Copyrighted Materials
Copyright © 1996 John Wiley & Sons Retrieved from www.knovel.com

4 Transmission System Effects

As we saw in the previous chapter, the transmission network's incremental power losses may cause a bias in the optimal economic scheduling of the generators. The *coordination equations* include the effects of the incremental transmission losses and complicate the development of the proper schedule. The network elements lead to two other, important effects:

1. The total real power loss in the network increases the total generation demand, and
2. The generation schedule may have to be adjusted by shifting generation to reduce flows on transmission circuits because they would otherwise become overloaded.

It is the last effect that is the most difficult to include in optimum dispatching. In order to include constraints on flows through the network elements, the flows must be evaluated as an integral part of the scheduling effort. This means we must solve the power flow equations along with the generation scheduling equations. (Note that earlier texts, papers, and even the first edition of this book referred to these equations as the "load flow" equations.)

If the constraints on flows in the networks are ignored, then it is feasible to use what are known as *loss formulae* that relate the total and incremental, real power losses in the network to the power generation magnitudes. Development of loss formulae is an art that requires knowledge of the power flows in the network under numerous "typical" conditions. Thus, there is no escaping the need to understand the methods involved in formulating and solving the power flow equations for an AC transmission system.

When the complete transmission system model is included in the development of generation schedules, the process is usually imbedded in a set of computer algorithms known as the *optimal power flow* (or OPF). The complete OPF is capable of establishing schedules for many controllable quantities in the bulk power system (i.e., the generation and transmission systems), such as transformer tap positions, VAR generation schedules, etc. We shall defer a detailed examination of the OPF until Chapter 13.

Another useful set of data that are obtainable when the transmission network is incorporated in the scheduling process is the incremental cost of power at various points in the network. With no transmission effects considered (that is, ignoring all incremental losses and any constraints on power flows), the network is assumed to be a single node and the incremental cost of power is equal to λ everywhere. That is,

$$\frac{dF_i}{dP_i} = \lambda$$

Including the effect of incremental losses will cause the incremental cost of real power to vary throughout the network. Consider the arrangement in Figure 3.2 and assume that the coordination equations have been solved so the values of dF_i/dP_i and λ are known. Let the "penalty factor" of bus i be defined as

$$Pf_i = \frac{1}{\left(1 - \frac{\partial P_{\text{loss}}}{\partial P_i}\right)}$$

so that the relationship between the incremental costs at any two buses, i and j, is

$$Pf_i F'_i = Pf_j F'_j$$

where $F'_k = dF_k/dP_k$ is the bus incremental cost. There is no requirement that bus i is a generator bus. If the network effects are included using a network model or a loss formula, bus i might be a load bus or a point where power is delivered to an interconnected system. The incremental cost (or "value") of power at bus i is then,

$$\text{Incremental cost at } i = F'_i = (Pf_j/Pf_i)F'_j$$

where j is any real generator bus where the incremental cost of production is known. So if we can develop a network model to be used in optimum generation scheduling that includes all of the buses, or at least those that are of importance, and if the incremental losses ($\partial P_L/\partial P_k$) can be evaluated, the coordination equations can be used to compute the incremental cost of power at any point of delivery.

When the schedule is determined using a complete power flow model by using an OPF, the flow constraints can be included and they may affect the value of the incremental cost of power in parts of the network. Rather than attempt a mathematical demonstration, consider a system in which most of the low cost generation is in the north, most of the load is in the south along with some higher cost generating units, and the northern and southern areas are interconnected by a relatively low capacity transmission network. The network north-to-south transfer capability limits the power that can be delivered from the northern area to satisfy the higher load demands. Under a schedule that is constrained by this transmission flow limitation, the southern area's generation would need to be increased above an unconstrained, optimal level in order to satisfy some of the load in that region. The constrained economic schedule would split the system into two regions with a higher incremental cost in the southern area. In most actual cases where transmission does constrain the economic schedule, the effect of the constraints is much more significant than the effects of incremental transmission losses.

This chapter develops the power flow equations and outlines methods of solution. Operations control centers frequently use a version of the power flow equations known as the "decoupled power flow." The power flow equations form the basis for the development of loss formulae. Scheduling methods frequently use penalty factors to incorporate the effect of incremental real power losses in dispatch. These can be developed from the loss formulae or directly from the power flow relationships.

Power flow is the name given to a network solution that shows currents, voltages, and real and reactive power flows at every bus in the system. It is normally assumed that the system is balanced and the common use of the term power flow implies a positive sequence solution only. Full three-phase power-flow solution techniques are available for special-purpose calculations. As used here, we are only interested in balanced solutions. Power flow is not a single calculation such as $E = IR$ or $\mathbf{E} = [Z]\mathbf{I}$ involving linear circuit analysis. Such circuit analysis problems start with a given set of currents or voltages, and one must solve for the linearly dependent unknowns. In the power-flow problem we are given a nonlinear relationship between voltage and current at each bus and we must solve for all voltages and currents such that these nonlinear relationships are met. The nonlinear relationships involve, for example, the real and reactive power consumption at a bus, or the generated real power and scheduled voltage magnitude at a generator bus. As such, the power flow gives us the electrical response of the transmission system to a particular set of loads and generator unit outputs. Power flows are an important part of power system design procedures (system planning). Modern digital computer power-flow programs are routinely run for systems with up to 5000 or more buses and also are used widely in power system control centers to study unique operating problems and to provide accurate calculations of bus penalty factors. Present, state-or-the-art system control centers use the power flow as a key, central element in the scheduling of generation, monitoring of the system, and development of interchange transactions. OPF programs are used to develop optimal economic schedules and control settings that will result in flows that are within the capabilities of the elements of the system, including the transmission network, and bus voltage magnitudes that are within acceptable tolerances.

4.1 THE POWER FLOW PROBLEM AND ITS SOLUTION

The power flow problem consists of a given transmission network where all lines are represented by a Pi-equivalent circuit and transformers by an ideal voltage transformer in series with an impedance. Generators and loads represent the boundary conditions of the solution. Generator or load real and reactive power involves products of voltage and current. Mathematically, the power flow requires a solution of a system of simultaneous nonlinear equations.

4.1.1 The Power Flow Problem on a Direct Current Network

The problems involved in solving a power flow can be illustrated by the use of direct current (DC) circuit examples. The circuit shown in Figure 4.1 has a resistance of 0.25 Ω tied to a constant voltage of 1.0 V (called the *reference voltage*). We wish to find the voltage at bus 2 that results in a net inflow of 1.2 W. *Buses* are electrical nodes. Power is said to be "injected" into a network; therefore, loads are simply negative injections.

The current from bus 2 to bus 1 is

$$I_{21} = (E_2 - 1.0) \times 4 \tag{4.1}$$

Power P_2 is

$$P_2 = 1.2 = E_2 I_{21} = E_2(E_2 - 1) \times 4 \tag{4.2}$$

or

$$4E_2^2 - 4E_2 - 1.2 = 0 \tag{4.3}$$

The solutions to this quadratic equation are $E_2 = 1.24162$ V and $E_2 = -0.24162$ V. Note that 1.2 W enter bus 2, producing a current of 0.96648 A ($E_2 = 1.24162$), which means that 0.96648 W enter the reference bus and 0.23352 W are consumed in the 0.25-Ω resistor.

Let us complicate the problem by adding a third bus and two more lines (see Figure 4.2). The problem is more complicated because we cannot simply write out the solutions using a quadratic formula. The admittance equations are

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 14 & -4 & -10 \\ -4 & 9 & -5 \\ -10 & -5 & 15 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \tag{4.4}$$

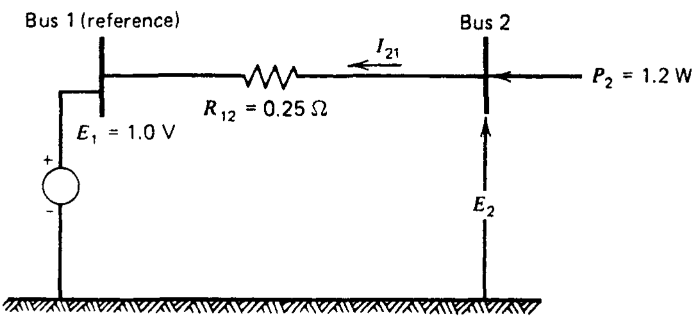

FIG. 4.1 Two-bus DC network.

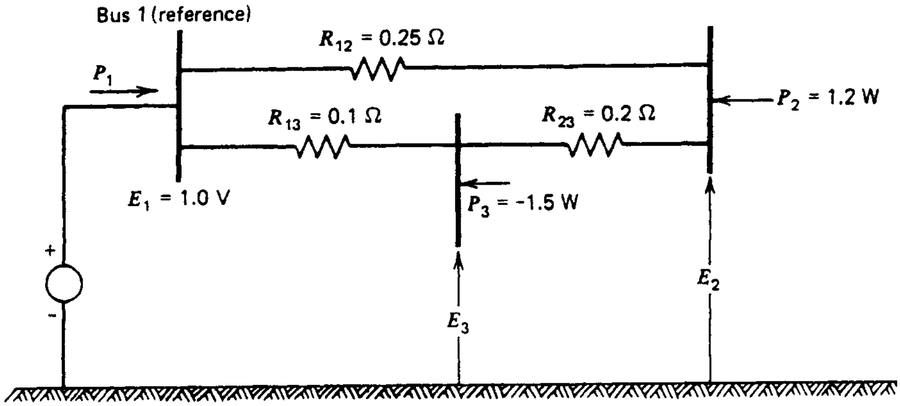

FIG. 4.2 Three-bus DC network.

In this case, we know the power injected at buses 2 and 3 and we know the voltage at bus 1. To solve for the unknowns (E_2, E_3 and P_1), we write Eqs. 4.5, 4.6, and 4.7. The solution procedure is known as the *Gauss–Seidel procedure*, wherein a calculation for a new voltage at each bus is made, based on the most recently calculated voltages at all neighbouring buses.

Bus 2:

$$I_2 = \frac{P_2}{E_2} = -4(1.0) + 9E_2 - 5E_3$$

$$E_2^{\text{new}} = \frac{1}{9}\left(\frac{1.2}{E_2^{\text{old}}} + 4 + 5E_3^{\text{old}}\right) \tag{4.5}$$

where E_2^{old} and E_3^{old} are the initial values for E_2 and E_3, respectively.

Bus 3:

$$I_3 = \frac{P_3}{E_3} = -10(1.0) - 5E_2^{\text{new}} + 15E_3$$

$$E_3^{\text{new}} = \frac{1}{15}\left[\frac{-1.5}{E_3^{\text{old}}} + 10 + 5E_2^{\text{new}}\right] \tag{4.6}$$

where E_2^{new} is the voltage found in solving Eq. 4.5, and E_3^{old} is the initial value of E_3.

Bus 1:

$$P_1 = E_1 I_1^{\text{new}} = 1.0 I_1^{\text{new}} = 14 - 4E_2^{\text{new}} - 10E_3^{\text{new}} \tag{4.7}$$

The Gauss–Seidel method first assumes a set of voltages at buses 2 and 3 and then uses Eqs. 4.5 and 4.6 to solve for new voltages. The new voltages are compared to the voltage's most recent values, and the process continues until

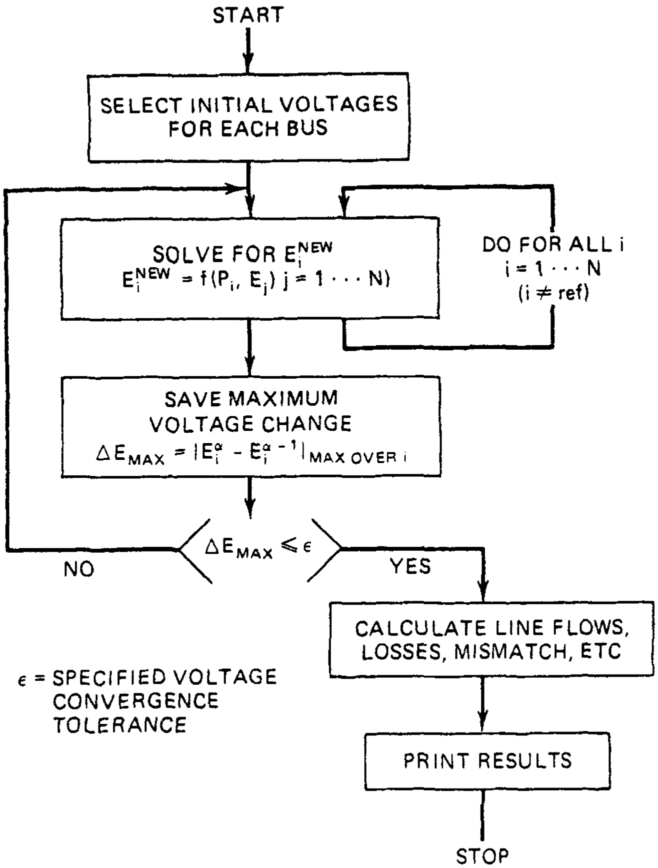

FIG. 4.3 Gauss–Seidel power-flow solution.

the change in voltage is very small. This is illustrated in the flowchart in Figure 4.3 and in Eqs. 4.8 and 4.9.

First iteration:

$$E_2^{(0)} = E_3^{(0)} = 1.0$$

$$\begin{aligned} E_2^{(1)} &= \frac{1}{9}\left(\frac{1.2}{1.0} + 4 + 5\right) = 1.133 \\ E_3^{(1)} &= \frac{1}{15}\left[\frac{-1.5}{1.0} + 10 + 5(1.133)\right] = 0.944 \end{aligned} \tag{4.8}$$

$$\Delta E_{\max} = 0.133 \text{ too large}$$

Note: In calculating $E_3^{(1)}$ we used the new value of E_2 found in the first correction.

Second iteration: $E_2^{(2)} = \frac{1}{9}\left[\frac{1.2}{1.133} + 4 + 5(0.944)\right] = 1.087$

$$E_3^{(2)} = \frac{1}{15}\left[\frac{-1.5}{0.944} + 10 + 5(1.087)\right] = 0.923 \qquad (4.9)$$

$$\Delta E_{max} = 0.046$$

And so forth until $\Delta E_{max} < \varepsilon$.

4.1.2 The Formulation of the AC Power Flow

AC power flows involve several types of bus specifications, as shown in Figure 4.4. Note that $[P, \theta]$, $[Q, |E|]$, and $[Q, \theta]$ combinations are generally not used.

The transmission network consists of complex impedances between buses and from the buses to ground. An example is given in Figure 4.5. The equations are written in matrix form as

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \left[\begin{array}{c|c|c|c} y_{12} & -y_{12} & 0 & 0 \\ \hline -y_{12} & (y_{12} + y_{2g} + y_{23}) & -y_{23} & 0 \\ \hline 0 & -y_{23} & (y_{23} + y_{3g} + y_{34}) & -y_{34} \\ \hline 0 & 0 & -y_{34} & (y_{34} + y_{4g}) \end{array}\right] \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} \qquad (4.10)$$

(All I's, E's, y's complex)

This matrix is called the *network Y matrix*, which is written as

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} \qquad (4.11)$$

The rules for forming a Y matrix are

If a line exists from i to j

$$Y_{ij} = -y_{ij}$$

and

$$Y_{ii} = \sum_j y_{ij} + y_{ig}$$

j over all lines connected to i.

Bus Type	P	Q	$\lvert E \rvert$	θ	Comments
Load	✓	✓			Usual load representation
Voltage Controlled	✓		✓		Assume $\lvert E \rvert$ is held constant no matter what Q is
Generator or Synchronous Condenser	✓		✓ when $Q^- < Q_g < Q^+$		Generator or synchronous condenser ($P = 0$) has VAR limits
	✓	✓ when $Q_g < Q^-$ $Q_g > Q^+$			Q^- minimum VAR limit; Q^+ maximum VAR limit; $\lvert E \rvert$ is held as long as Q_g is within limit
Fixed Z to Ground					Only Z is given
Reference			✓	✓	"Swing bus" must adjust net power to hold voltage constant (essential for solution)

FIG. 4.4 Power-flow bus specifications (quantities checked are the bus boundary conditions).

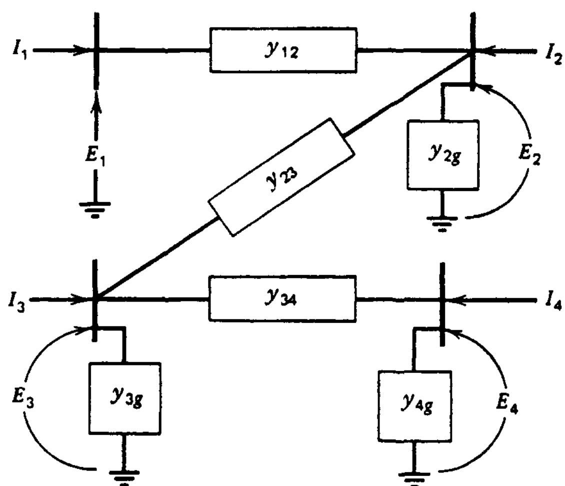

FIG. 4.5 Four-bus AC network.

The equation of net power injection at a bus is usually written as

$$\frac{P_k - jQ_k}{E_k^*} = \sum_{\substack{j=1 \\ j \neq k}}^{n} Y_{kj} E_j + Y_{kk} E_k \tag{4.12}$$

4.1.2.1 The Gauss–Seidel Method

The voltages at each bus can be solved for by using the Gauss–Seidel method. The equation in this case is

$$E_k^{(\alpha)} = \frac{1}{Y_{kk}} \frac{(P_k - jQ_k)}{E_k^{(\alpha-1)*}} - \frac{1}{Y_{kk}} \left[\sum_{j<k} Y_{kj} E_j^{(\alpha)} + \sum_{j>k} Y_{kj} E_j^{(\alpha-1)} \right] \tag{4.13}$$

Voltage at
iteration α

The Gauss–Seidel method was the first AC power-flow method to be developed for solution on digital computers. This method is characteristically long in solving due to its slow convergence and often difficulty is experienced with unusual network conditions such as negative reactance branches. The solution procedure is the same as shown in Figure 4.3.

4.1.2.2 The Newton–Raphson Method

One of the disadvantages of the Gauss–Seidel method lies in the fact that each bus is treated independently. Each correction to one bus requires subsequent correction to all the buses to which it is connected. The Newton–Raphson method is based on the idea of calculating the corrections while taking account of all the interactions.

Newton's method involves the idea of an error in a function $f(x)$ being driven to zero by making adjustments Δx to the independent variable associated with the function. Suppose we wish to solve

$$f(x) = K \tag{4.14}$$

In Newton's method, we pick a starting value of x and call it x^0. The error is the difference between K and $f(x^0)$. Call the error ε. This is shown in Figure 4.6 and given in Eq. 4.15.

$$f(x^0) + \varepsilon = K \tag{4.15}$$

To drive the error to zero, we use a Taylor expansion of the function about x^0.

$$f(x^0) + \frac{df(x^0)}{dx} \Delta x + \varepsilon = K \tag{4.16}$$

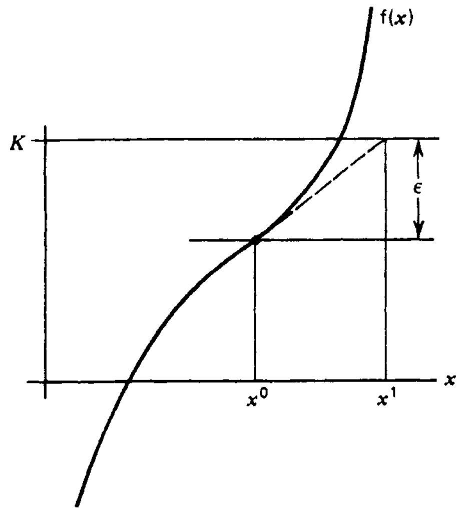

FIG. 4.6 Newton's method.

Setting the error to zero, we calculate

$$\Delta x = \left(\frac{\mathrm{df}(x^0)}{\mathrm{d}x}\right)^{-1} [K - \mathrm{f}(x^0)] \tag{4.17}$$

When we wish to solve a load flow, we extend Newton's method to the multivariable case (the multivariable case is called the *Newton–Raphson method*). An equation is written for each bus "*i*."

$$P_i + jQ_i = E_i I_i^* \tag{4.18}$$

where

$$I_i = \sum_{k=1}^{N} Y_{ik} E_k$$

then

$$P_i + jQ_i = E_i \left(\sum_{k=1}^{N} Y_{ik} E_k \right)^*$$

$$= |E_i|^2 Y_{ii}^* + \sum_{\substack{k=1 \\ k \neq i}}^{N} Y_{ik}^* E_i E_k^*$$

As in the Gauss–Seidel method, a set of starting voltages is used to get things going. The $P + jQ$ calculated is subtracted from the scheduled $P + jQ$ at the bus, and the resulting errors are stored in a vector. As shown in the following, we will assume that the voltages are in polar coordinates and that we are going to adjust each voltage's magnitude and phase angle as separate independent variables. Note that at this point, two equations are written for each bus: one for real power and one for reactive power. For each bus,

$$\Delta P_i = \sum_{k=1}^{N} \frac{\partial P_i}{\partial \theta_k} \Delta\theta_k + \sum_{k=1}^{N} \frac{\partial P_i}{\partial |E_k|} \Delta|E_k|$$
$$\Delta Q_i = \sum_{k=1}^{N} \frac{\partial Q_i}{\partial \theta_k} \Delta\theta_k + \sum_{k=1}^{N} \frac{\partial Q_i}{\partial |E_k|} \Delta|E_k| \qquad (4.19)$$

All the terms are arranged in a matrix (the Jacobian matrix) as follows.

$$\begin{bmatrix} \Delta P_1 \\ \Delta Q_1 \\ \Delta P_2 \\ \Delta Q_2 \\ \vdots \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial P_1}{\partial \theta_1} & \frac{\partial P_1}{\partial |E_1|} & \cdots \\ \frac{\partial Q_1}{\partial \theta_1} & \frac{\partial Q_1}{\partial |E_1|} & \cdots \\ \vdots & \vdots & \vdots \end{bmatrix}}_{\text{Jacobian matrix}} \begin{bmatrix} \Delta\theta_1 \\ \Delta|E_1| \\ \vdots \end{bmatrix} \qquad (4.20)$$

The Jacobian matrix in Eq. 4.20 starts with the equation for the real and reactive power at each bus. This equation, Eq. 4.18, is repeated below:

$$P_i + jQ_i = E_i \sum_{k=1}^{N} Y_{ik}^* E_k^*$$

This can be expanded as:

$$\begin{aligned} P_i + jQ_i &= \sum_{k=1}^{N} |E_i||E_k|(G_{ik} - jB_{ik})\varepsilon^{j(\theta_i - \theta_k)} \\ &= \sum_{k=1}^{N} \{|E_i||E_k|[G_{ik}\cos(\theta_i - \theta_k) + B_{ik}\sin(\theta_i - \theta_k)] \\ &\quad + j[|E_i||E_k|[G_{ik}\sin(\theta_i - \theta_k) - B_{ik}\cos(\theta_i - \theta_k)]]\} \end{aligned} \qquad (4.21)$$

where

θ_i, θ_k = the phase angles at buses i and k, respectively;

$|E_i|, |E_k|$ = the bus voltage magnitudes, respectively

$G_{ik} + jB_{ik}$ = Y_{ik} is the ik term in the Y matrix of the power system.

The general practice in solving power flows by Newton's method has been to use

$$\frac{\Delta|E_i|}{|E_i|}$$

instead of simply $\Delta|E_i|$; this simplifies the equations. The derivatives are:

$$\begin{aligned}
\frac{\partial P_i}{\partial \theta_k} &= |E_i||E_k|[G_{ik}\sin(\theta_i-\theta_k) - B_{ik}\cos(\theta_i-\theta_k)] \\
\frac{\partial P_i}{\left(\dfrac{\partial|E_k|}{|E_k|}\right)} &= |E_i||E_k|[G_{ik}\cos(\theta_i-\theta_k) + B_{ik}\sin(\theta_i-\theta_k)] \\
\frac{\partial Q_i}{\partial \theta_k} &= -|E_i||E_k|[G_{ik}\cos(\theta_i-\theta_k) + B_{ik}\sin(\theta_i-\theta_k)] \\
\frac{\partial Q_i}{\left(\dfrac{\partial|E_i|}{|E_i|}\right)} &= |E_i||E_k|[G_{ik}\sin(\theta_i-\theta_k) - B_{ik}\cos(\theta_i-\theta_k)]
\end{aligned} \tag{4.22}$$

For $i = k$:

$$\begin{aligned}
\frac{\partial P_i}{\partial \theta_i} &= -Q_i - B_{ii}E_i^2 \\
\frac{\partial P_i}{\left(\dfrac{\partial|E_i|}{|E_i|}\right)} &= P_i + G_{ii}E_i^2 \\
\frac{\partial Q_i}{\partial \theta_i} &= P_i - G_{ii}E_i^2 \\
\frac{\partial Q_i}{\left(\dfrac{\partial|E_i|}{|E_i|}\right)} &= Q_i - B_{ii}E_i^2
\end{aligned}$$

Equation 4.20 now becomes

$$\begin{bmatrix} \Delta P_1 \\ \Delta Q_1 \\ \Delta P_2 \\ \Delta Q_2 \\ \vdots \end{bmatrix} = [J] \begin{bmatrix} \Delta\theta_1 \\ \dfrac{\Delta|E_1|}{|E_1|} \\ \Delta\theta_2 \\ \dfrac{\Delta|E_2|}{|E_2|} \\ \vdots \end{bmatrix} \tag{4.23}$$

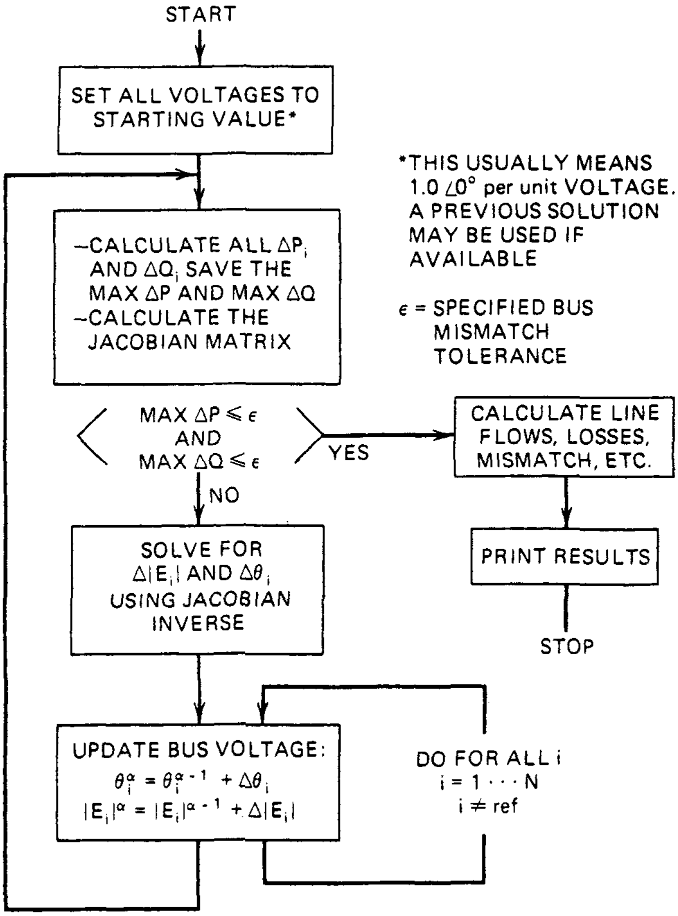

FIG. 4.7 Newton–Raphson power-flow solution.

The solution to the Newton–Raphson power flow runs according to the flowchart in Figure 4.7. Note that solving for $\Delta\theta$ and $\Delta|E|$ requires the solution of a set of linear equations whose coefficients make up the Jacobian matrix. The Jacobian matrix generally has only a few percent of its entries that are nonzero. Programs that solve an AC power flow using the Newton–Raphson method are successful because they take advantage of the Jacobian's "sparsity." The solution procedure uses Gaussian elimination on the Jacobian matrix and does not calculate J^{-1} explicitly. (See reference 3 for introduction to "sparsity" techniques.)

EXAMPLE 4A

The six-bus network shown in Figure 4.8 will be used to demonstrate several aspects of load flows and transmission loss factors. The voltages and flows

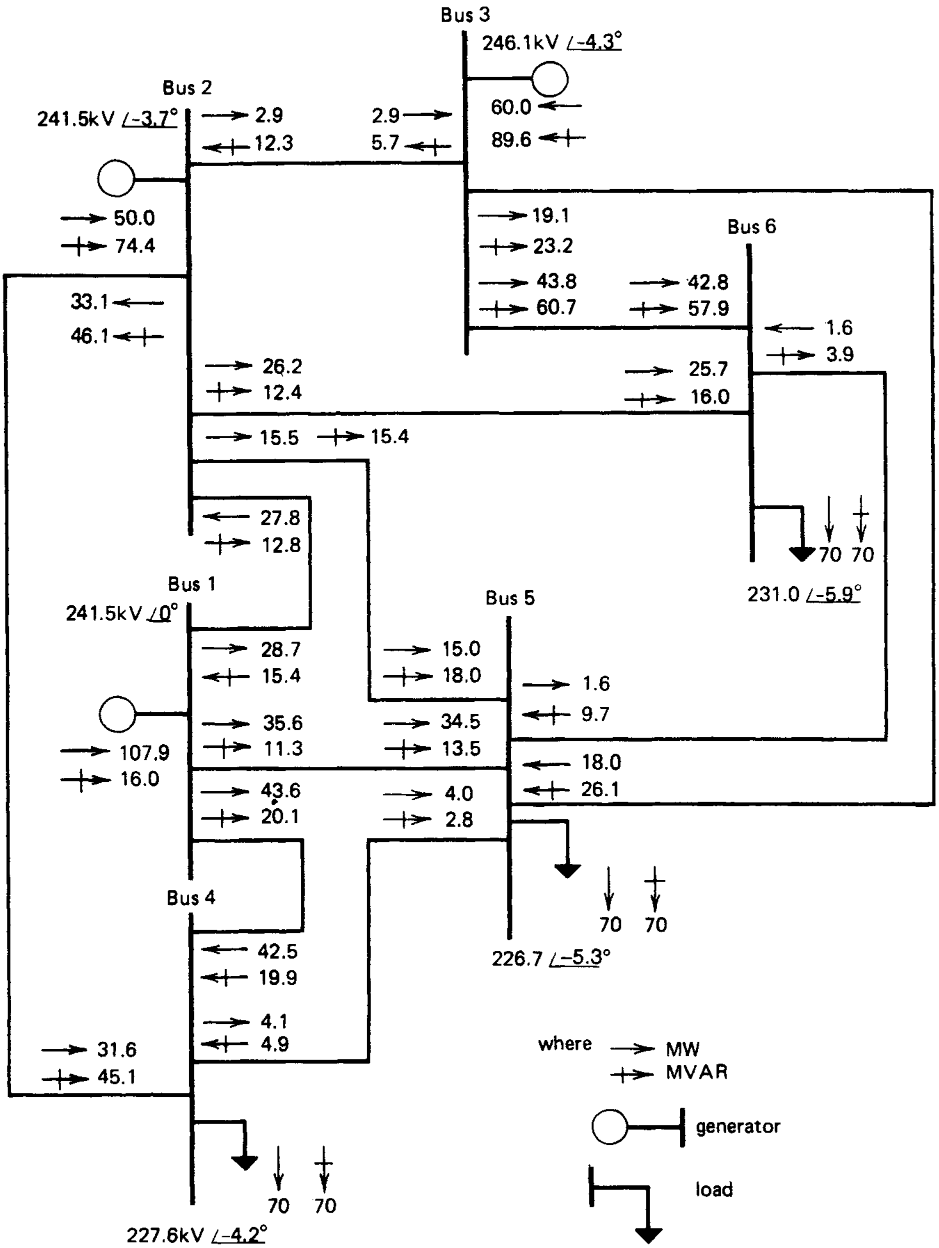

FIG. 4.8 Six-bus network base case AC power flow.

shown are for the "base case" of 210 MW total load. The impedance values and other data for this system may be found in the appendix of this chapter.

4.1.3 The Decoupled Power Flow

The Newton power flow is the most robust power flow algorithm used in practice. However, one drawback to its use is the fact that the terms in the Jacobian matrix must be recalculated each iteration, and then the entire set of linear equations in Eq. 4.23 must also be resolved each iteration.

Since thousands of complete power flows are often run for a planning or operations study, ways to speed up this process were sought. Reference 11 shows the development of a technique known as the "fast decoupled power flow" (it is often referred to as the "Stott decoupled power flow," in reference to its first author).

Starting with the terms in the Jacobian matrix (see Eq. 4.22), the following simplications are made:

- Neglect and interaction between P_i and any $|E_k|$ (it was observed by power system engineers that real power was little influenced by changes in voltage magnitude—so this effect was incorporated in the algorithm). Then, all the derivatives

$$\frac{\partial P_i}{\left(\frac{\partial |E_k|}{|E_k|}\right)}$$

  will be considered to be zero.
- Neglect any interaction between Q_i and θ_k (see the note above—a similar observation was made on the insensitivity of reactive power to changes in phase angle). Then, all the derivatives

$$\frac{\partial Q_i}{\partial \theta_k}$$

  are also considered to be zero.
- Let $\cos(\theta_i - \theta_j) \cong 1$ which is a good approximation since $(\theta_i - \theta_j)$ is usually small.
- Assume that

$$G_{ik} \sin(\theta_i - \theta_k) \ll B_{ik}$$

- Assume that

$$Q_i \ll B_{ii}|E_i|^2$$

This leaves the derivatives as:

$$\frac{\partial P_i}{\partial \theta_k} = -|E_i||E_k|B_{ik} \tag{4.24}$$

$$\frac{\partial Q_i}{\left(\dfrac{\partial |E_k|}{|E_k|}\right)} = -|E_i||E_k|B_{ik} \tag{4.25}$$

If we now write the power flow adjustment equations as:

$$\Delta P_i = \left(\frac{\partial P_i}{\partial \theta_k}\right)\Delta\theta_k \tag{4.26}$$

$$\Delta Q_i = \left[\frac{\partial Q_i}{\left(\dfrac{\partial |E_k|}{|E_k|}\right)}\right]\frac{\Delta|E_k|}{|E_k|} \tag{4.27}$$

then, substituting Eq. 4.24 into Eq. 4.26, and Eq. 4.25 into Eq. 4.27, we obtain:

$$\Delta P_i = -|E_i||E_k|B_{ik}\Delta\theta_k \tag{4.28}$$

$$\Delta Q_i = -|E_i||E_k|B_{ik}\frac{\Delta|E_k|}{|E_k|} \quad (4.29$$

Further simplification can then be made:

- Divide Eqs. 4.28 and 4.29 by $|E_i|$.
- Assume $|E_k| \cong 1$ in Eq. 4.28.

which results in:

$$\frac{\Delta P_i}{|E_i|} = -B_{ik}\Delta\theta_k \tag{4.30}$$

$$\frac{\Delta Q_i}{|E_i|} = -B_{ik}\Delta|E_k| \tag{4.31}$$

We now build Eqs. 4.30 and 4.31 into two matrix equations:

$$\begin{bmatrix} \dfrac{\Delta P_1}{|E_1|} \\ \dfrac{\Delta P_2}{|E_2|} \\ \vdots \end{bmatrix} = \begin{bmatrix} -B_{11} & -B_{12} & \cdots \\ -B_{21} & -B_{22} & \cdots \\ \vdots & & \end{bmatrix} \begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \\ \vdots \end{bmatrix} \tag{4.32}$$

$$\begin{bmatrix} \dfrac{\Delta Q_1}{|E_1|} \\ \dfrac{\Delta Q_2}{|E_2|} \\ \vdots \end{bmatrix} = \begin{bmatrix} -B_{11} & -B_{12} & \cdots \\ -B_{21} & -B_{22} & \cdots \\ \vdots \end{bmatrix} \begin{bmatrix} \Delta|E_1| \\ \Delta|E_2| \\ \vdots \end{bmatrix} \tag{4.33}$$

Note that both Eqs. 4.32 and 4.33 use the same matrix. Further simplification, however, will make them different.

Simplifying the $\Delta P - \Delta\theta$ relationship of Eq. 4.32:

- Assume $r_{ik} \ll x_{ik}$; this changes $-B_{ik}$ to $-1/x_{ik}$.
- Eliminate all shunt reactances to ground.
- Eliminate all shunts to ground which arise from autotransformers.

Simplifying the $\Delta Q - \Delta|E|$ relationship of Eq. 4.33:

- Omit all effects from phase shift transformers.

The resulting equations are:

$$\begin{bmatrix} \dfrac{\Delta P_1}{|E_1|} \\ \dfrac{\Delta P_2}{|E_2|} \\ \vdots \end{bmatrix} = [B'] \begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \\ \vdots \end{bmatrix} \tag{4.34}$$

$$\begin{bmatrix} \dfrac{\Delta Q_1}{|E_1|} \\ \dfrac{\Delta Q_2}{|E_2|} \\ \vdots \end{bmatrix} = [B''] \begin{bmatrix} \Delta|E_1| \\ \Delta|E_2| \\ \vdots \end{bmatrix} \tag{4.35}$$

where the terms in the matrices are:

$$B'_{ik} = -\frac{1}{x_{ik}}, \text{ assuming a branch from } i \text{ to } k \text{ (zero otherwise)}$$

$$B'_{ii} = \sum_{k=1}^{N} \frac{1}{x_{ik}}$$

$$B''_{ik} = -B_{ik} = -\frac{x_{ik}}{r_{ik}^2 + x_{ik}^2}$$

$$B''_{ii} = \sum_{k=1}^{N} -B_{ik}$$

The decoupled power flow has several advantages and disadvantages over the Newton power flow. (Note: Since the introduction and widespread use of the decoupled power flow, the Newton power flow is often referred to as the "full Newton" power flow.)

Advantages:

- B' and B'' are constant; therefore, they can be calculated once and, except for changes to B'' resulting from generation VAR limiting, they are not updated.
- Since B' and B'' are each about one-quarter of the number of terms in $[J]$ (the full Newton power flow Jacobian matrix), there is much less arithmetic to solve Eqs. 4.34 and 4.35.

Disadvantages:

- The decoupled power flow algorithm may fail to converge when some of the underlying assumptions (such as $r_{ik} \ll x_{ik}$) do not hold. In such cases, one must switch to using the full Newton power flow.

Note that Eq. 4.34 is often referred to as the P–θ Eq. and Eq. 4.35 as the Q–E (or Q–V) equation.

A flowchart of the algorithm is shown in Figure 4.9. A comparison of the convergence of the Gauss–Seidel, the full Newton and the decoupled power flow algorithms is shown in Figure 4.10.

4.1.4 The "DC" Power Flow

A further simplification of the power flow algorithm involves simply dropping the Q–V equation (Eq. 4.35) altogether. This results in a completely linear, noniterative, power flow algorithm. To carry this out, we simply assume that all $|E_i| = 1.0$ per unit. Then Eq. 4.34 becomes:

$$\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \vdots \end{bmatrix} = [B'] \begin{bmatrix} \Delta \theta_1 \\ \Delta \theta_2 \\ \vdots \end{bmatrix} \tag{4.36}$$

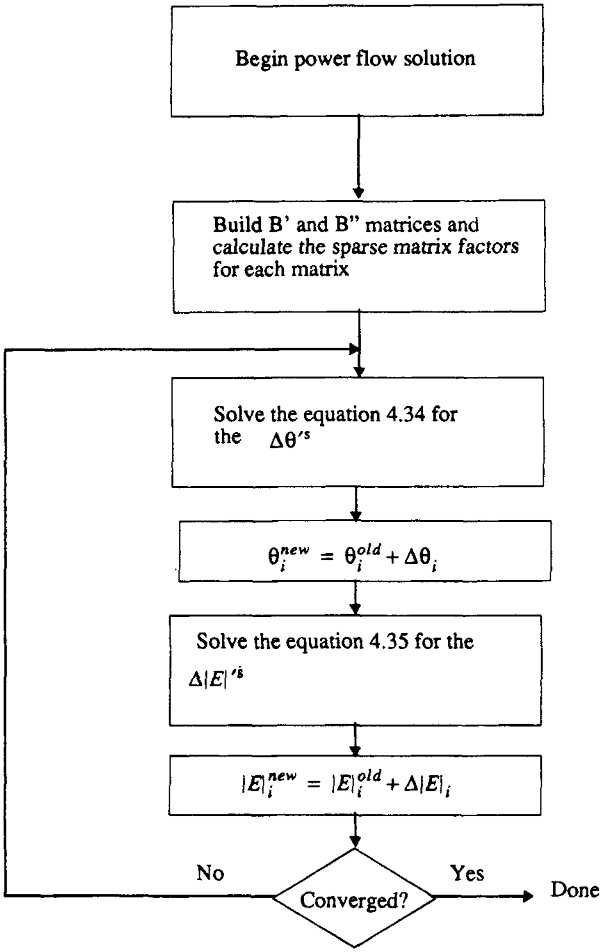

FIG. 4.9 Decoupled power flow algorithm.

where the terms in B' are as described previously. The DC power flow is only good for calculating MW flows on transmission lines and transformers. It gives no indication of what happens to voltage magnitudes, or MVAR or MVA flows. The power flowing on each line using the DC power flow is then:

$$P_{ik} = \frac{1}{x_{ik}}(\theta_i - \theta_k) \tag{4.37}$$

and

$$P_i = \sum_{\substack{k = \text{buses} \\ \text{connected to } i}}^{N} P_k$$

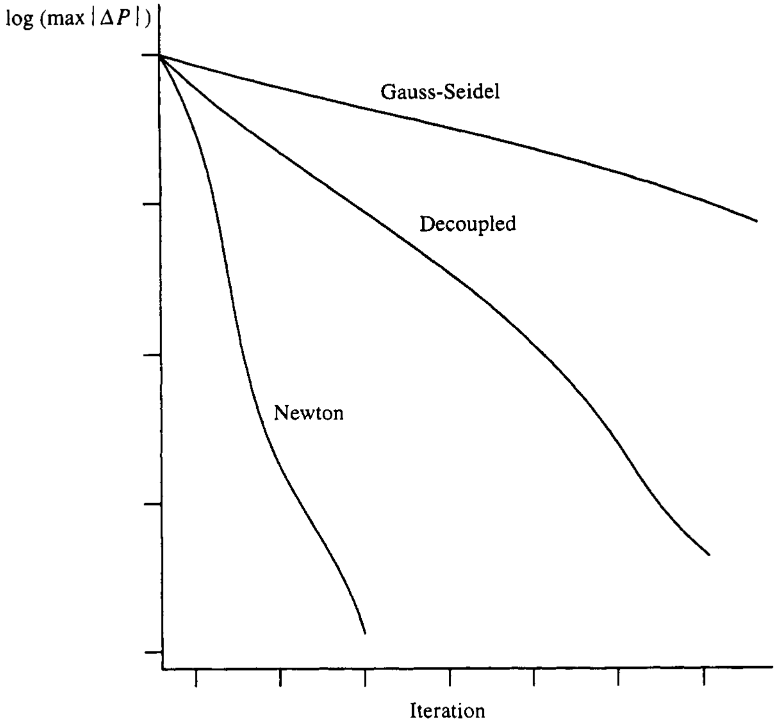

FIG. 4.10 Comparison of three power flow algorithm convergence characteristics.

EXAMPLE 4B

The megawatt flows on the network in Figure 4.11 will be solved using the DC power flow. The B' matrix equation is:

$$\begin{bmatrix} 7.5 & -5.0 \\ -5.0 & 9.0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

$$\theta_3 = 0$$

Note that all megawatt quantities and network quantities are expressed in pu (per unit on 100 MVA base). All phase angles will then be in radians.

The solution to the preceding matrix equation is:

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0.2118 & 0.1177 \\ 0.1177 & 0.1765 \end{bmatrix} \begin{bmatrix} 0.65 \\ -1.00 \end{bmatrix} = \begin{bmatrix} 0.02 \\ -0.1 \end{bmatrix}$$

The resulting flows are shown in Figure 4.12 and calculated using Eq. 4.37. Note that all flows in Figure 4.12 were converted to actual megawatt values.

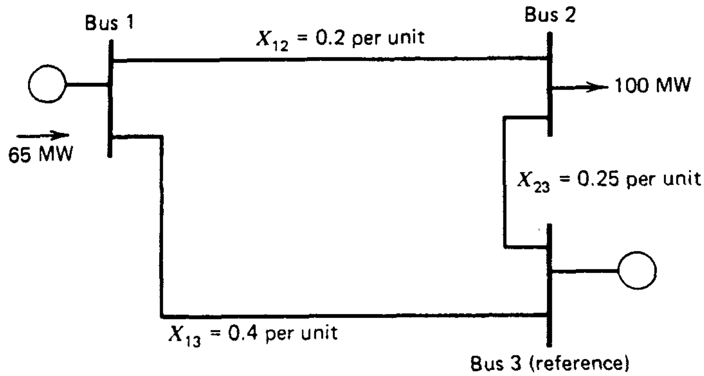

FIG. 4.11 Three-bus network.

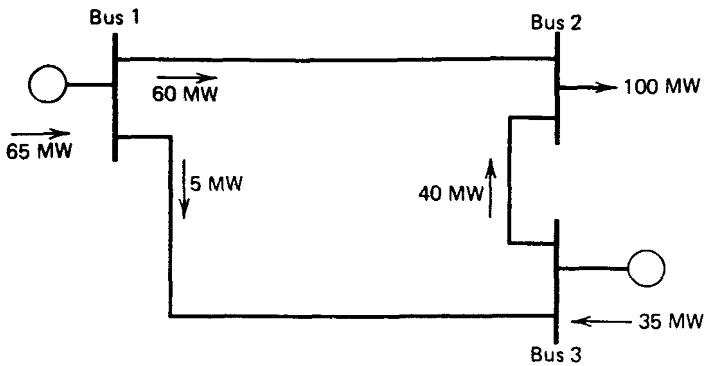

FIG. 4.12 Three-bus network showing flows calculated by DC power flow.

EXAMPLE 4C

The network of Example 4A was solved using the DC power flow with resulting power flows as shown in Figure 4.13. The DC power flow is useful for rapid calculations of real power flows, and, as will be shown later, it is very useful in security analysis studies.

4.2 TRANSMISSION LOSSES

4.2.1 A Two-Generator System

We are given the power system in Figure 4.14. The losses on the transmission line are proportional to the square of the power flow. The generating units are identical, and the production cost is modeled using a quadratic equation. If both units were loaded to 250 MW, we would fall short of the 500 MW load value by 12.5 MW lost on the transmission line, as shown in Figure 4.15.

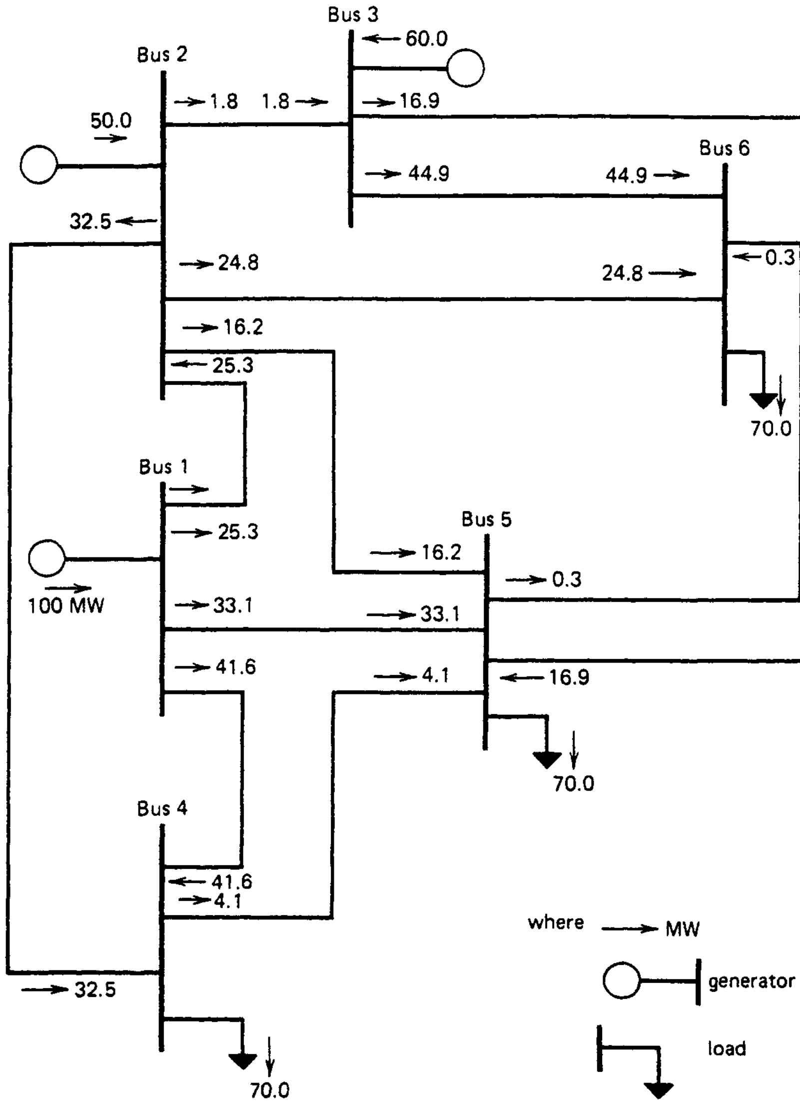

FIG. 4.13 Six-bus network base case DC power flow for Example 4C.

Where should the extra 12.5 MW be generated? Solve the Lagrange equation that was given in Chapter 3.

$$\mathscr{L} = F_1(P_1) + F_2(P_2) + \lambda(500 + P_{\text{loss}} - P_1 - P_2) \tag{4.38}$$

where

$$P_{\text{loss}} = 0.0002P_1^2$$

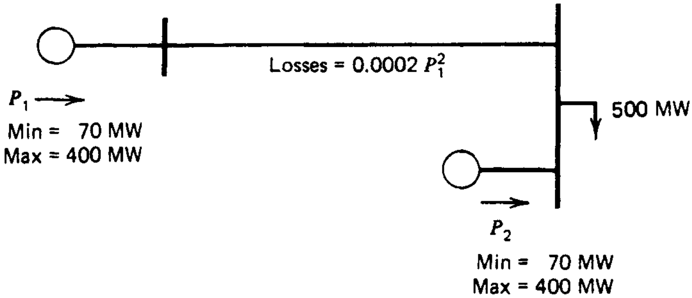

FIG. 4.14 Two-generator system.

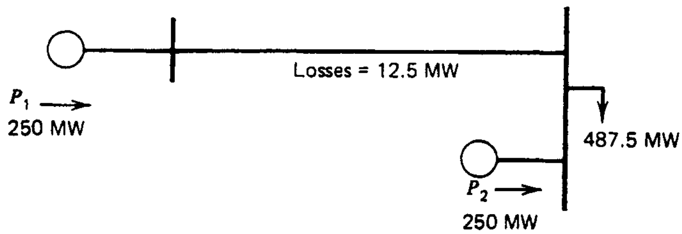

FIG. 4.15 Two-generator system with both generators at 250 MW output.

then

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial P_1} &= \frac{dF_1(P_1)}{dP_1} - \lambda\left(1 - \frac{\partial P_{loss}}{\partial P_1}\right) = 0 \\ \frac{\partial \mathcal{L}}{\partial P_2} &= \frac{dF_2(P_2)}{dP_2} - \lambda\left(1 - \frac{\partial P_{loss}}{\partial P_2}\right) = 0 \\ P_1 + P_2 &- 500 - P_{loss} = 0 \end{aligned} \tag{4.39}$$

Substituting into Eq. 4.39,

$$7.0 + 0.004P_1 - \lambda(1 - 0.0004P_1) = 0$$

$$7.0 + 0.004P_2 - \lambda = 0$$

$$P_1 + P_2 - 500 - 0.0002P_1^2 = 0$$

Solution: $P_1 = 178.882$

$P_2 = 327.496$

Production cost: $F_1(P_1) + F_2(P_2) = 4623.15$R/h

Losses: 6.378 MW

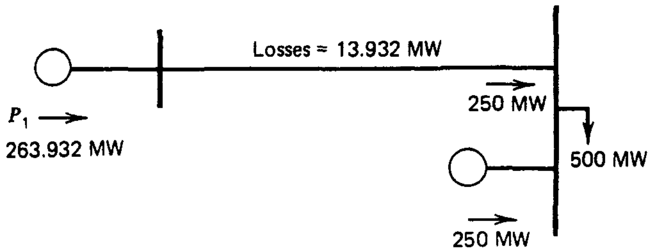

FIG. 4.16 Two-generator system with generator 1 supplying all losses.

Suppose we had decided simply to ignore the economic influence of losses and ran unit 1 up until it supplied all the losses. It would need to be run at 263.932 MW, as shown in Figure 4.16. In this case, the total production cost would be

$$F_1(263.932) + F_2(250) = 4661.84 \text{ Ɽ/h}$$

Note that the optimum dispatch tends toward supplying the losses from the unit close to the load, and it also resulted in a lower value of losses. Also note that best economics are not necessarily attained at minimum losses. The minimum loss solution for this case would simply run unit 1 down and unit 2 up as far as possible. The result is unit 2 on high limit.

$$P_1 = 102.084 \text{ MW}$$
$$P_2 = 400.00 \text{ MW (high limit)}$$

The minimum loss production cost would be

$$F_1(102.084) + F_2(400) = 4655.43 \text{ Ɽ/h}$$
$$\text{Min losses} = 2.084 \text{ MW}$$

4.2.2 Coordination Equations, Incremental Losses, and Penalty Factors

The classic Lagrange multiplier solution to the economic dispatch problem was given in Chapter 3. This is repeated here and expanded.

Minimize: $\mathscr{L} = F_T + \lambda\phi$

Where: $F_T = \sum_{i=1}^{N} F_i(P_i)$

$$\phi = P_{\text{load}} + P_{\text{loss}}(P_1, P_2 \ldots P_N) - \sum_{i=1}^{N} P_i$$

Solution: $\dfrac{\partial \mathscr{L}}{\partial P_i} = 0 \quad \text{for all } P_{i\min} \le P_i \le P_{i\max}$

Then

$$\frac{\partial \mathscr{L}}{\partial P_1} = \frac{\mathrm{d}F_i}{\mathrm{d}P_i} - \lambda\left(1 - \frac{\partial P_{\text{loss}}}{\partial P_i}\right) = 0$$

The equations are rearranged

$$\left(\frac{1}{1 - \dfrac{\partial P_{\text{loss}}}{\partial P_i}}\right)\frac{\mathrm{d}F_i(P_i)}{\mathrm{d}P_i} = \lambda \tag{4.40}$$

where

$$\frac{\partial P_{\text{loss}}}{\partial P_i}$$

is called the *incremental loss* for bus i, and

$$Pf_i = \left(\frac{1}{1 - \dfrac{\partial P_{\text{loss}}}{\partial P_i}}\right)$$

is called the *penalty factor* for bus i. Note that if the losses increase for an increase in power from bus i, the incremental loss is positive and the penalty factor is greater than unity.

When we did not take account of transmission losses, the economic dispatch problem was solved by making the incremental cost at each unit the same. We can still use this concept by observing that the penalty factor, Pf_i, will have the following effect. For $Pf_i > 1$ (positive increase in P_i results in increase in losses)

$$Pf_i \frac{\mathrm{d}F_i(P_i)}{\mathrm{d}P_i}$$

acts as if

$$\frac{\mathrm{d}F_i(P_i)}{\mathrm{d}P_i}$$

had been slightly increased (moved up). For $Pf_i < 1$ (positive increase in P_i results in decrease in losses)

$$Pf_i \frac{\mathrm{d}F_i(P_i)}{\mathrm{d}P_i}$$

acts as if

$$\frac{\mathrm{d}F_i(P_i)}{\mathrm{d}P_i}$$

had been slightly decreased (moved down). The resulting set of equations look like

$$Pf_i \frac{dF_i(P_i)}{dP_i} = \lambda \qquad \text{for all } P_{i\min} \leq P_i \leq P_{i\max} \tag{4.41}$$

and are called *coordination equations*. The P_i values that result when penalty factors are used will be somewhat different from the dispatch which ignores the losses (depending on the Pf_i and $dF_i(P_i)/dP_i$ values). This is illustrated in Figure 4.17.

4.2.3 The *B* Matrix Loss Formula

The *B* matrix loss formula was originally introduced in the early 1950s as a practical method for loss and incremental loss calculations. At the time, automatic dispatching was performed by analog computers and the loss formula was "stored" in the analog computers by setting precision potentiometers. The equation for the *B* matrix loss formula is as follows.

$$P_{\text{loss}} = \mathbf{P}^T[B]\mathbf{P} + B_0^T P + B_{00} \tag{4.42}$$

where

$\mathbf{P}$ = vector of all generator bus net MW

$[B]$ = square matrix of the same dimension as $\mathbf{P}$

B_0 = vector of the same length as $\mathbf{P}$

B_{00} = constant

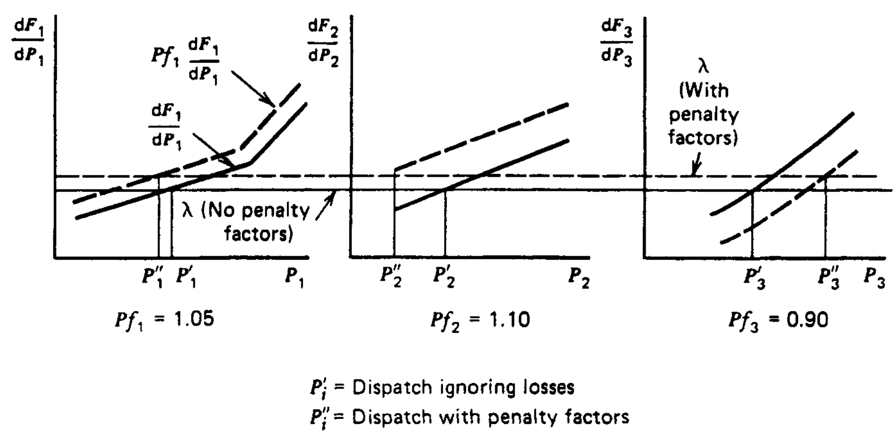

FIG. 4.17 Economic dispatch, with and without penalty factors.

This can be written:

$$P_{\text{loss}} = \sum_i \sum_j P_i B_{ij} P_j + \sum_i B_{i0} P_i + B_{00} \tag{4.43}$$

Before we discuss the calculation of the B coefficients, we will discuss how the coefficients are used in an economic dispatch calculation. Substitute Eq. 4.43 into Eqs. 3.7, 3.8, and 3.9.

$$\phi = -\sum_{i=1}^{N} P_i + P_{\text{load}} + \left(\sum_i \sum_j P_i B_{ij} P_j + \sum_i B_{i0} P_i + B_{00} \right) \tag{4.44}$$

Then

$$\frac{\partial \mathscr{L}}{\partial P_i} = \frac{\mathrm{d}F_i(P_i)}{\mathrm{d}P_i} - \lambda \left(1 - 2 \sum_j B_{ij} P_j - B_{i0} \right) \tag{4.45}$$

Note that the presence of the incremental losses has coupled the coordination equations; this makes solution somewhat more difficult. A method of solution that is often used is shown in Figure 4.18.

EXAMPLE 4D

The B matrix loss formula for the network in Example 4A is given here. (Note that all P_i values must be per unit on 100 MVA base, which results in P_{loss} in per unit on 100 MVA base.)

$$P_{\text{loss}} = [P_1 \quad P_2 \quad P_3] \begin{bmatrix} 0.0676 & 0.00953 & -0.00507 \\ 0.00953 & 0.0521 & 0.00901 \\ -0.00507 & 0.00901 & 0.0294 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$+ [-0.0766 \quad -0.00342 \quad 0.0189] \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} + 0.040357$$

From the base case power flow we have

$$P_1 = 107.9 \text{ MW}$$

$$P_2 = 50.0 \text{ MW}$$

$$P_2 = 60.0 \text{ MW}$$

$$P_{\text{loss}} = 7.9 \text{ MW (as calculated by the power flow)}$$

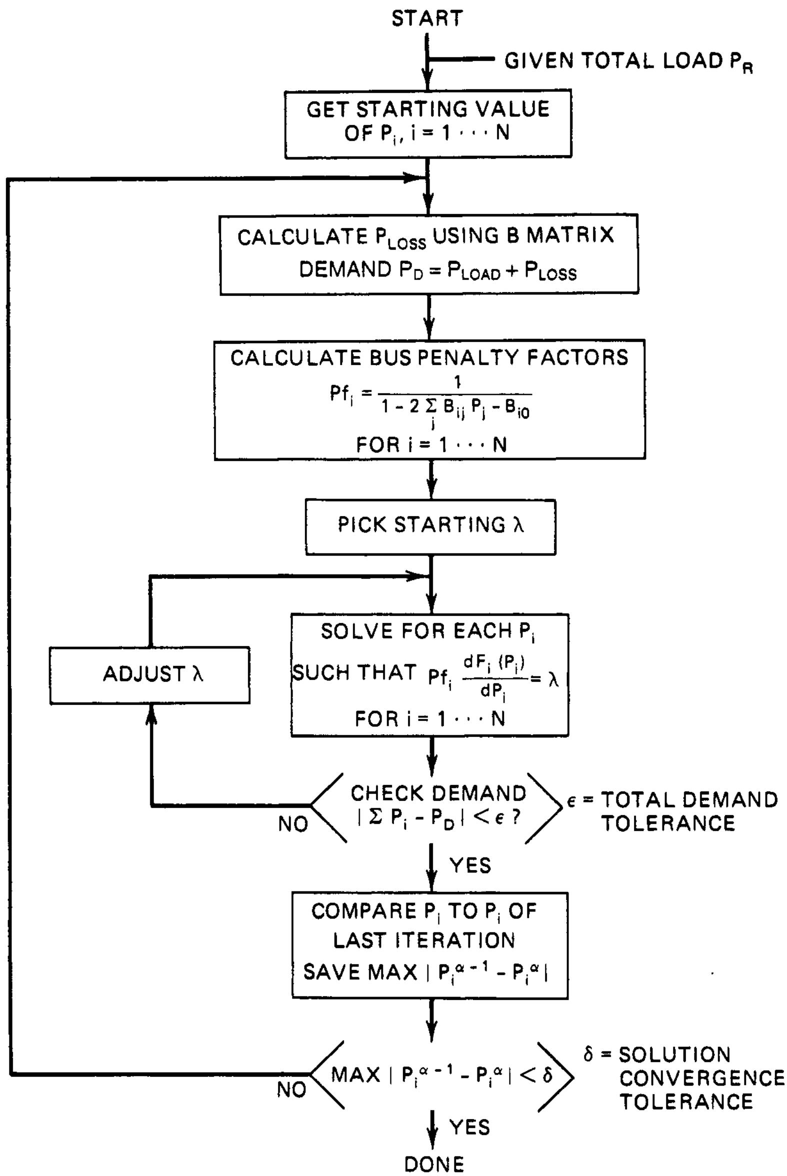

FIG. 4.18 Economic dispatch with updated penalty factors.

With these generation values placed in the B matrix, we see a very close agreement with the power flow calculation.

$$P_{\text{loss}} = [1.079 \quad 0.50 \quad 0.60]\begin{bmatrix} 0.0676 & 0.00953 & -0.00507 \\ 0.00953 & 0.0521 & 0.00901 \\ -0.00507 & 0.00901 & 0.0294 \end{bmatrix}\begin{bmatrix} 1.079 \\ 0.50 \\ 0.60 \end{bmatrix}$$

$$+ [-0.0766 \quad -0.00342 \quad 0.0189]\begin{bmatrix} 1.079 \\ 0.50 \\ 0.60 \end{bmatrix} + 0.040357$$

$$= 0.07877 \text{ pu (or 7.877 MW) loss}$$

EXAMPLE 4E

Let the fuel cost curves for the three units in the six-bus network of Example 4A be given as

$$F_1(P_1) = 213.1 + 11.669P_1 + 0.00533P_1^2 \text{ ₽/h}$$
$$F_2(P_2) = 200.0 + 10.333P_2 + 0.00889P_2^2 \text{ ₽/h}$$
$$F_3(P_3) = 240.0 + 10.833P_3 + 0.00741P_3^2 \text{ ₽/h}$$

with unit dispatch limits

$$50.0 \text{ MW} \le P_1 \le 200 \text{ MW}$$
$$37.5 \text{ MW} \le P_2 \le 150 \text{ MW}$$
$$45.0 \text{ MW} \le P_3 \le 180 \text{ MW}$$

A computer program using the method of Figure 4.17 was run using:

$$P_{\text{load}} \text{ (total load to be supplied)} = 210 \text{ MW}$$

The resulting iterations (Table 4.1) show how the program must redispatch again and again to account for the changes in losses and penalty factors.

Note that the flowchart of Figure 4.18 shows a "two-loop" procedure. The "inner" loop adjusts λ until total demand is met; then the outer loop recalculates the penalty factors. (Under some circumstances the penalty factors are quite sensitive to changes in dispatch. If the incremental costs are relatively "flat," this procedure may be unstable and special precautions may need to be employed to insure convergence.)

TABLE 4.1 Iterations for Example 4E

Iteration	λ	P_{loss}	P_D	P_1	P_2	P_3
1	12.8019	17.8	227.8	50.00	85.34	92.49
2	12.7929	11.4	221.4	74.59	71.15	75.69
3	12.8098	9.0	219.0	73.47	70.14	75.39
4	12.8156	8.8	218.8	73.67	69.98	75.18
5	12.8189	8.8	218.8	73.65	69.98	75.18
6	12.8206	8.8	218.8	73.65	69.98	75.18

4.2.4 Exact Methods of Calculating Penalty Factors

4.2.4.1 A Discussion of Reference Bus Versus Load Center Penalty Factors

The B matrix assumes that all load currents conform to an equivalent total load current and that the equivalent load current is the negative of the sum of all generator currents. When incremental losses are calculated, something is implied.

$$\text{Total loss} = \mathbf{P}^T[B]\mathbf{P} + \mathbf{B}_0^T\mathbf{P} + B_{00}$$

$$\text{Incremental loss at generator bus } i = \frac{\partial P_{loss}}{\partial P_i}$$

The incremental loss is the change in losses when an increment is made in generation output. As just derived, the incremental loss for bus i assumed that all the other generators remained fixed. By the original assumption, however, the load currents all conform to each other and always balance with the generation; then the implication in using a B matrix is that an *incremental increase in generator output is matched by an equivalent increment in load.*

An alternative approach to economic dispatch is to use a reference bus that always moves when an increment in generation is made. Figure 4.19 shows a

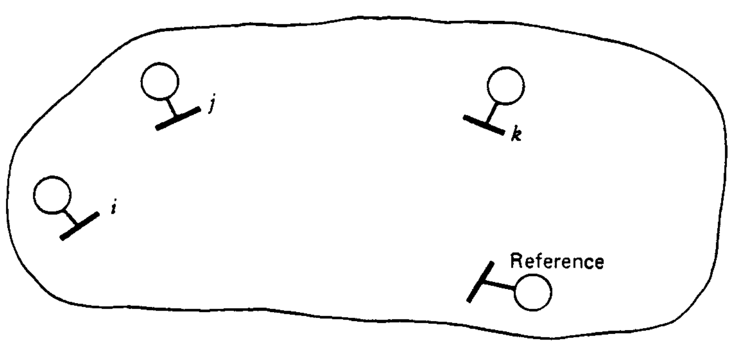

FIG. 4.19 Power system with reference generator.

power system with several generator buses and a reference-generator bus. Suppose we change the generation on bus i by ΔP_i,

$$P_i^{\text{new}} = P_i^{\text{old}} + \Delta P_i \tag{4.46}$$

Furthermore, we will assume that *load stays constant* and that to compensate for the increase in ΔP_i, the reference bus just drops off by ΔP_{ref}.

$$P_{\text{ref}}^{\text{new}} = P_{\text{ref}}^{\text{old}} + \Delta P_{\text{ref}} \tag{4.47}$$

If nothing else changed, ΔP_{ref} would be the negative of ΔP_i; however, the flows on the system can change as a result of the two generation adjustments. The change in flow is apt to cause a change in losses so that ΔP_{ref} is not necessarily equal to ΔP_i. That is,

$$\Delta P_{\text{ref}} = -\Delta P_i + \Delta P_{\text{loss}} \tag{4.48}$$

Next, we can define β_i as the ratio of the negative change in the reference-bus power to the change ΔP_i.

$$\beta_i = \frac{-\Delta P_{\text{ref}}}{\Delta P_i} = \frac{(\Delta P_i - \Delta P_{\text{loss}})}{\Delta P_i} \tag{4.49}$$

or

$$\beta_i = 1 - \frac{\partial P_{\text{loss}}}{\partial P_i} \tag{4.50}$$

We can define economic dispatch as follows.

> All generators are in economic dispatch when a shift of ΔP MW from any generator to the reference bus results in no change in net production cost; where ΔP is arbitrarily small.

That is, if

$$\text{Total production cost} = \sum F_i(P_i)$$

then the change in production cost with a shift ΔP_i from plant i is

$$\Delta\text{Production cost} = \frac{\mathrm{d}F_i(P_i)}{\mathrm{d}P_i}\,\Delta P_i + \frac{\mathrm{d}F_{\text{ref}}(P_{\text{ref}})}{\mathrm{d}P_{\text{ref}}}\,\Delta P_{\text{ref}} \tag{4.51}$$

but

$$\Delta P_{\text{ref}} = -\beta_i \Delta P_i$$

then

$$\Delta\text{Production cost} = \frac{\mathrm{d}F_i(P_i)}{\mathrm{d}P_i}\,\Delta P_i - \beta_i\,\frac{\mathrm{d}F_{\text{ref}}(P_{\text{ref}})}{\mathrm{d}P_{\text{ref}}}\,\Delta P_i \tag{4.52}$$

To satisfy the economic conditions,

$$\Delta\text{Production cost} = 0$$

or

$$\frac{\mathrm{d}F_i(P_i)}{\mathrm{d}P_i} = \beta_i\,\frac{\mathrm{d}F_{\text{ref}}(P_{\text{ref}})}{\mathrm{d}P_{\text{ref}}} \tag{4.53}$$

which could be written as

$$\frac{1}{\beta_i}\,\frac{\mathrm{d}F_i(P_i)}{\mathrm{d}P_i} = \frac{\mathrm{d}F_{\text{ref}}(P_{\text{ref}})}{\mathrm{d}P_{\text{ref}}} \tag{4.54}$$

This is very similar to Eq. 4.40. To obtain an economic dispatch solution, pick a value of generation on the reference bus and then set all other generators according to Eq. 4.54, and check for total demand and readjust reference generation as needed until a solution is reached.

Note further that this method is exactly the first-order gradient method with losses.

$$\Delta F_T = \sum_{i \neq \text{ref}} \left[\frac{\mathrm{d}F_i}{\mathrm{d}P_i} - \beta_i\,\frac{\mathrm{d}F_{\text{ref}}}{\mathrm{d}P_{\text{ref}}}\right]\Delta P_i \tag{4.55}$$

4.2.4.2 Reference-Bus Penalty Factors Direct from the AC Power Flow

The reference-bus penalty factors may be derived using the Newton–Raphson power flow. What we wish to know is the ratio of change in power on the reference bus when a change ΔP_i is made.

Where P_{ref} is a function of the voltage magnitude and phase angle on the network, when a change in ΔP_i is made, all phase angles and voltages in the network will change. Then

$$\begin{aligned}\Delta P_{\text{ref}} &= \sum_i \frac{\partial P_{\text{ref}}}{\partial \theta_i}\,\Delta\theta_i + \sum_i \frac{\partial P_{\text{ref}}}{\partial |E_i|}\,\Delta|E_i| \\ &= \sum_i \frac{\partial P_{\text{ref}}}{\partial \theta_i}\,\frac{\partial \theta_i}{\partial P_i}\,\Delta P_i + \sum_i \frac{\partial P_{\text{ref}}}{\partial |E_i|}\,\frac{\partial |E_i|}{\partial P_i}\,\Delta P_i\end{aligned} \tag{4.56}$$

To carry out the matrix manipulations, we will also need the following.

$$\begin{aligned}\Delta P_{\text{ref}} &= \sum_i \frac{\partial P_{\text{ref}}}{\partial \theta_i}\,\Delta\theta_i + \sum_i \frac{\partial P_{\text{ref}}}{\partial |E_i|}\,\Delta|E_i| \\ &= \sum_i \frac{\partial P_{\text{ref}}}{\partial \theta_i}\,\frac{\partial \theta_i}{\partial Q_i}\,\Delta Q_i + \sum_i \frac{\partial P_{\text{ref}}}{\partial |E_i|}\,\frac{\partial |E_i|}{\partial Q_i}\,\Delta Q_i\end{aligned} \tag{4.57}$$

The terms $\partial P_{\text{ref}}/\partial \theta_i$ and $\partial P_{\text{ref}}/|E_i|$ *are derived by differentiating Eq.* 4.18 *for the reference bus. The terms* $\partial \theta_i/\partial P_i$ and $\partial |E_i|/\partial P_i$ are from the inverse Jacobian matrix (see Eq. 4.20). We can write Eqs. 4.56 and 4.57 for every bus i in the network. The resulting equation is

$$\left[\frac{\partial P_{\text{ref}}}{\partial P_1}\ \frac{\partial P_{\text{ref}}}{\partial Q_1}\ \frac{\partial P_{\text{ref}}}{\partial P_2}\ \frac{\partial P_{\text{ref}}}{\partial Q_2} \cdots \frac{\partial P_{\text{ref}}}{\partial P_N}\ \frac{\partial P_{\text{ref}}}{\partial Q_N}\right] = \left[\frac{\partial P_{\text{ref}}}{\partial \theta_1}\ \frac{\partial P_{\text{ref}}}{\partial |E_1|}\ \frac{\partial P_{\text{ref}}}{\partial \theta_2}\ \frac{\partial P_{\text{ref}}}{\partial |E_2|} \cdots \frac{\partial P_{\text{ref}}}{\partial \theta_N}\ \frac{\partial P_{\text{ref}}}{\partial |E_N|}\right][J^{-1}] \tag{4.58}$$

By transposing we get

$$\begin{bmatrix} \frac{\partial P_{\text{ref}}}{\partial P_1} \\ \frac{\partial P_{\text{ref}}}{\partial Q_1} \\ \frac{\partial P_{\text{ref}}}{\partial P_2} \\ \frac{\partial P_{\text{ref}}}{\partial Q_2} \\ \vdots \\ \frac{\partial P_{\text{ref}}}{\partial P_N} \\ \frac{\partial P_{\text{ref}}}{\partial Q_N} \end{bmatrix} = [J^{T-1}] \begin{bmatrix} \frac{\partial P_{\text{ref}}}{\partial \theta_1} \\ \frac{\partial P_{\text{ref}}}{\partial |E_1|} \\ \frac{\partial P_{\text{ref}}}{\partial \theta_2} \\ \frac{\partial P_{\text{ref}}}{\partial |E_2|} \\ \vdots \\ \frac{\partial P_{\text{ref}}}{\partial \theta_N} \\ \frac{\partial P_{\text{ref}}}{\partial |E_N|} \end{bmatrix} \tag{4.59}$$

In practice, instead of calculating J^{T-1} explicitly, we use Gaussian elimination on J^T in the same way we operate on J in the Newton power flow solution.

APPENDIX
Power Flow Input Data for Six-Bus System

Figure 4.20 lists the input data for the six-bus sample system used in the examples in Chapter 4. The impedances are per unit on a base of 100 MVA. The generation cost functions are contained in Example 4E.

Line Data

From bus	To bus	R(pu)	X(pu)	BCAP[a] (pu)
1	2	0.10	0.20	0.02
1	4	0.05	0.20	0.02
1	5	0.08	0.30	0.03
2	3	0.05	0.25	0.03
2	4	0.05	0.10	0.01
2	5	0.10	0.30	0.02
2	6	0.07	0.20	0.025
3	5	0.12	0.26	0.025
3	6	0.02	0.10	0.01
4	5	0.20	0.40	0.04
5	6	0.10	0.30	0.03

[a] BCAP = half total line charging suseptance.

Bus Data

Bus number	Bus type	Voltage schedule (pu V)	P_{gen} (pu MW)	P_{load} (pu MW)	Q_{load} (pu MVAR)
1	Swing	1.05			
2	Gen.	1.05	0.50	0.0	0.0
3	Gen.	1.07	0.60	0.0	0.0
4	Load		0.0	0.7	0.7
5	Load		0.0	0.7	0.7
6	Load		0.0	0.7	0.7

FIG. 4.20 Input data for six-bus sample power system.

PROBLEMS

4.1 The circuit elements in the 138 kV circuit in Figure 4.21 are in per unit on a 100 MVA base with the nominal 138 kV voltage as base. The $P + jQ$ load is scheduled to be 170 MW and 50 MVAR.

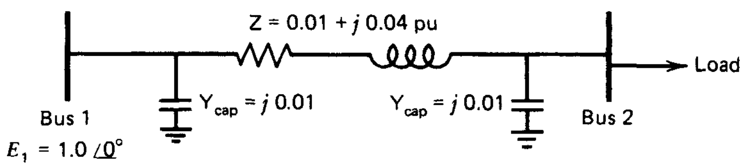

FIG. 4.21 Two-bus AC system for Problem 4.1.

a. Write the Y matrix for this two-bus system.

b. Assume bus 1 as the reference bus and set up the Gauss–Seidel correction equation for bus 2. (Use $1.0 \angle 0°$ as the initial voltage on bus 2.) Carry out two or three iterations and show that you are converging.

c. Apply the "DC" load flow conventions to this circuit and solve for the phase angle at bus 2 for the same load real power of 1.7 per unit.

4.2 Given the network in Figure 4.22 (base = 100 MVA):

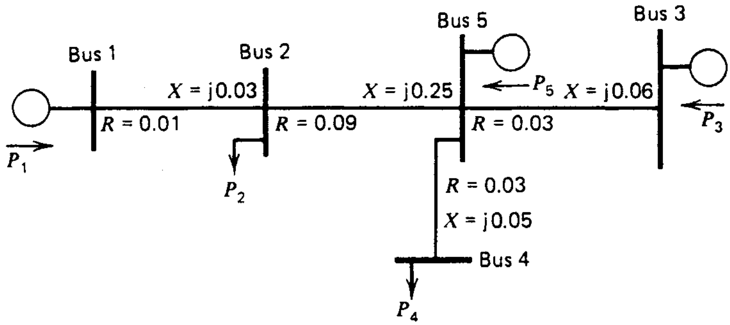

FIG. 4.22 Five-bus network for Problem 4.2.

a. Develop the $[B']$ matrix for this system.

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = [B'] \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix} \quad \begin{matrix} P \text{ in per unit MW} \\ \theta \text{ in radians (rad)} \end{matrix}$$

b. Assume bus 5 as the reference bus. To carry out a "DC" load flow, we will set $\theta_5 = 0$ rad. Row 5 and column 5 will be zeroed.

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} & & & & 0 \\ & & \text{Remainder} & & 0 \\ & & \text{of } B' & & 0 \\ & & & & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix}$$

Solve for the $[B']^{-1}$ matrix.

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix} = [B']^{-1} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix}$$

c. Calculate the phase angles for the set of power injections.

$P_1 = 100$ MW generation

$P_2 = 120$ MW load

$P_3 = 150$ MW generation

$P_4 = 200$ MW load

d. Calculate P_5 according to the "DC" load flow.

e. Calculate all power flows on the system using the phase angles found in part c.

f. (Optional) Calculate the reference-bus penalty factors for buses 1, 2, 3, and 4. Assume all bus voltage magnitudes are 1.0 per unit.

4.3 Given the following loss formula (use P values in MW):

$$B_{ij} = \begin{matrix} & 1 & 2 & 3 \\ 1 \\ 2 \\ 3 \end{matrix} \begin{bmatrix} 1.36255 \times 10^{-4} & 1.753 \times 10^{-5} & 1.8394 \times 10^{-4} \\ 1.754 \times 10^{-5} & 1.5448 \times 10^{-4} & 2.82765 \times 10^{-4} \\ 1.8394 \times 10^{-4} & 2.82765 \times 10^{-4} & 1.6147 \times 10^{-3} \end{bmatrix}$$

B_{i0} and B_{00} are neglected. Assume three units are on-line and have the following characteristics.

Unit 1:

$$H_1 = 312.5 + 8.25P_1 + 0.005P_1^2, \text{ MBu/h}$$

$$50 \le P_1 \le 250 \text{ MW}$$

$$\text{Fuel cost} = 1.05 \text{ Ɽ/MBtu}$$

Unit 2:

$$H_2 = 112.5 + 8.25P_2 + 0.005P_2^2, \text{ MBtu/h}$$

$$5 \le P_2 \le 150 \text{ MW}$$

$$\text{Fuel cost} = 1.217 \text{ Ɽ/MBtu}$$

Unit 3: $H_3 = 50 + 8.25P_3 + 0.005P_3^2$, MBtu/h

$15 \leq P_3 \leq 100$ MW

Fuel cost = 1.1831 ₽/MBtu

a. No Losses Used in Scheduling

i. Calculate the optimum dispatch and total cost neglecting losses for $P_D = 190$ MW.*

ii. Using this dispatch and the loss formula, calculate the system losses.

b. Losses Included in Scheduling

i. Find the optimum dispatch for a total generation of $P_D = 190$ MW* using the coordination equations and the loss formula.

ii. Calculate the cost rate.

iii. Calculate the total losses using the loss formula.

iv. Calculate the resulting load supplied.

4.4 All parts refer to the three-bus system shown in Figure 4.23.

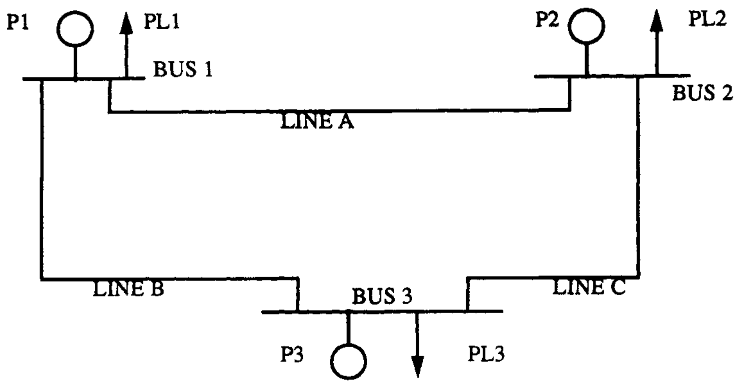

FIG. 4.23 Network for Problem 4.4.

Data for this problem is as follows:

Unit 1: $P_1 = 570$ MW

Unit 2: $P_2 = 330$ MW

Unit 3: $P_3 = 200$ MW

Loads:

$P_{L1} = 200$ MW

$P_{L2} = 400$ MW

$P_{L3} = 500$ MW

* $P_{demand} = P_1 + P_2 + P_3 = P_D$

P_{loss} = power loss

$P_{load} = P_D - P_{loss}$ = net load

Transmission line data:

P_{loss} in line A $= 0.02P_A^2$ (where $P_A = P$ flow from bus 1 to bus 2)

P_{loss} in line B $= 0.02P_B^2$ (where $P_B = P$ flow from bus 1 to bus 3)

P_{loss} in line C $= 0.02P_C^2$ (where $P_C = P$ flow from bus 2 to bus 3)

Note: the above data are for P_{loss} in per unit when power flows P_A or P_B or P_C are in per unit.

Line reactances:

$$X_A = 0.2 \text{ per unit}$$

$$X_B = 0.3333 \text{ per unit}$$

$$X_C = 0.05 \text{ per unit}$$

(assume 100-MVA base when converting to per unit).

a. Find how the power flows distribute using the DC power flow approximation. Use bus 3 as the reference.

b. Calculate the total losses.

c. Calculate the incremental losses for bus 1 and bus 2 as follows: assume that ΔP_1 is balanced by an equal change on the reference bus. Use the DC power flow data from part a and calculate the change in power flow on all three lines ΔP_A, ΔP_B, and ΔP_C. Now calculate the line incremental loss as:

$$\Delta P_{\text{loss}_A} = \left(\frac{\partial}{\partial P_A}\right)\Delta P_A = (0.04P_A)\Delta P_A$$

Similarly, calculate for lines B and C.

d. Find the bus penalty factors calculated from the line incremental losses found in part c.

4.5 The three-bus, two-generator power system shown in Figure 4.24 is to be dispatched to supply the 500-MW load. Each transmission line has losses

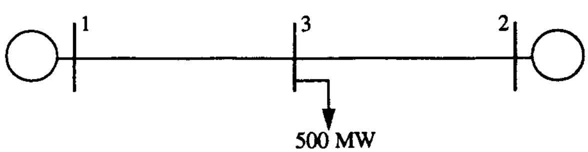

FIG. 4.24 Circuit for Problem 4.5.

that are given by the equations below.

$$P_{\text{loss}_{13}} = 0.0001P_1^2$$

$$P_{\text{loss}_{23}} = 0.0002P_2^2$$

$$F_1(P_1) = 500 + 8P_1 + 0.002P_1^2$$

$$50\ \text{MW} < P_1 < 500\ \text{MW}$$

$$F_2(P_2) = 400 + 7.9P_2 + 0.0025P_2^2$$

$$50\ \text{MW} < P_2 < 500\ \text{MW}$$

You are to attempt to solve for both the economic dispatch of this system and the "power flow." The power flow should show what power enters and leaves each bus of the network. If you use an iterative solution, show at least two complete iterations. You may use the following initial conditions: $P_1 = 250$ MW and $P_2 = 250$ MW.

FURTHER READING

The basic papers on solution of the power flow can be found in references 1–5. The development of the loss-matrix equations is based on the work of Kron (reference 6), who developed the reference-frame transformation theory. Other developments of the transmission-loss formula are seen in references 7 and 8. Meyer's paper (9) is representative of recent adaptation of sparsity programming methods to calculation of the loss matrix.

The development of the reference-bus penalty factor method can be seen in references 10 and 11. Reference 12 gives an excellent derivation of the reference-bus penalty factors derived from the Newton power-flow equations. Reference 12 provides an excellent summary of recent developments in power system dispatch.

1. Ward, J. B., Hale, H. W., "Digital Computer Solution of Power-Flow Problems," *AIEE Transactions, Part III Power Apparatus and Systems*, Vol. 75, June 1956, pp. 398–404.
2. VanNess, J. E., "Iteration Methods for Digital Load Flow Studies," *AIEE Transactions on Power Apparatus and Systems*, Vol. 78A, August 1959, pp. 583–588.
3. Tinney, W. F., Hart, C. E., "Power Flow Solution by Newton's Method," *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-86, November 1967, pp. 1449–1460.
4. Stott, B., Alsac, O., "Fast Decoupled Load Flow," *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-93, May/June 1974, pp. 859–869.
5. Stott, B., "Review of Load-Flow Calculation Methods," *Proceedings of the IEEE*, Vol. 62, No. 2, July 1974, pp. 916–929.
6. Kron, G., "Tensorial Analysis of Integrated Transmission Systems—Part I: The Six Basic Reference Frames," *AIEE Transactions*, Vol. 70, Part I, 1951, pp. 1239–1248.
7. Kirchmayer, L. K., Stagg, G. W., "Analysis of Total and Incremental Losses in Transmission Systems," *AIEE Transactions*, Vol. 70, Part I, 1951, pp. 1179–1205.

8. Early, E. D., Watson, R. E., "A New Method of Determining Constants for the General Transmission Loss Equation," *AIEE Transactions on Power Apparatus and Systems*, Vol. PAS-74, February 1956, pp. 1417–1423.
9. Meyer, W. S., "Efficient Computer Solution for Kron and Kron Early-Loss Formulas," *Proceedings of the* 1973 *PICA Conference*, IEEE 73 CHO 740-1, PWR, pp. 428–432.
10. Shipley, R. B., Hochdorf, M., "Exact Economic Dispatch—Digital Computer Solution," *AIEE Transactions on Power Apparatus and Systems*, Vol. PAS-75, November 1956, pp. 1147–1152.
11. Dommel, H. W., Tinney, W. F., "Optimal Power Flow Solutions," *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-87, October 1968, pp. 1866–1876.
12. Happ, H. H., "Optimal Power Dispatch," *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-93, May/June 1974, pp. 820–830.