chapter 13.

Together, the functions of system monitoring, contingency analysis, and corrective action analysis comprise a very complex set of tools that can aid in the secure operation of a power system. This chapter concentrates on contingency analysis.

11.2 FACTORS AFFECTING POWER SYSTEM SECURITY

As a consequence of many widespread blackouts in interconnected power systems, the priorities for operation of modern power systems have evolved to the following.

- Operate the system in such a way that power is delivered reliably.
- Within the constraints placed on the system operation by reliability considerations, the system will be operated most economically.

The greater part of this book is devoted to developing methods to operate a power system to gain maximum economy. But what factors affect its operation from a reliability standpoint? We will assume that the engineering groups who have designed the power system's transmission and generation systems have done so with reliability in mind. This means that adequate generation has been installed to meet the load and that adequate transmission has been installed to deliver the generated power to the load. If the operation of the system went on without sudden failures or without experiencing unanticipated operating states, we would probably have no reliability problems. However, any piece of equipment in the system can fail, either due to internal causes or due to external causes such as lightning strikes, objects hitting transmission towers, or human errors in setting relays. It is highly uneconomical, if not impossible, to build a power system with so much redundancy (i.e., extra transmission lines, reserve generation, etc.) that failures never cause load to be dropped on a system. Rather, systems are designed so that the probability of dropping load is acceptably small. Thus, most power systems are designed to have sufficient redundancy to withstand all major failure events, but this does not guarantee that the system will be 100% reliable.

Within the design and economic limitations, it is the job of the operators to try to maximize the reliability of the system they have at any given time. Usually, a power system is never operated with all equipment "in" (i.e., connected) since failures occur or maintenance may require taking equipment out of service. Thus, the operators play a considerable role in seeing that the system is reliable.

In this chapter, we will not be concerned with all the events that can cause trouble on a power system. Instead, we will concentrate on the possible consequences and remedial actions required by two major types of failure events—transmission-line outages and generation-unit failures.

Transmission-line failures cause changes in the flows and voltages on the transmission equipment remaining connected to the system. Therefore, the analysis of transmission failures requires methods to predict these flows and voltages so as to be sure they are within their respective limits. Generation failures can also cause flows and voltages to change in the transmission system, with the addition of dynamic problems involving system frequency and generator output.

11.3 CONTINGENCY ANALYSIS: DETECTION OF NETWORK PROBLEMS

We will briefly illustrate the kind of problems we have been describing by use of the six-bus network used in Chapter 4. The base-case power flow results for Example 4A are shown in Figure 11.1 and indicate a flow of 43.8 MW and 60.7 MVAR on the line from bus 3 to bus 6. The limit on this line can be expressed in MW or in MVA. For the purpose of this discussion, assume that we are only interested in the MW loading on the line. Now let us ask what will happen if the transmission line from bus 3 to bus 5 were to open. The resulting flows and voltages are shown in Figure 11.2. Note that the flow on the line from bus 3 to bus 6 has increased to 54.9 MW and that most of the other transmission lines also experienced changes in flow. Note also that the bus voltage magnitudes changed, particularly at bus 5, which is now almost 5% below nominal. Figures 11.3 and 11.4 are examples of generator outages and serve to illustrate the fact that generation outages can also result in changes in flows and voltages on a transmission network. In the example shown in Figure 11.3, all the generation lost from bus 3 is picked up on the generator at bus 1. Figure 11.4 shows the case when the loss of generation on bus 3 is made up by an increase in generation at buses 1 and 2. Clearly, the differences in flows and voltages show that how the lost generation is picked up by the remaining units is imporant.

If the system being modeled is part of a large interconnected network, the lost generation will be picked up by a large number of generating units outside the system's immediate control area. When this happens, the pickup in generation is seen as an increase in flow over the tie lines to the neighboring systems. To model this, we can build a network model of our own system plus an equivalent network of our neighbor's system and place the swing bus or reference bus in the equivalent system. A generator outage is then modeled so that all lost generation is picked up on the swing bus, which then appears as an increase on the tie flows, thus approximately modeling the generation loss when interconnected. If, however, the system of interest is not interconnected, then the loss of generation must be shown as a pickup in output on the other generation units within the system. An approximate method of doing this is shown in Section 11.3.2.

Operations personnel must know which line or generation outages will cause flows or voltages to fall outside limits. To predict the effects of outages,

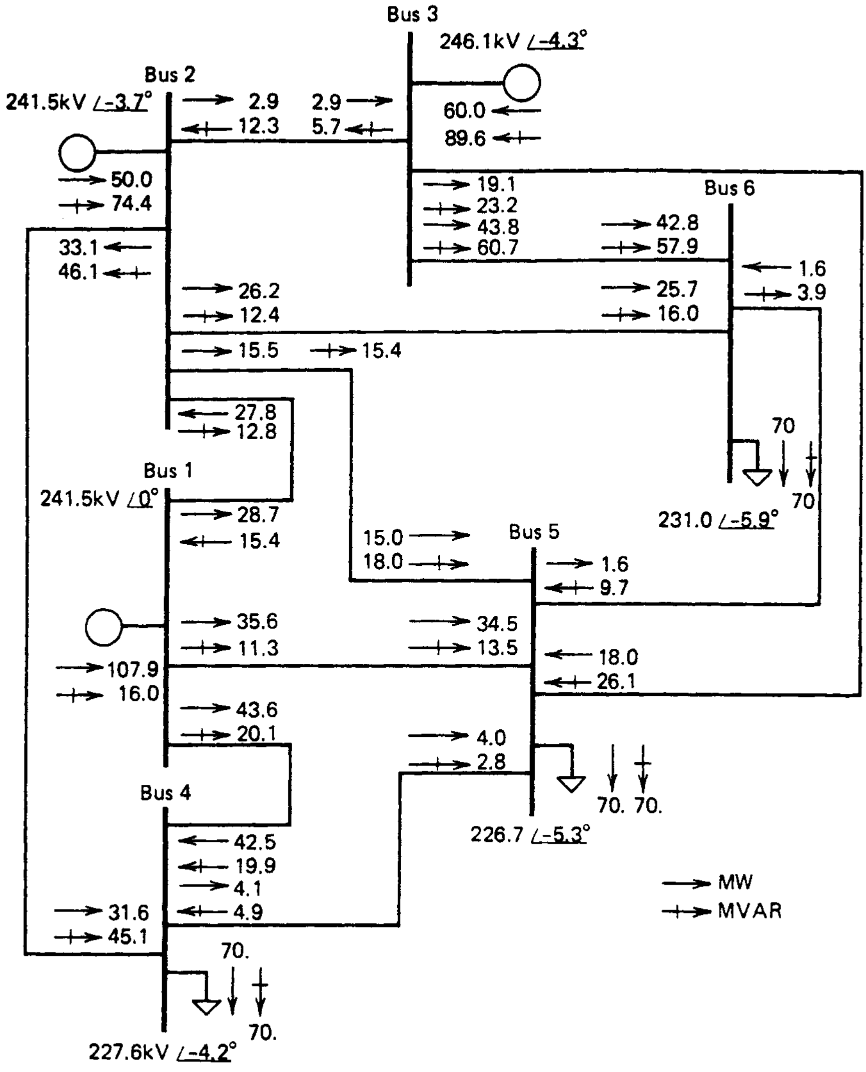


FIG. 11.1 Six-bus network base case AC power flow (see Example 4A).

contingency analysis techniques are used. Contingency analysis procedures model single failure events (i.e., one-line outage or one-generator outage) or multiple equipment failure events (i.e., two transmission lines, one transmission line plus one generator, etc.), one after another in sequence until "all credible outages" have been studied. For each outage tested, the contingency analysis procedure checks all lines and voltages in the network against their respective limits. The simplest form of such a contingency analysis technique is shown in Figure 11.5.

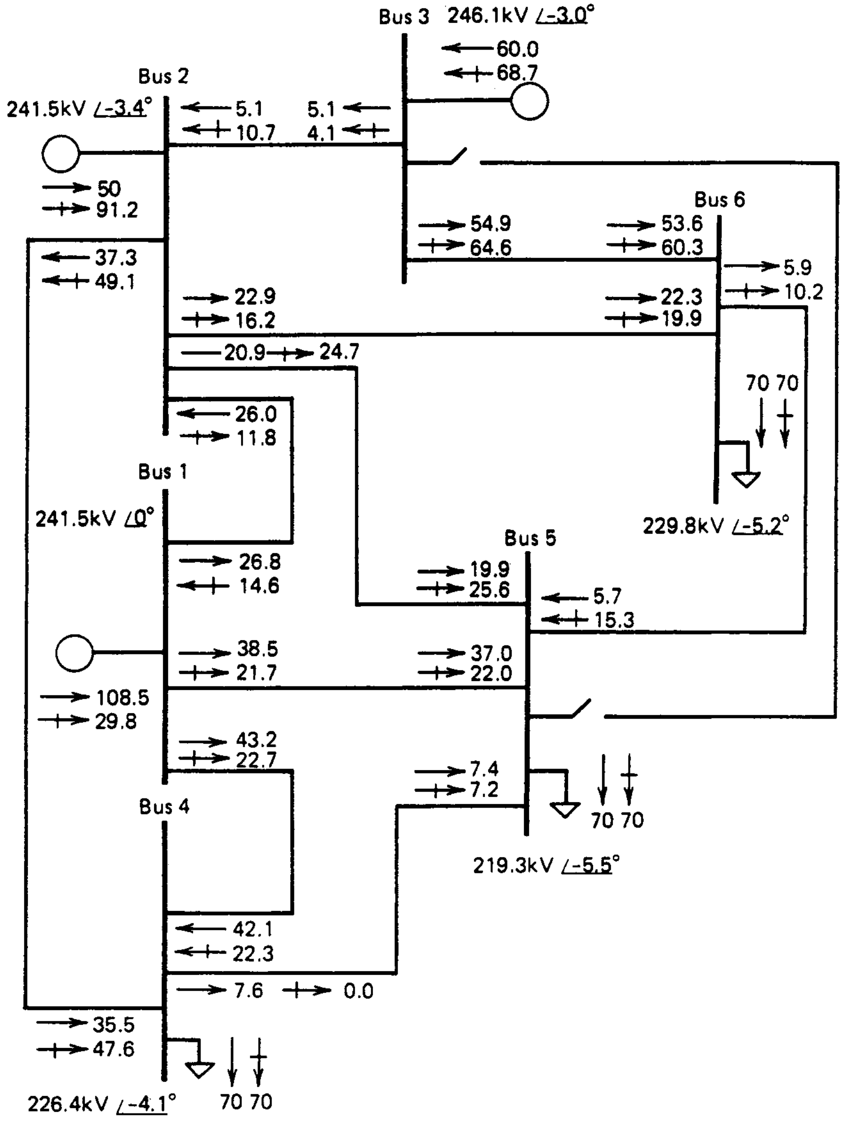


FIG. 11.2 Six-bus network line outage case; line from bus 3 to bus 5 opened.

The most difficult methodological problem to cope with in contingency analysis is the speed of solution of the model used. The most difficult logical problem is the selection of "all credible outages." If each outage case studied were to solve in 1 sec and several thousand outages were of concern, it would take close to 1 h before all cases could be reported. This would be useful if the system conditions did not change over that period of time. However, power

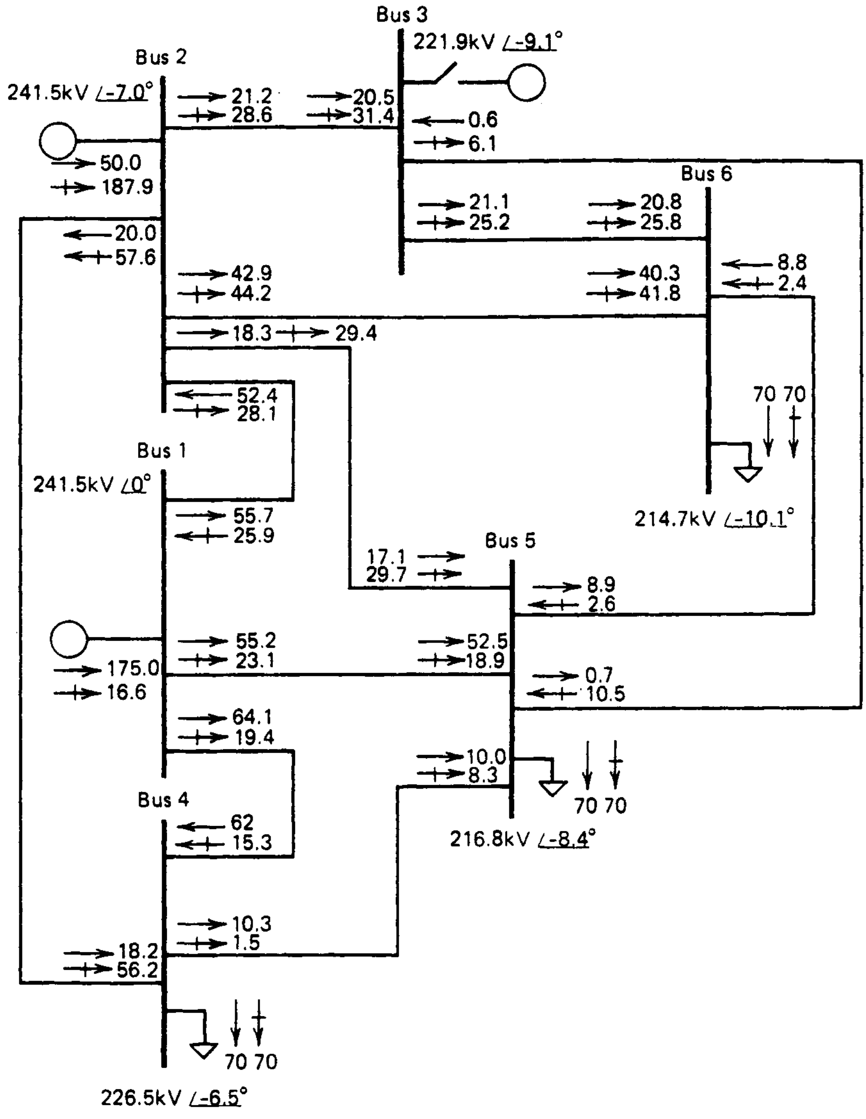


FIG. 11.3 Six-bus network generator outage case. Outage of generator on bus 3; lost generation picked up on generator 1.

systems are constantly undergoing changes and the operators usually need to know if the present operation of the system is safe, without waiting too long for the answer. Contingency analysis execution times of less than 1 min for several thousand outage cases are typical of computer and analytical technology as of 1995.

One way to gain speed of solution in a contingency analysis procedure is to

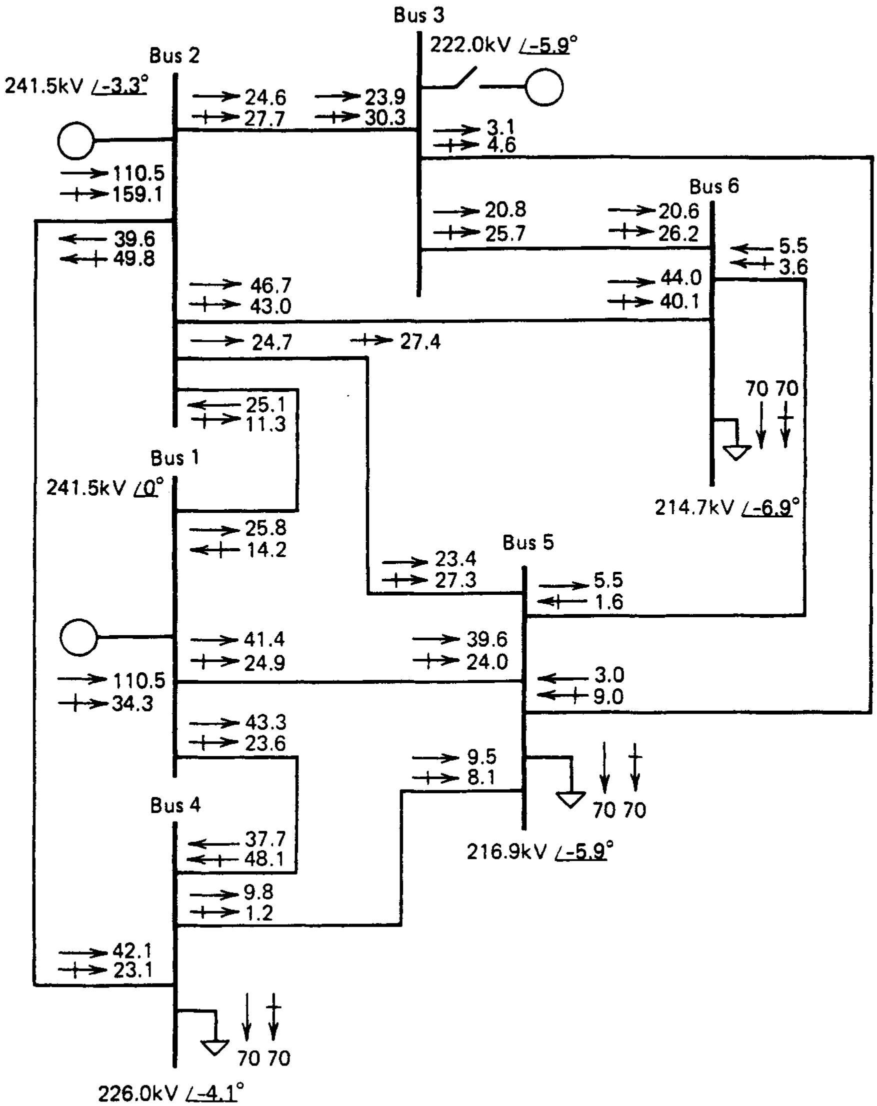


FIG. 11.4 Six-bus network generator outage case. Outage of generator on bus 3; lost generation picked up on generator 1 and generator 2.

use an approximate model of the power system. For many systems, the use of DC load flow models provides adequate capability. In such systems, the voltage magnitudes may not be of great concern and the DC load flow provides sufficient accuracy with respect to the megawatt flows. For other systems, voltage is a concern and full AC load flow analysis is required.

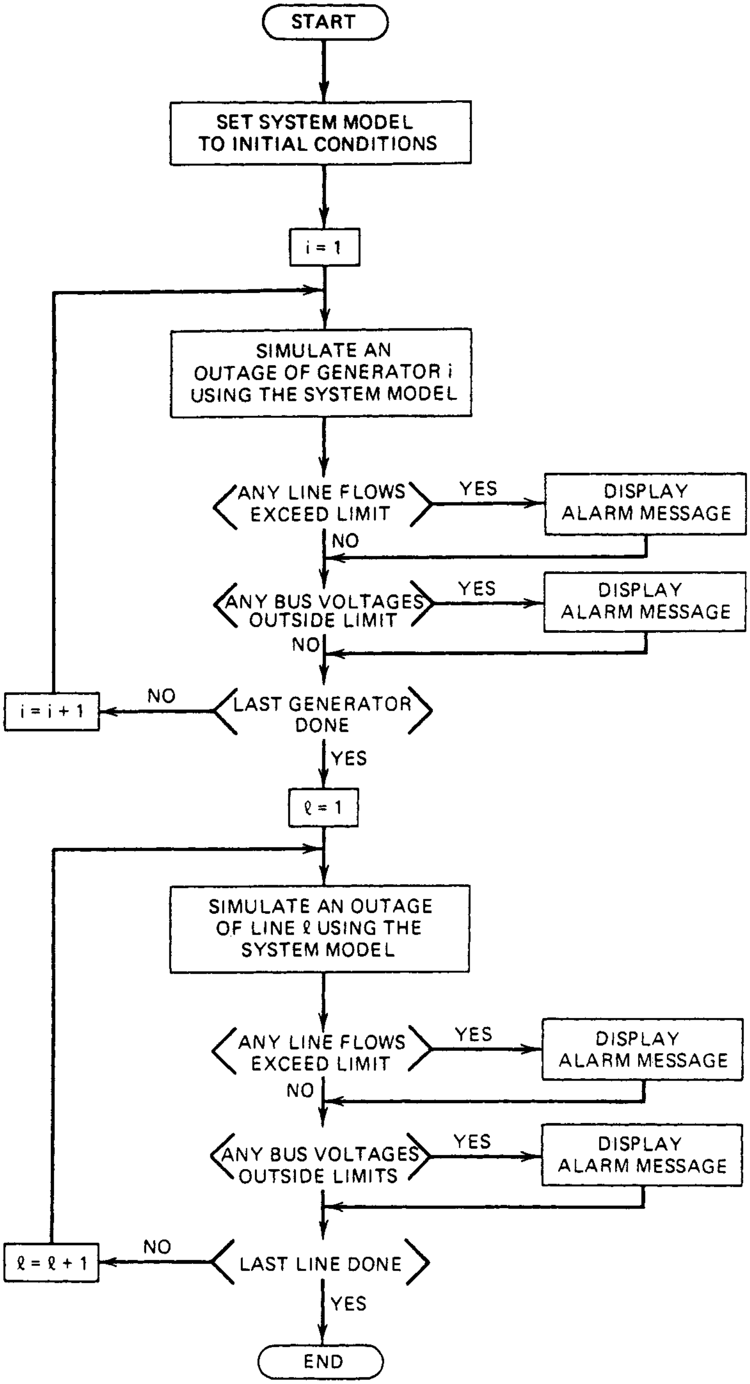


FIG. 11.5 Contingency analysis procedure.

11.3.1 An Overview of Security Analysis

A security analysis study which is run in an operations center must be executed very quickly in order to be of any use to operators. There are three basic ways to accomplish this.

- Study the power system with approximate but very fast algorithms.
- Select only the important cases for detailed analysis.
- Use a computer system made up of multiple processors or vector processors to gain speed.

The first method has been in use for many years and goes under various names such as "D factor methods," "linear sensitivity methods," "DC power flow methods," etc. This approach is useful if one only desires an approximate analysis of the effect of each outage. This text presents these methods under the name linear sensitivity factors and uses the same derivation as was presented in Chapter 4 under the DC power flow methods. It has all the limitations attributed to the DC power flow; that is, only branch MW flows are calculated and these are only within about 5% accuracy. There is no knowledge of MVAR flows or bus voltage magnitudes. Linear sensitivity factors are presented in Section 11.3.2.

If it is necessary to know a power system's MVA flows and bus voltage magnitudes after a contingency outage, then some form of complete AC power flow must be used. This presents a great deal of difficulty when thousands of cases must be checked. It is simply impossible, even on the fastest processors in existence today (1995) to execute thousands of complete AC power flows quickly enough. Fortunately, this need not be done as most of the cases result in power flow results which do not have flow or voltage limit violations. What is needed are ways to eliminate all or most of the nonviolation cases and only run complete power flows on the "critical" cases. These techniques go under the names of "contingency selection" or "contingency screening" and are introduced in Section 11.3.4.

Last of all, it must be mentioned that there are ways of running thousands of contingency power flows if special computing facilities are used. These facilities involve the use of many processors running separate cases in parallel, or vector processors which achieve parallel operation by "unwinding" the looping instruction sets in the computer code used. As of the writing of this edition (1995), such techniques are still in the research stage.

11.3.2 Linear Sensitivity Factors

The problem of studying thousands of possible outages becomes very difficult to solve if it is desired to present the results quickly. One of the easiest ways to provide a quick calculation of possible overloads is to use *linear sensitivity factors*. These factors show the approximate change in line flows for changes in generation on the network configuration and are derived from the DC load flow presented in Chapter 4. These factors can be derived in a variety of ways and basically come down to two types:

1. Generation shift factors.
2. Line outage distribution factors.

Here, we shall describe how these factors are used. The derivation of sensitivity factors is given in Appendix 11A.

The generation shift factors are designated $a_{\ell i}$ and have the following definition:

$$a_{\ell i} = \frac{\Delta f_\ell}{\Delta P_i} \tag{11.1}$$

where

ℓ = line index

i = bus index

Δf_ℓ = change in megawatt power flow on line ℓ when a change in generation, ΔP_i, occurs at bus i

ΔP_i = change in generation at bus i

It is assumed in this definition that the change in generation, ΔP_i, is exactly compensated by an opposite change in generation at the reference bus, and that all other generators remain fixed. The $a_{\ell i}$ factor then represents the sensitivity of the flow on line ℓ to a change in generation at bus i. Suppose one wanted to study the outage of a large generating unit and it was assumed that all the generation lost would be made up by the reference generation (we will deal with the case where the generation is picked up by many machines shortly). If the generator in question was generating P_i^0 MW and it was lost, we would represent ΔP_i as

$$\Delta P_i = -P_i^0 \tag{11.2}$$

and the new power flow on each line in the network could be calculated using a precalculated set of "a" factors as follows:

$$\hat{f}_\ell = f_\ell^0 + a_{\ell i}\Delta P_i \quad \text{for } \ell = 1 \ldots L \tag{11.3}$$

where

$\hat{f}_\ell$ = flow on line ℓ after the generator on bus i fails

f_ℓ^0 = flow before the failure

The "outage flow," $\hat{f}_\ell$, on each line can be compared to its limit and those exceeding their limit flagged for alarming. This would tell the operations personnel that the loss of the generator on bus i would result in an overload on line ℓ.

The generation shift sensitivity factors are linear estimates of the change in flow with a change in power at a bus. Therefore, the effects of simultaneous changes on several generating buses can be calculated using superposition. Suppose, for example, that the loss of the generator on bus i were compensated by governor action on machines throughout the interconnected system. One frequently used method assumes that the remaining generators pick up in proportion to their maximum MW rating. Thus, the proportion of generation pickup from unit j ($j \neq i$) would be

$$\gamma_{ji} = \frac{P_j^{\max}}{\sum_{\substack{k \\ k \neq i}} P_k^{\max}} \tag{11.4}$$

where

$P_k^{\max}$ = maximum MW rating for generator k

γ_{ji} = proportionality factor for pickup on generating unit j when unit i fails

Then, to test for the flow on line ℓ, under the assumption that all the generators in the interconnection participate in making up the loss, use the following:

$$\hat{f}_\ell = f_\ell^0 + a_{\ell i}\Delta P_i - \sum_{j \neq i} [a_{\ell j}\gamma_{ji}\Delta P_i] \tag{11.5}$$

Note that this assumes that no unit will actually hit its maximum. If this is apt to be the case, a more detailed generation pickup algorithm that took account of generation limits would be required.

The line outage distribution factors are used in a similar manner, only they apply to the testing for overloads when transmission circuits are lost. By definition, the line outage distribution factor has the following meaning:

$$d_{\ell,k} = \frac{\Delta f_\ell}{f_k^0} \tag{11.6}$$

where

$d_{\ell,k}$ = line outage distribution factor when monitoring line ℓ after an outage on line k

Δf_ℓ = change in MW flow on line ℓ

f_k^0 = original flow on line k before it was outaged (opened)

If one knows the power on line ℓ and line k, the flow on line ℓ with line k out can be determined using "d" factors.

$$\hat{f}_\ell = f_\ell^0 + d_{\ell,k} f_k^0 \tag{11.7}$$

where

$$f_\ell^0, f_k^0 = \text{preoutage flows on lines } \ell \text{ and } k\text{, respectively}$$

$$\hat{f}_\ell = \text{flow on line } \ell \text{ with line } k \text{ out}$$

By precalculating the line outage distribution factors, a very fast procedure can be set up to test all lines in the network for overload for the outage of a particular line. Furthermore, this procedure can be repeated for the outage of each line in turn, with overloads reported to the operations personnel in the form of alarm messages.

Using the generator and line outage procedures described earlier, one can program a digital computer to execute a contingency analysis study of the power system as shown in Figure 11.6. Note that a line flow can be positive or negative so that, as shown in Figure 11.6, we must check f against $-f_\ell^{max}$ as well as f_ℓ^{max}. This figure makes several assumptions; first, it assumes that the generator output for each of the generators in the system is available and that the line flow for each transmission line in the network is also available. Second, it assumes that the sensitivity factors have been calculated and stored, and that they are correct. The assumption that the generation and line flow MWs are available can be satisfied with telemetry systems or with state estimation techniques. The assumption that the sensitivity factors are correct is valid as long as the transmission network has not undergone any significant switching operations that would change its structure. For this reason, control systems that use sensitivity factors must have provision for updating the factors when the network is switched. A third assumption is that all generation pickup will be made on the reference bus. If this is not the case, substitute Eq. 11.5 in the generator outage loop.

EXAMPLE 11A

The $[X]$ matrix for our six-bus sample network is shown in Figure 11.7, together with the generation shift distribution factors and the line outage distribution factors.

The generation shift distribution factors that give the fraction of generation shift that is picked up on a transmission line are designated $a_{\ell i}$. The a factor is obtained by finding line ℓ along the rows and then finding the generator to be shifted along the columns. For instance, the shift factor for a change in the flow on line 1-4 when making a shift in generation on bus 3 is found in the second row, third column.

The line outage distribution factors are stored such that each row and column corresponds to one line in the network. The distribution factor $d_{\ell,k}$ is obtained by finding line ℓ along the rows and then finding line k along that row in the appropriate column. For instance, the line outage distribution factor that gives the fraction of flow picked up on line 3-5 for an outage on line 3-6

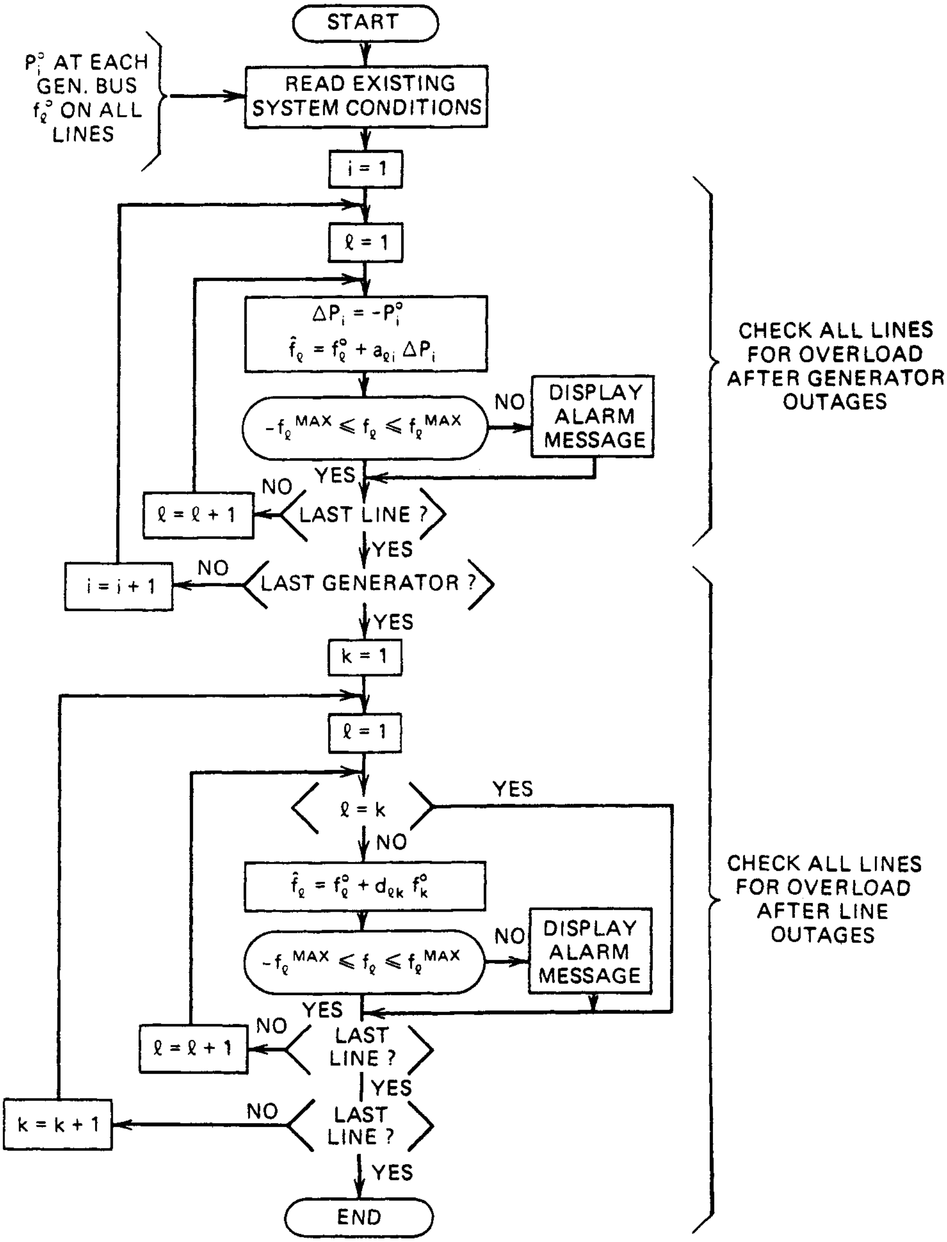


FIG. 11.6 Contingency analysis using sensitivity factors.

X Matrix for Six-Bus Sample System (Reference at Bus 1)

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.09412 & 0.08051 & 0.06298 & 0.06435 & 0.08129 \\
0 & 0.08051 & 0.16590 & 0.05897 & 0.09077 & 0.12895 \\
0 & 0.06298 & 0.05897 & 0.10088 & 0.05422 & 0.05920 \\
0 & 0.06435 & 0.09077 & 0.05422 & 0.12215 & 0.08927 \\
0 & 0.08129 & 0.12895 & 0.05920 & 0.08927 & 0.16328
\end{bmatrix}
$$

Generation Shift Factors For Six-Bus Sample System

	Bus 1	Bus 2	Bus 3
$\ell = 1$ (line 1-2)	0	−0.47	−0.40
$\ell = 2$ (line 1-4)	0	−0.31	−0.29
$\ell = 3$ (line 1-5)	0	−0.21	−0.30
$\ell = 4$ (line 2-3)	0	0.05	−0.34
$\ell = 5$ (line 2-4)	0	0.31	0.22
$\ell = 6$ (line 2-5)	0	0.10	−0.03
$\ell = 7$ (line 2-6)	0	0.06	−0.24
$\ell = 8$ (line 3-5)	0	0.06	0.29
$\ell = 9$ (line 3-6)	0	−0.01	0.37
$\ell = 10$ (line 4-5)	0	0	−0.08
$\ell = 11$ (line 5-6)	0	−0.06	−0.13

Line Outage Distribution Factors for Six-Bus Sample System

	$k=1$ (Line 1-2)	$k=2$ (Line 1-4)	$k=3$ (Line 1-5)	$k=4$ (Line 2-3)	$k=5$ (Line 2-4)	$k=6$ (Line 2-5)	$k=7$ (Line 2-6)	$k=8$ (Line 3-5)	$k=9$ (Line 3-6)	$k=10$ (Line 4-5)	$k=11$ (Line 5-6)
$\ell = 1$ (line 1-2)		0.64	0.54	−0.11	−0.50	−0.21	−0.12	−0.14	0.01	0.01	0.13
$\ell = 2$ (line 1-4)	0.59		0.46	−0.03	0.61	−0.06	−0.04	−0.04	0	−0.33	0.04
$\ell = 3$ (line 1-5)	0.41	0.36		0.15	−0.11	0.27	0.16	0.18	−0.02	0.32	−0.17
$\ell = 4$ (line 2-3)	−0.10	−0.03	0.18		0.12	0.23	0.47	−0.40	−0.53	0.17	0.13
$\ell = 5$ (line 2-4)	−0.59	0.76	−0.17	0.16		0.30	0.17	0.19	−0.02	−0.67	−0.19
$\ell = 6$ (line 2-5)	−0.19	−0.06	0.33	0.22	0.23		0.24	0.27	−0.03	0.31	−0.26
$\ell = 7$ (line 2-6)	−0.12	−0.04	0.21	0.51	0.15	0.27		−0.20	0.58	0.20	0.44
$\ell = 8$ (line 3-5)	−0.12	−0.04	0.20	−0.38	0.14	0.27	−0.17		0.47	0.19	−0.42
$\ell = 9$ (line 3-6)	0.01	0	−0.03	−0.62	−0.02	−0.03	0.64	0.60		−0.02	0.56
$\ell = 10$ (line 4-5)	0.01	−0.24	0.29	0.13	−0.39	0.24	0.14	0.15	−0.02		−0.15
$\ell = 11$ (line 5-6)	0.11	0.03	−0.18	0.12	−0.13	−0.23	0.36	−0.40	0.42	−0.18	

FIG. 11.7 Outage factors for a six-bus system.

is found in the eighth row and ninth column. Figure 11.3 shows an outage of the generator on bus 3 with all pickup of lost generation coming on the generator at bus 1. To calculate the flow on line 1-4 after the outage of the generator on bus 3, we need (see Figure 11.1):

$$\text{Base-case flow on line 1-4} = 43.6 \text{ MW}$$

$$\text{Base-case generation on bus 3} = 60 \text{ MW}$$

$$\text{Generation shift distribution factor} = a_{1\text{-}4,3} = -0.29$$

Then the flow on line 1-4 after generator outage is = base-case $\text{flow}_{1\text{-}4}$ + $a_{1\text{-}4,3} \Delta P_{\text{gen}_3} = 43.6 + (-0.29)(-60 \text{ MW}) = 61$ MW.

To show how the line outage and generation shift factors are used, calculate some flows for the outages shown in Figures 11.2 and 11.3. Figure 11.2 shows an outage of line 3-5. If we wish to calculate the power flowing on line 3-6 with line 3-5 opened, we would need the following.

$$\text{Base-case flow on line 3-5} = 19.1 \text{ MW}$$

$$\text{Base-case flow on line 3-6} = 43.8 \text{ MW}$$

$$\text{Line outage distribution factor: } d_{3\text{-}6,3\text{-}5} = 0.60$$

Then the flow on 3-6 after the outage is = base $\text{flow}_{3\text{-}6} + d_{3\text{-}6,3\text{-}5} \times$ base $\text{flow}_{3\text{-}5} = 43.8 + (0.60) \times (19.1) = 55.26$ MW.

In both outage cases, the flows calculated by the sensitivity methods are reasonably close to the values calculated by the full AC load flows as shown in Figures 11.2 and 11.3.

11.3.3 AC Power Flow Methods

The calculations made by network sensitivity methods are faster than those made by AC power flow methods and therefore find wide use in operations control systems. However, there are many power systems where voltage magnitudes are the critical factor in assessing contingencies. In addition, there are some systems where VAR flows predominate on some circuits, such as underground cables, and an analysis of only the MW flows will not be adequate to indicate overloads. When such situations present themselves, the network sensitivity methods may not be adequate and the operations control system will have to incorporate a full AC power flow for contingency analysis.

When an AC power flow is to be used to study each contingency case, the speed of solution and the number of cases to be studied are critical. To repeat what was said before, if the contingency alarms come too late for operators to act, they are worthless. Most operations control centers that use an AC power flow program for contingency analysis use either a Newton–Raphson or the decoupled power flow. These solution algorithms are used because of their speed of solution and the fact that they are reasonably reliable in convergence when solving difficult cases. The decoupled load flow has the further advantage that a matrix alteration formula can be incorporated into it to simulate the outage of transmission lines without reinverting the system Jacobian matrix at each iteration.

The simplest AC security analysis procedure consists of running an AC power flow analysis for each possible generator, transmission line, and transformer outage as shown in Figure 11.8. This procedure will determine the overloads and voltage limit violations accurately (at least within the accuracy of the power flow program, the accuracy of the model data, and the accuracy with which we have obtained the initial conditions for the power flow). It does suffer a major drawback, however, and that concerns the time such a program takes to execute. If the list of outages has several thousand entries, then the total time to test for all of the outages can be too long.

We are thus confronted with a dilemma. Fast, but inaccurate, methods involving the *a* and *d* factors can be used to give rapid analysis of the system, but they cannot give information about MVAR flows and voltages. Slower, full AC power flow methods give full accuracy but take too long.

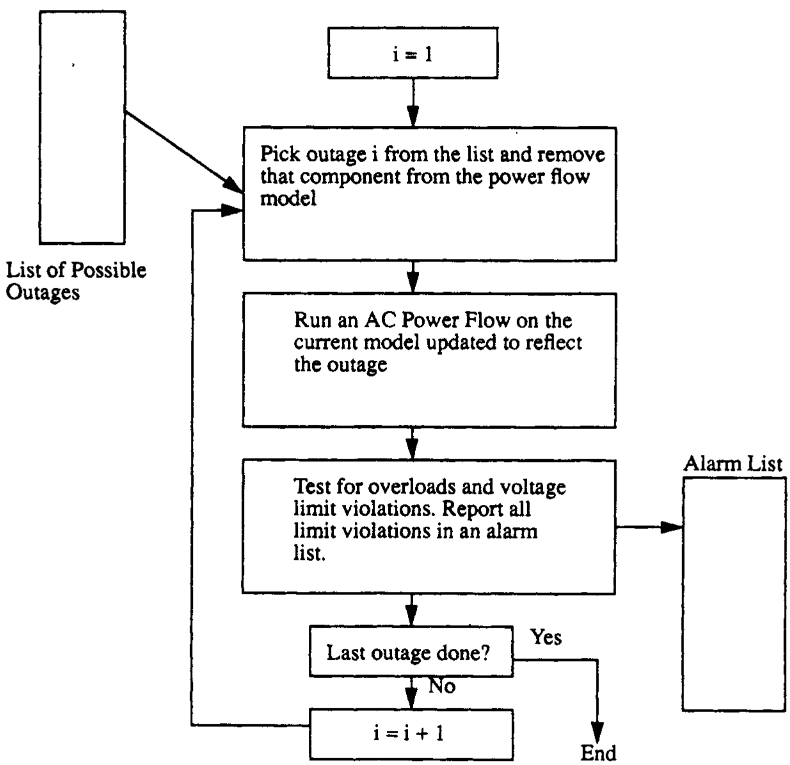


FIG. 11.8 AC power flow security analysis.

Fortunately, there is a way out of this dilemma. Because of the way the power system is designed and operated, very few of the outages will actually cause trouble. That is, most of the time spent running AC power flows will go for solutions of the power flow model that discover that there are no problems. Only a few of the power flow solutions will, in fact, conclude that an overload or voltage violation exists.

The solution to this dilemma is to find a way to select contingencies in such a way that only those that are likely to result in an overload or voltage limit violation will actually be studied in detail and the other cases will go unanalyzed. A flowchart for a process like this appears in Figure 11.9. Selecting

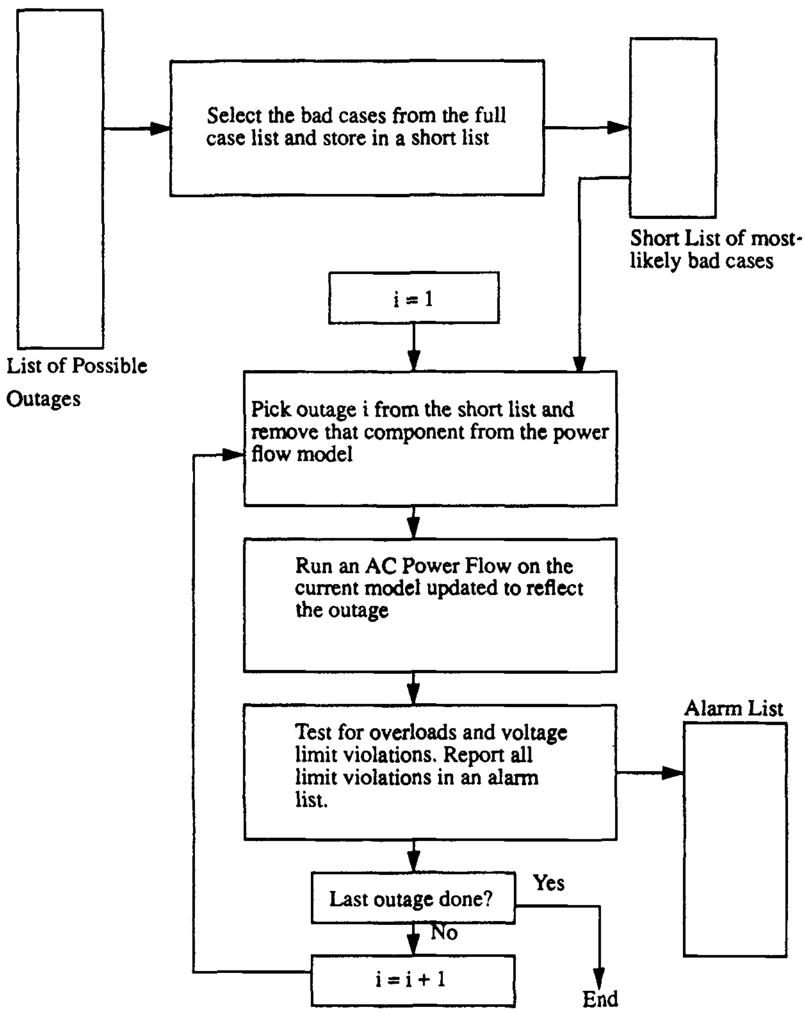


FIG. 11.9 AC power flow security analysis with contingency case selection.

the bad or likely trouble cases from the full outage case list is not an exact procedure and has been the subject of intense research for the past 15 years. Two sources of error can arise.

1. **Placing too many cases on the short list:** this is essentially the "conservative" approach and simply leads to longer run times for the security analysis procedure to execute.
2. **Skipping cases:** here, a case that would have shown a problem is not placed on the short list and results in possibly having that outage take place and cause trouble without the operators being warned.

11.3.4 Contingency Selection

We would like to get some measure as to how much a particular outage might affect the power system. The idea of a performance index seems to fulfill this need. The definition for the overload performance index (PI) is as follows:

$$\mathrm{PI} = \sum_{\substack{\text{all branches} \\ l}} \left(\frac{P_{\text{flow } l}}{P_l^{\max}} \right)^{2n} \tag{11.8}$$

If n is a large number, the PI will be a small number if all flows are within limit, and it will be large if one or more lines are overloaded. The problem then is how to use this performance index.

Various techniques have been tried to obtain the value of PI when a branch is taken out. These calculations can be made exactly if $n = 1$; that is, a table of PI values, one for each line in the network, can be calculated quite quickly. The selection procedure then involves ordering the PI table from largest value to least. The lines corresponding to the top of the list are then the candidates for the short list. One procedure simply ordered the PI table and then picked the top N_c entries from this list and placed them on the short list (see reference 8).

However when $n = 1$, the PI does not snap from near zero to near infinity as the branch exceeds its limit. Instead, it rises as a quadratic function. A line that is just below its limit contributes to PI almost equal to one that is just over its limit. The result is a PI that may be large when many lines are loaded just below their limit. Thus the PI's ability to distinguish or detect bad cases is limited when $n = 1$. Ordering the PI values when $n = 1$ usually results in a list that is not at all representative of one with the truly bad cases at the top. Trying to develop an algorithm that can quickly calculate PI when $n = 2$ or larger has proven extremely difficult.

One way to perform an outage case selection is to perform what has been called the *1P1Q method* (see references 9 and 10). Here, a decoupled power flow is used. As shown in Figure 11.10, the solution procedure is interrupted after one iteration (one $P - \theta$ calculation and one $Q - V$ calculation; thus, the name 1P1Q). With this procedure, the PI can use as large an n value as desired, say $n = 5$. There appears to be sufficient information in the solution at the end of

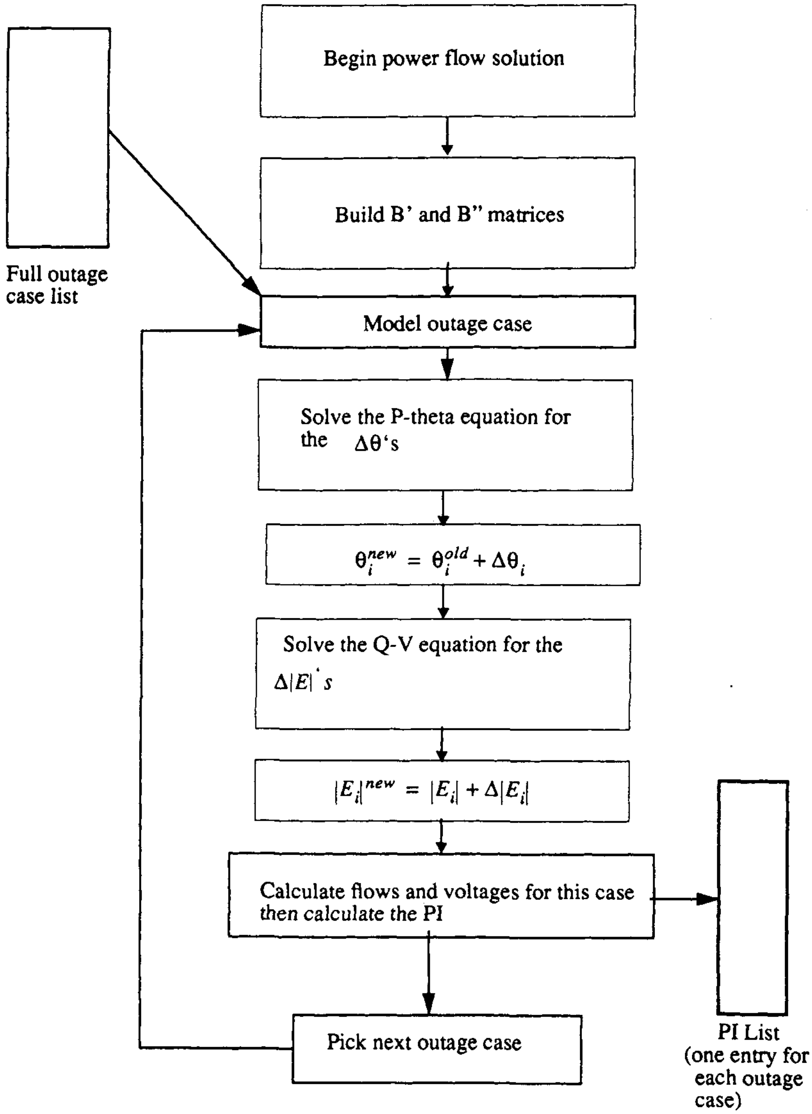


FIG. 11.10 The 1P1Q contingency selection procedure.

the first iteration of the decoupled power flow to give a reasonable PI. Another advantage to this procedure is the fact that the voltages can also be included in the PI. Thus, a different PI can be used, such as:

$$\mathrm{PI} = \sum_{\substack{\text{all branches}\\ l}} \left(\frac{P_{\text{flow } l}}{P_l^{\max}}\right)^{2n} + \sum_{\substack{\text{all buses}\\ i}} \left(\frac{\Delta|E_i|}{\Delta|E|^{\max}}\right)^{2m} \tag{11.9}$$

where $\Delta|E_i|$ is the difference between the voltage magnitude as solved at the end of the 1P1Q procedure and the base-case voltage magnitude. $\Delta|E|^{max}$ is a value set by utility engineers indicating how much they wish to limit a bus voltage from changing on one outage case.

To complete the security analysis, the PI list is sorted so that the largest PI appears at the top. The security analysis can then start by executing full power flows with the case which is at the top of the list, then solve the case which is second, and so on down the list. This continues until either a fixed number of cases is solved, or until a predetermined number of cases are solved which do not have any alarms.

11.3.5 Concentric Relaxation

Another idea to enter the field of security analysis in power systems is that an outage only has a limited geographical effect. The loss of a transmission line does not cause much effect a thousand miles away; in fact, we might hope that it doesn't cause much trouble beyond 20 miles from the outage, although if the line were a heavily loaded, high-voltage line, its loss will most likely be felt more than 20 miles away.

To realize any benefit from the limited geographical effect of an outage, the power system must be divided into two parts: the affected part and the part that is unaffected. To make this division, the buses at the end of the outaged line are marked as layer zero. The buses that are one transmission line or transformer from layer zero are then labeled layer one. This same process can be carried out, layer by layer, until all the buses in the entire network are included. Some arbitrary number of layers is chosen and all buses included in that layer and lower-numbered layers are solved as a power flow with the outage in place. The buses in the higher-numbered layers are kept as constant voltage and phase angle (i.e., as reference buses).

This procedure can be used in two ways: either the solution of the layers included becomes the final solution of that case and all overloads and voltage violations are determined from this power flow, or the solution simply is used to form a performance index for that outage. Figure 11.11 illustrates this layering procedure.

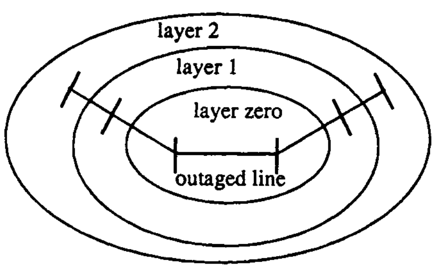


FIG. 11.11 Layering of outage effects.

The concentric relaxation procedure was originally proposed by Zaborsky (see reference 13). The trouble with the concentric relaxation technique is that it requires more layers for circuits whose influence is felt further from the outage.

11.3.6 Bounding

A paper by Brandwajn (reference 11) solves at least one of the problems in using the concentric relaxation method. Namely, it uses an adjustable region around the outage to solve for the outage case overloads. In reference 11, this is applied only to the linear (DC) power flow; it has subsequently been extended for AC network analysis.

To perform the analysis in the bounding technique we define three subsystems of the power system as follows:

N1 = the subsystem immediately surrounding the outaged line

N2 = the external subsystem that we shall not solve in detail

N3 = the set of boundary buses that separate N1 and N2

The subsystems appear as shown in Figure 11.12. The bounding method is based on the fact that we can make certain assumptions about the phase angle spread across the lines in N2, given the injections in N1 and the maximum phase angle appearing across any two buses in N3. In Appendix 11A of this chapter we show how to calculate the ΔP_k and the ΔP_m injections that will make the phase angles on buses k and m simulate the outage of line k–m.

If we are given a transmission line in N2 with flow f_{pq}^0, then there is a maximum amount that the flow on pq can shift. That is, it can increase from

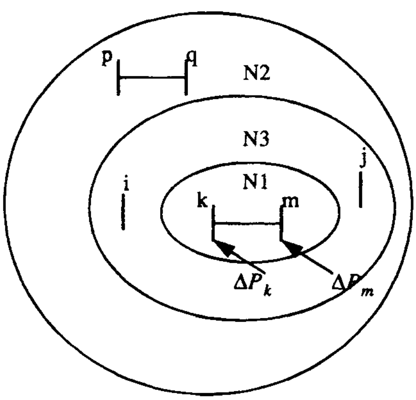


FIG. 11.12 Layers used in bounding analysis.

f^0_{pq} to its upper limit or it can decrease to its lower limit. Then,

$$\Delta f^{\max}_{pq} = \text{smaller of } [(f^+_{pq} - f^0_{pq}), (f^0_{pq} - f^-_{pq})] \tag{11.10}$$

Further, we can translate this into a maximum change in phase angle difference as follows:

$$f_{pq} = \frac{1}{x_{pq}} (\theta_p - \theta_q) \tag{11.11}$$

or

$$\Delta f_{pq} = \frac{1}{x_{pq}} (\Delta\theta_p - \Delta\theta_q) \tag{11.12}$$

and finally:

$$(\Delta\theta_p - \Delta\theta_q)^{\max} = \Delta f^{\max}_{pq} x_{pq} \tag{11.13}$$

Thus, we can define the maximum change in the phase angle difference across pq. Reference 11 develops the theorem that:

$$|\Delta\theta_p - \Delta\theta_q| < |\Delta\theta_i - \Delta\theta_j| \tag{11.14}$$

where i and j are any pair of buses in N3, $\Delta\theta_i$ is the largest $\Delta\theta$ in N3, and $\Delta\theta_j$ is the smallest $\Delta\theta$ in N3 (see Appendix 11B).

Equation 11.14 is interpreted as follows: the right-hand side, $|\Delta\theta_i - \Delta\theta_j|$, provides an upper limit to the maximum change in angular spread across any circuit in N2. Thus, it provides us with a limit as to how far any of the N2 circuits can change their flow. By combining Eqs. 11.13 and 11.14 we obtain:

$$\Delta f^{\max}_{pq} x_{pq} < |\Delta\theta_i - \Delta\theta_j| \tag{11.15}$$

Figure 11.13 shows a graphical interpretation of the bounding process. There are two cases represented in Figure 11.13: a circuit on the top of the figure that

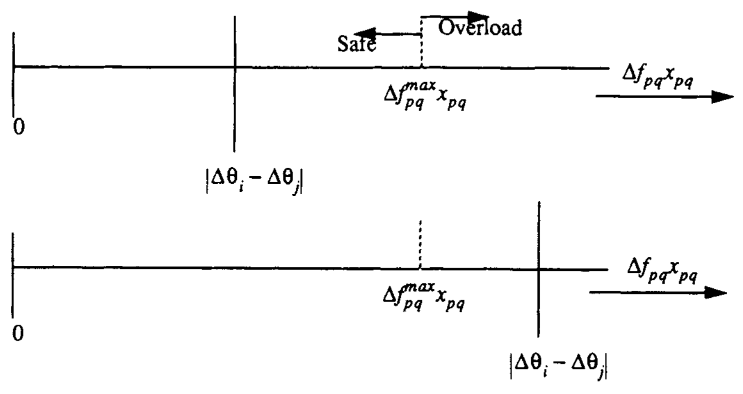


FIG. 11.13 Interpretation of bounding.

cannot go over limit, while that on the bottom could. In each case, the horizontal line represents the change in flow on circuit pq times its reactance, $\Delta f_{pq} x_{pq}$; the dotted line, labeled $\Delta f_{pq}^{max} x_{pq}$, represents the point where circuit pq will go into overload and is determined as explained previously. Any value of $\Delta f_{pq} x_{pq}$ to the right of the dotted line represents an overload.

The solid line labeled $|\Delta\theta_i - \Delta\theta_j|$ represents the upper limit on $\Delta f_{pq} x_{pq}$. Thus, if the solid line is below (to the left) of the dotted line, then the circuit theory upper limit predicts that the circuit cannot go into overload; if on the other hand, the solid line is above (to the right of) the dotted line, the circuit may be shifted in flow due to the outage so as to violate a limit.

A completely safe N2 region would be one in which the maximum $|\Delta\theta_i - \Delta\theta_j|$ upper limit is small enough to be less than all of the $\Delta f_{pq}^{max} x_{pq}$ limits. In fact, as the N1 region is enlarged, the value of $|\Delta\theta_i - \Delta\theta_j|$ will become smaller and smaller. Therefore, the test to determine whether the N1 region encompasses all possible overloaded circuits should be as follows:

> All circuits in N2 are safe from overload if the value of $|\Delta\theta_i - \Delta\theta_j|$ is less than the smallest value of $\Delta f_{pq}^{max} x_{pq}$ over all pairs pq, where pq corresponds to the buses at the ends of circuits in N2

If this condition fails, then we have to expand N1, calculate a new $|\Delta\theta_i - \Delta\theta_j|$ in N3, and rerun the test over the newly defined N2 region circuits. When an N2 is found which passes the test, we are done and only region N1 need be studied in detail.

References 10 and 12 extend this concept to screening for AC contingency effects. Such contingency selection/screening techniques form the foundation for many real-time computer security analysis algorithms.

EXAMPLE 11B

In this example, we shall take the six-bus sample system used previously and show how the bounding technique works so that not all of the circuits in the system need be analyzed. Note that this is a small system so that the net savings in computer time may not be that great. Nonetheless, it demonstrates the principles used in the bounding technique quite well.

We shall study the outage of transmission line 3-6. The DC power flow will be used throughout and the initial conditions will be those shown in Figure 4.12. The MW limits on the transmission lines are shown in the table at the top of the next page.

Line	MW Limit
1-2	30
1-4	50
1-5	40
2-3	20
2-4	40
2-5	20
2-6	30
3-5	20
3-6	60
4-5	20
5-6	20

In this example, we shall proceed in steps. Step A will analyze the system as if the N1 and N3 regions consist of only line 3-6 itself, as shown in Figure 11.14. If the bounding criteria is met, no other analysis need be done as it will establish that no overloads exist anywhere in the system. If the bounding criteria fails, we still proceed to step B. Step B expands the bounded region from line 3-6 to include all buses which are once removed from buses 3 and 6; that is, it includes buses 2, 3, 5, and 6 as shown in Figure 11.15, and in this case the boundary of the region, N3, consists of buses 2 and 5.

To start, we need to calculate $\Delta f_{pq}^{\max}$ and then $\Delta f_{pq}^{\max} x_{pq}$ as given in Eqs. 11.10 through 11.13. These values are given below where the flows and flow limits are all converted to per unit on a 100 MVA base. (The line reactances are found in the appendix to Chapter 4.)

Line	MW Limit (per unit)	f_{pq}^{0} (per unit)	$\Delta f_{pq}^{\max}$	x_{pq}	$\Delta f_{pq}^{\max} x_{pq}$
1-2	0.30	0.253	0.047	0.20	0.0094
1-4	0.50	0.416	0.084	0.20	0.0168
1-5	0.40	0.331	0.069	0.30	0.0207
2-3	0.20	0.018	0.182	0.25	0.0455
2-4	0.40	0.325	0.075	0.10	0.0075
2-5	0.20	0.162	0.038	0.30	0.0114
2-6	0.30	0.248	0.052	0.20	0.0104
3-5	0.20	0.169	0.031	0.26	0.00806
3-6	0.60	0.449	—	—	—
4-5	0.20	0.041	0.159	0.40	0.0636
5-6	0.20	0.003	0.197	0.30	0.0591

For step A, we use Eq. 11A.13 from Appendix 11A to calculate $\delta_{3,36}$ and $\delta_{6,36}$ as

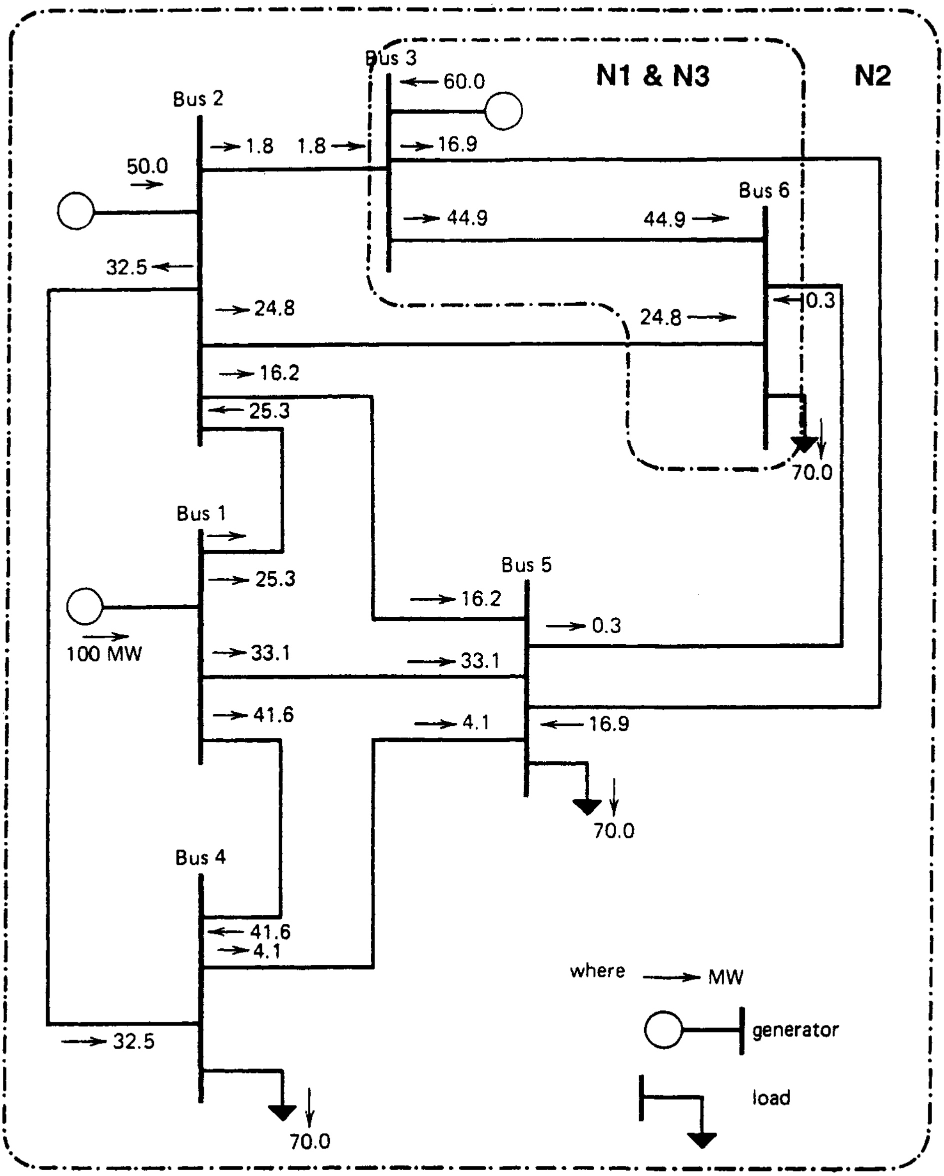


FIG. 11.14 Step A of Example 11B.

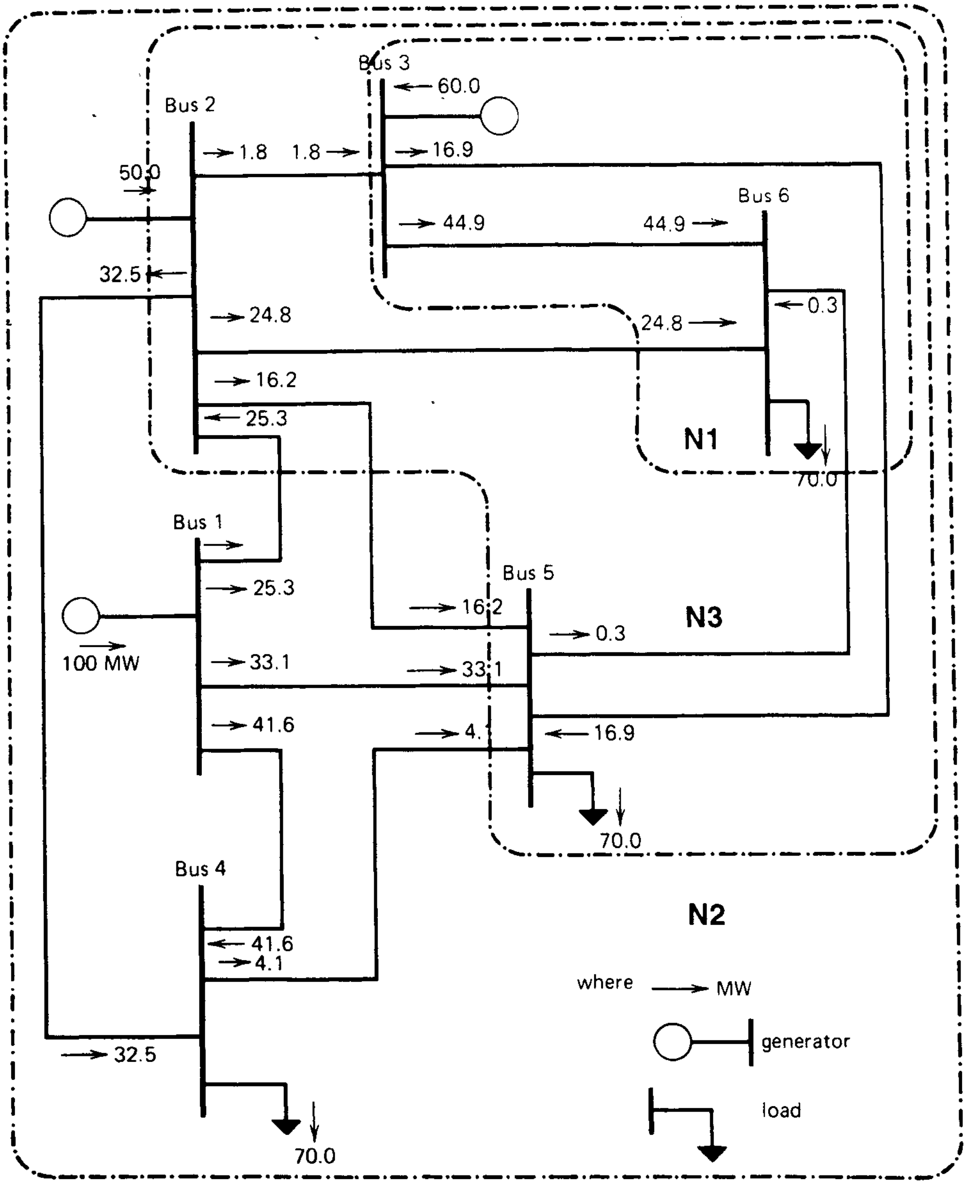


FIG. 11.15 Step B of Example 11B.

shown below.

$$\delta_{3,36} = \frac{(X_{33} - X_{36})x_{36}}{x_{36} - (X_{33} + X_{66} - 2X_{36})} = 0.12865$$

$$\delta_{6,36} = \frac{(X_{63} - X_{66})x_{36}}{x_{36} - (X_{ss} + X_{66} - X_{36})} = -0.11953$$

Then using Eq. 11A.11

$$|\Delta\theta_3 - \Delta\theta_6| = 0.111437$$

According to the criterion in Eq. 11.14, the value $|\Delta\theta_i - \Delta\theta_j|$ must be less that the smallest value of $|\Delta\theta_p - \Delta\theta_q|$ which equals $\Delta f_{pq}^{\max} x_{pq}$ and is found in the table above to be at line 2-4. Since $|\Delta\theta_3 - \Delta\theta_6| = 0.111437$ and the minimum $|\Delta\theta_i - \Delta\theta_j|$ is $|\Delta\theta_2 - \Delta\theta_4|$ which has a value of 0.0075, the criteria fails. We must proceed to step B.

Step B requires that we calculate $|\Delta\theta_i - \Delta\theta_j|$ for buses 2 and 5. This value is 0.003564 and the bounding criteria is satisfied.

If we had used the *d* factors for the six-bus system as shown in Example 11A, we could simply find all the line flows for the 3-6 outage as shown in the table below.

Line	MW Limit (per unit)	f_{pq}^0 (per unit)	$f_{pq}^{3\text{-}6\text{ out}}$	
1-2	0.30	0.253	0.257	
1-4	0.50	0.416	0.416	
1-5	0.40	0.331	0.322	
2-3	0.20	0.018	−0.220	overload
2-4	0.40	0.325	0.316	
2-5	0.20	0.162	0.148	
2-6	0.30	0.248	0.508	overload
3-5	0.20	0.169	0.380	overload
3-6	0.60	0.449	—	
4-5	0.20	0.041	0.320	
5-6	0.20	0.003	0.191	

Note that three overloads exist on lines 2-3, 2-6, and 3-5, which are all within the bounded region N1 + N3 in Figure 11.15.

APPENDIX 11A
Calculation of Network Sensitivity Factors

First, we show how to derive the generation-shift sensitivity factors. To start, repeat Eq. 4.36.

$$\boldsymbol{\theta} = [X]\mathbf{P} \tag{11A.1}$$

This is the standard matrix calculation for the DC load flow. Since the DC power-flow model is a linear model, we may calculate perturbations about a given set of system conditions by use of the same model. Thus, if we are interested in the changes in bus phase angles, $\Delta\theta$, for a given set of changes in the bus power injections, $\Delta\mathbf{P}$, we can use the following calculation.

$$\Delta\boldsymbol{\theta} = [X]\Delta\mathbf{P} \tag{11A.2}$$

In Eq. 11A.1, it is assumed that the power on the swing bus is equal to the sum of the injections of all the other buses. Similarly, the net perturbation of the swing bus in Eq. 11A.2 is the sum of the perturbations on all the other buses.

Suppose that we are interested in calculating the generation shift sensitivity factors for the generator on bus i. To do this, we will set the perturbation on bus i to $+1$ and the perturbation on all the other buses to zero. We can then solve for the change in bus phase angles using the matrix calculation in Eq. 11A.3.

$$\Delta\boldsymbol{\theta} = [X]\begin{bmatrix} +1 \\ -1 \end{bmatrix}\begin{matrix} \text{—row } i \\ \text{—ref row} \end{matrix} \tag{11A.3}$$

The vector of bus power injection perturbations in Eq. 11A.3 represents the situation when a 1 pu power increase is made at bus i and is compensated by a 1 pu decrease in power at the reference bus. The $\Delta\theta$ values in Eq. 11A.3 are thus equal to the derivative of the bus angles with respect to a change in power injection at bus i. Then, the required sensitivity factors are

$$\begin{aligned} a_{\ell i} &= \frac{df_\ell}{dP_i} = \frac{d}{dP_i}\left[\frac{1}{x_\ell}(\theta_n - \theta_m)\right] \\ &= \frac{1}{x_\ell}\left(\frac{d\theta_n}{dP_i} - \frac{d\theta_m}{dP_i}\right) = \frac{1}{x_\ell}(X_{ni} - X_{mi}) \end{aligned} \tag{11A.4}$$

where

$$X_{ni} = \frac{d\theta_n}{dP_i} = n^{\text{th}} \text{ element from the } \Delta\boldsymbol{\theta} \text{ vector in Eq. 11A.3}$$

$$X_{mi} = \frac{d\theta_m}{dP_i} = m^{\text{th}} \text{ element from the } \Delta\boldsymbol{\theta} \text{ vector in Eq. 11A.3}$$

$$x_\ell = \text{line reactance for line } \ell$$

A line outage may be modeled by adding two power injections to a system, one at each end of the line to be dropped. The line is actually left in the system and the effects of its being dropped are modeled by injections. Suppose line k

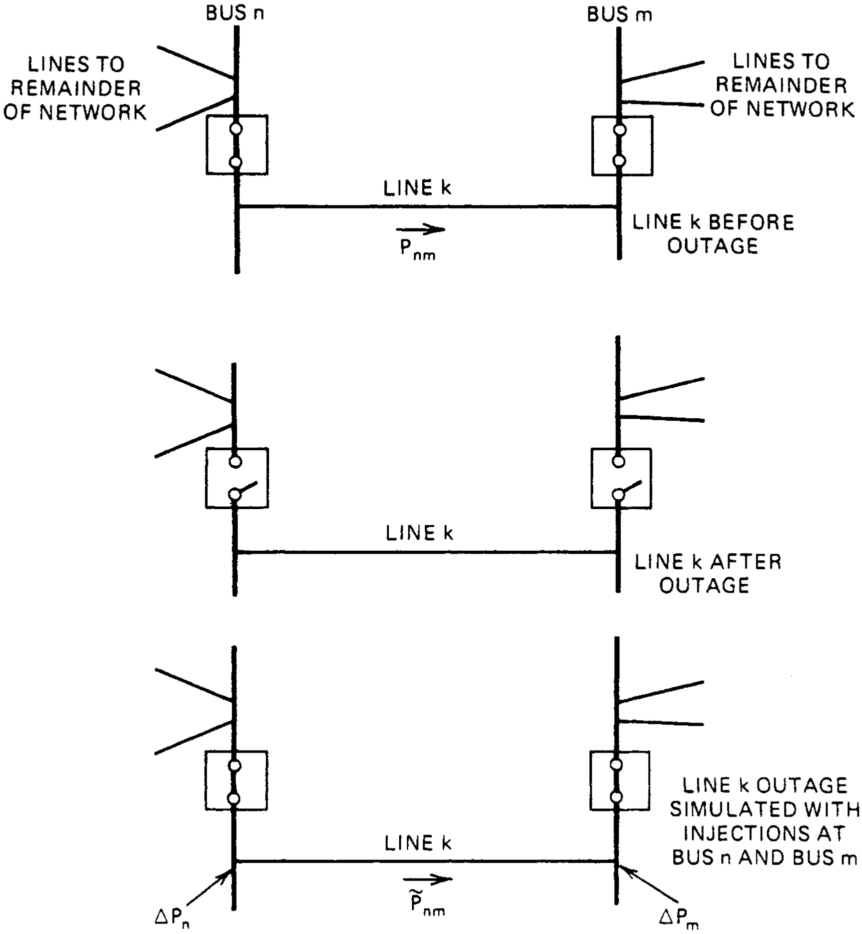


FIG. 11.16 Line outage modeling using injections.

from bus n to bus m were opened by circuit breakers as shown in Figure 11.16. Note that when the circuit breakers are opened, no current flows through them and the line is completely isolated from the remainder of the network. In the bottom part of Figure 11.16, the breakers are still closed but injections ΔP_n and ΔP_m have been added to bus n and bus m, respectively. If $\Delta P_n = \tilde{P}_{nm}$, where $\tilde{P}_{nm}$ is equal to the power flowing over the line, and $\Delta P_m = -\tilde{P}_{nm}$, we will still have no current flowing through the circuit breakers even though they are closed. As far as the remainder of the network is concerned, the line is disconnected.

Using Eq. 11A.2 relating to $\mathbf{\Delta\theta}$ and $\mathbf{\Delta P}$, we have

$$\mathbf{\Delta\theta} = [X]\mathbf{\Delta P}$$

where

$$\Delta\mathbf{P} = \begin{bmatrix} \vdots \\ \Delta P_n \\ \vdots \\ \Delta P_m \end{bmatrix}$$

so that

$$\begin{aligned} \Delta\theta_n &= X_{nn}\Delta P_n + X_{nm}\Delta P_m \\ \Delta\theta_m &= X_{mn}\Delta P_n + X_{mm}\Delta P_m \end{aligned} \tag{11A.5}$$

define

$\theta_n, \theta_m, P_{nm}$	to exist before the outage, where P_{nm} is the flow on line k from bus n to bus m
$\Delta\theta_n, \Delta\theta_m, \Delta P_{nm}$	to be the incremental changes resulting from the outage
$\tilde{\theta}_n, \tilde{\theta}_m, \tilde{P}_{nm}$	to exist after the outage

The outage modeling criteria requires that the incremental injections ΔP_n and ΔP_m equal the power flowing over the outaged line *after* the injections are imposed. Then, if we let the line reactance be x_k

$$\tilde{P}_{nm} = \Delta P_n = -\Delta P_m \tag{11A.6}$$

where

$$\tilde{P}_{nm} = \frac{1}{x_k}(\tilde{\theta}_n - \tilde{\theta}_m)$$

then

$$\begin{aligned} \Delta\theta_n &= (X_{nn} - X_{nm})\Delta P_n \\ \Delta\theta_m &= (X_{mm} - X_{mn})\Delta P_n \end{aligned} \tag{11A.7}$$

and

$$\begin{aligned} \tilde{\theta}_n &= \theta_n + \Delta\theta_n \\ \tilde{\theta}_m &= \theta_m + \Delta\theta_m \end{aligned} \tag{11A.8}$$

giving

$$\tilde{P}_{nm} = \frac{1}{x_k}(\tilde{\theta}_n - \tilde{\theta}_m) = \frac{1}{x_k}(\theta_n - \theta_m) + \frac{1}{x_k}(\Delta\theta_n - \Delta\theta_m)$$

or (11A.9)

$$\tilde{P}_{nm} = P_{nm} + \frac{1}{x_k}(X_{nn} + X_{mm} - 2X_{nm})\Delta P_n$$

Then (using the fact that $\tilde{P}_{nm}$ is set to ΔP_n)

$$\Delta P_n = \left[\frac{1}{1 - \frac{1}{x_k}(X_{nn} + X_{mm} - 2X_{nm})} \right] P_{nm} \tag{11A.10}$$

Define a sensitivity factor δ as the ratio of the change in phase angle θ, anywhere in the system, to the original power P_{nm} flowing over a line nm before it was dropped. That is,

$$\delta_{i,nm} = \frac{\Delta\theta_i}{P_{nm}} \tag{11A.11}$$

If neither n or m is the system reference bus, two injections, ΔP_n and ΔP_m, are imposed at buses n and m, respectively. This gives a change in phase angle at bus i equal to

$$\Delta\theta_i = X_{in}\Delta P_n + X_{im}\Delta P_m \tag{11A.12}$$

Then using the relationship between ΔP_n and ΔP_m, the resulting δ factor is

$$\delta_{i,nm} = \frac{(X_{in} - X_{im})x_k}{x_k - (X_{nn} + X_{mm} - 2X_{nm})} \tag{11A.13}$$

If either n or m is the reference bus, only one injection is made. The resulting δ factors are

$$\begin{aligned} \delta_{i,nm} &= \frac{X_{in}x_k}{(x_k - X_{nn})} \qquad \text{for } m = \text{ref} \\ &= \frac{-X_{im}x_k}{(x_k - X_{mm})} \qquad \text{for } n = \text{ref} \end{aligned} \tag{11A.14}$$

If bus i itself is the reference bus, then $\delta_{i,nm} = 0$ since the reference bus angle is constant.

The expression for $d_{\ell,k}$ is

$$\begin{aligned} d_{\ell,k} &= \frac{\Delta f_\ell}{f_k^0} = \frac{\frac{1}{x_\ell}(\Delta\theta_i - \Delta\theta_j)}{f_k^0} \\ &= \frac{1}{x_\ell}\left(\frac{\Delta\theta_i}{P_{nm}} - \frac{\Delta\theta_j}{P_{nm}}\right) \\ &= \frac{1}{x_\ell}(\delta_{i,nm} - \delta_{j,nm}) \end{aligned} \tag{11A.15}$$

if neither i nor j is a reference bus

$$d_{\ell,k} = \frac{1}{x_\ell}\left(\frac{(X_{in} - X_{im})x_k - (X_{jn} - X_{jm})x_k}{x_k - (X_{nn} + X_{mm} - 2X_{nm})}\right)$$

$$= \frac{\frac{x_k}{x_\ell}(X_{in} - X_{jn} - X_{im} + X_{jm})}{x_k - (X_{nn} + X_{mm} - 2X_{nm})} \tag{11A.16}$$

The fact that the a and d factors are linear models of the power system allows us to use superposition to extend them. One very useful extension is to use the a and d factors to model the power system in its post-outage state; that is, to generate factors that model the system's sensitivity after a branch has been lost.

Suppose one desired to have the sensitivity factor between line ℓ and generator bus i when line k was opened. This is calculated by first assuming that the change in generation on bus i, ΔP_i, has a direct effect on line ℓ and an indirect effect through its influence on the power flowing on line k, which, in turn, influences line ℓ when line k is out. Then

$$\Delta f_\ell = a_{\ell i}\Delta P_i + d_{\ell,k}\Delta f_k \tag{11A.17}$$

However, we know that

$$\Delta f_k = a_{ki}\Delta P_i \tag{11A.18}$$

therefore,

$$\Delta f_\ell = a_{\ell i}\Delta P_i + d_{\ell,k}a_{ki}\Delta P_i = (a_{\ell i} + d_{\ell,k}a_{ki})\Delta P_i \tag{11A.19}$$

We can refer to $a_{\ell i} + d_{\ell,k}a_{ki}$ as the "compensated generation shift sensitivity."

The compensated sensitivity factors are useful in finding corrections to the generation dispatch that will make the post-contingency state of the system secure from overloads. This will be dealt with in Chapter 13 under the topic of "security-constrained optimal power flow."

APPENDIX 11B
Derivation of Equation 11.14

Equation 11.14, repeated here as Eq. 11B.1:

$$|\Delta\theta_p - \Delta\theta_q| < |\Delta\theta_i - \Delta\theta_j| \tag{11B.1}$$

is proved as shown in reference 11 (the proof is attributed to Moslehi).

Suppose that buses i and j have the highest and lowest values of $\Delta\theta$ in the N3 region. Then the following both hold:

$$\Delta\theta_i > \Delta\theta_f$$

and

$$\Delta\theta_j < \Delta\theta_f$$

for all buses f in N3. Taking any external bus in N2, call it bus e, we shall state that

$$\Delta\theta_e < \Delta\theta_i \tag{11B.2}$$

and

$$\Delta\theta_e > \Delta\theta_j \tag{11B.3}$$

Proof: Suppose Eq. 11B.2 is not true and there exists a bus e' such that

$$\Delta\theta_{e'} > \Delta\theta_i$$

and, further, suppose that

$$\Delta\theta_{e'} > \Delta\theta_e \tag{11B.4}$$

for all the buses in N3. This implies that Eq. 11B.4 holds for the union of buses in N2 and N3. If we now look at the network as a DC power flow network, with no impedances to ground, and only the two injections at buses k and m, then all incremental power flows leaving node e' must be positive, since the incremental flows leaving node e' are found from

$$\Delta f_{e'e} = \frac{1}{x_{e'e}}(\Delta\theta_{e'} - \Delta\theta_e) \tag{11B.5}$$

However, since the network in N2 and N3 is strictly passive, and there are no impedances to ground, this violates Kirchoff's current law, which requires all branch flows incident to a bus to sum to zero. The only way for this to be true would be if all flows were zero; that is, all incremental angle spreads were equal. We can continue this reasoning to the neighbor buses of e' until we reach node i and conclude that

$$\Delta\theta_{e'} = \Delta\theta_i \tag{11B.6}$$

which contradicts Eq. 11B.4; thus, Eq. 11B.2 is proved. Equation 11B.3 is proved in a similar fashion. Then, as a result, Eq. 11B.1 is also proved.

PROBLEMS

11.1 Figure 11.17 shows a four-bus power system. Also given below are the impedance data for the transmission lines of the system as well as the generation and load values.

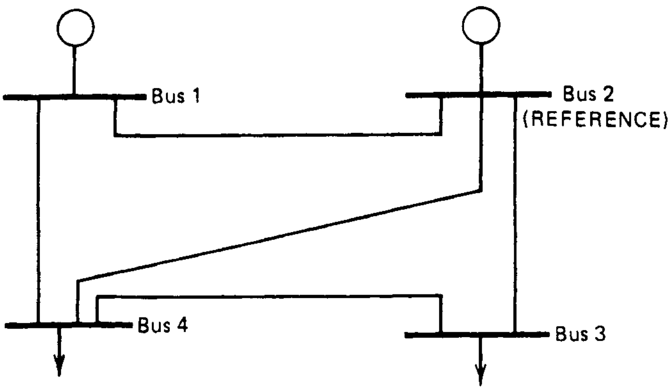


FIG. 11.17 Four-bus network for Problem 11.1.

Line	Line rectance (pu)
1-2	0.2
1-4	0.25
2-3	0.15
2-4	0.30
3-4	0.40

Bus	Load (MW)	Generation (MW)
1		150
2		350
3	220	
4	280	

a. Calculate the generation shift sensitivity coefficients for a shift in generation from bus 1 to bus 2.

b. Calculate the line outage sensitivity factors for outages on lines 1-2, 1-4, and 2-3.

11.2 In the system shown in Figure 11.18, three generators are serving a load of 1300 MW. The MW flow distribution, bus loads, and generator outputs are as shown. The generators have the following characteristics.

Generator No.	P_{min} (MW)	P_{max} (MW)
1	100	600
2	90	400
3	100	500

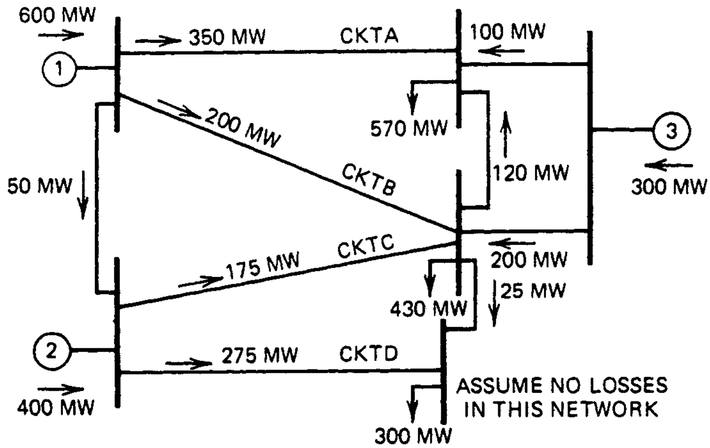


FIG. 11.18 Three-generator system for Problem 11.2.

The circuits have the following limits:

CKT A	600 MW max
CKT B	600 MW max
CKT C	450 MW max
CKT D	350 MW max

Throughout this problem we will only be concerned with flows on the circuit labeled A, B, C, and D. The generation shift sensitivity coefficients, $a_{\ell i}$, for circuits, A, B, C, and D are as follows.

CKT	Shift on Gen. 1	Shift on Gen. 2
A	0.7	0.08
B	0.2	0.02
C	0.06	0.54
D	0.04	0.36

Example: $$\Delta P_{\text{flow}_\ell} = a_{\ell,i} \times \Delta P_i$$

if

$$\ell = C \quad \text{and} \quad i = 2$$

$$\Delta P_{\text{flow}_c} = (0.54)\Delta P_2$$

Assume a shift on gen. 1 or gen. 2 will be compensated by an equal (opposite) shift on gen. 3. The line outage sensitivity factors $d_{\ell,k}$ are

$\ell \downarrow$ / $k \rightarrow$	A	B	C	D
A	X	0.8	0.21	0.14
B	0.9	X	0.06	0.04
C	0.06	0.12	X	0.82
D	0.04	0.08	0.73	X

As an example, suppose the loss of circuit k will increase the loading on circuit ℓ as follows.

$$P_{\text{flow}_\ell} = P_{\text{flow}_\ell}\text{ (before outage)} + d_{\ell,k} \times P_{\text{flow}_k}\text{ (before outage)}$$

if

$$\ell = \text{A} \quad \text{and} \quad k = \text{B}$$

The new flow on ℓ would be

$$P_{\text{flow}_\text{A}} = P_{\text{flow}_\text{A}} + (0.8)P_{\text{flow}_\text{B}}$$

a. Find the contingency (outage) flow distribution on circuits A, B, C, and D for an outage on circuit A. Repeat for an outage on B, then on C, then on D. (Only one circuit is lost at one time.) Are there any overloads?

b. Can you shift generation from gen. 1 to gen. 3, or from gen. 2 to gen. 3, so that no overloads occur? If so, how much shift?

11.3 Given the three-bus network shown in Figure 11.19 (see Example 4B), where

$$x_{12} = 0.2 \text{ pu}$$
$$x_{13} = 0.4 \text{ pu}$$
$$x_{23} = 0.25 \text{ pu}$$

the $[X]$ matrix is

$$\begin{bmatrix} 0.2118 & 0.1177 & 0 \\ 0.1177 & 0.1765 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

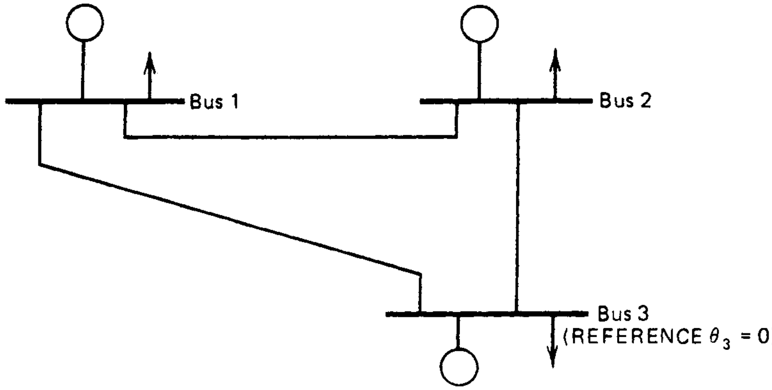


FIG. 11.19 Three-bus system for Problem 11.3.

Use a 100-MVA base. The base loads and generations are as follows.

Bus	Load (MW)	Gen. (MW)	Gen. min (MW)	Gen. max (MW)
1	100	150	50	250
2	300	180	60	250
3	100	170	60	300

a. Find base power flows on the transmission lines.

b. Calculate the generation shift factors for line 1-2. Calculate the shift in generation on bus 1 and 2 so as to force the flow on line 1-2 to zero MW. Assume for economic reasons that any shifts from base conditions are more expensive for shifts at the generator on bus 1 than for shifts on bus 2, and that the generator on bus 3 can be shifted without any penalty.

11.4 Using the system shown in Example 11B, find N1, N2 and N3 for the outage of the line from bus 2 to bus 4. Do you need to expand region N1? Where are the overloads, if any? (Use the same branch flow limits as shown in Example 11B.)

11.5 Using the data found in Figure 11.7, find the base-case bus phase angles and all line flows using the following bus loads and generators: all loads are 100 MW and all generators are also at 100 MW. Assume line flow limits as shown in the following table.

Line	MW Limit
1-2	70
1-4	90
1-5	70
2-3	20
2-4	50
2-5	40
2-6	60
3-5	30
3-6	70
4-5	30
5-6	20

For a line outage on line 1-4, find the change in phase angle across each of the remaining lines and see if the phase angle change across buses 1 and 4 meets the bounding criteria developed in the text.

11.6 Using the data from Problem 11.2, calculate the performance index, PI, for each outage case. Use a value of $n = 1$ and $n = 5$; that is for

$$\mathrm{PI} = \sum_{\text{all lines}} \left(\frac{\text{flow}_{ij}}{\text{flow max}_{ij}} \right)^{2n}$$

Which PI does a better job of predicting the case with the overload? Explain why.

FURTHER READING

The subject of power system security has received a great deal of attention in the engineering literature since the middle 1960s. The list of references presented here is therefore large but also quite limited nonetheless.

Reference 1 is a key paper on the topic of system security and energy control system philosophy. Reference 2 provides the basic theory for contingency assessment of power systems. Reference 3 covers contingency analysis using DC power flow methods. Reference 4 is a broad overview of security assessment and contains an excellent bibliography covering the literature on security assessment up to 1975.

The use of AC power flows in contingency analysis is possible with any AC load flow algorithm. However, the *fast-decoupled* power flow algorithm is generally recognized as the best for this purpose since its Jacobian matrix is constant and single-line outages can be modeled using the matrix inversion lemma. Reference 5 covers the fast-decoupled power flow algorithm and its application.

Correcting the generation dispatch by sensitivity methods is covered by reference 6. The use of linear programming to solve power systems problems is covered in reference 7.

References 8–12 cover some of the literature on contingency selection, and reference 13 gives a technique for solving the power flow using an approximation called concentric relaxation. References 14 and 15 give an indication of recent research on dynamic security assessment; that is, detecting fault cases that may cause dynamic or transient stability problems. Finally, reference 16 is concerned with the emerging area of voltage stability, which seeks to find contingencies which will cause such severe voltage problems as to bring on what is known as a "voltage collapse."

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