BAB I
OPERATOR NABLA

1.1. Fungsi Operator Nabla

operator nabla berfungsi sebagai diferensial dari suatu fungsi yang oleh Koordinat Cartesius dirumuskan sebagai

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

Ada 3 cara perkalian untuk operator nabla, seperti dalam vektor.

1. Bekerja pada fungsi skalar yang disebut : gradien

$$\nabla T = \hat{i}\frac{\partial T}{\partial x} + \hat{j}\frac{\partial T}{\partial y} + \hat{k}\frac{\partial T}{\partial z}$$

2. Bekerja pada fungsi vektor yang disebut divergensi melalui perkalian dot.

$$\nabla \cdot \vec{V} = \frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z}$$

3. Bekerja pada fungsi vektor melalui perkalian silang yang disebut rotasi / curl.

$$\nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix} = \hat{i}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) + \hat{j}\left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right) + \hat{k}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right).$$

Beberapa aturan dalam perkalian operator nabla

1. $\nabla(fg) = f(\nabla g) + g(\nabla f)$
2. $\nabla(\vec{A}\cdot\vec{B}) = \vec{A}\times(\nabla\times\vec{B}) + \vec{B}\times(\nabla\times\vec{A}) + (\vec{A}\cdot\nabla)\vec{B} + (\vec{B}\cdot\nabla)\vec{A}$
3. $\nabla(f\vec{A}) = f(\nabla\cdot\vec{A}) + \vec{A}\cdot(\nabla f)$
4. $\nabla\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{A}) - \vec{A}\cdot(\nabla\times\vec{B})$.
5. $\nabla\times(f\vec{A}) = f(\nabla\times\vec{A}) - \vec{A}\times(\nabla f)$.
6. $\nabla\times(\vec{A}\times\vec{B}) = (\vec{B}\cdot\nabla)\vec{A} - (\vec{A}\cdot\nabla)\vec{B} + \vec{A}\cdot(\nabla\cdot\vec{B}) - \vec{B}\cdot(\nabla\cdot\vec{A})$.

Perkalian tripel

1. $\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{C}(\vec{A}\times\vec{B}) = \vec{B}(\vec{C}\times\vec{A})$.
2. $\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$.

Turunan kedua.

1. $\nabla\cdot(\nabla\times\vec{A}) = 0 \rightarrow \vec{A}(\nabla\times\nabla) = 0$.
2. $\nabla\times(\nabla f) = 0 \rightarrow f$ = skalar, dng. $\nabla\times\nabla f = 0$.
3. $\nabla\times(\nabla\times\vec{A}) = \nabla(\nabla\cdot A) - \nabla^2 A$

Contoh Penggunaan operator nabla 2

1. Suatu fungsi dengan variabel, seperti suhu $T(x, y, z)$, yg menunjukkan suatu suhu pada suatu ruang.

Berdasarkan derivatif parsial:

$$dT = \frac{\partial T}{\partial x}dx + \frac{\partial T}{\partial y}dy + \frac{\partial T}{\partial z}dz$$

$$= \nabla T \cdot d\vec{l} \quad , \text{ dimana } \quad d\vec{l} = \hat{i}dx + \hat{j}dy + \hat{k}dz$$

$$dT = \nabla T \cdot d\vec{l} = |\nabla T||d\vec{l}|\cos\theta$$

misal: Vektor posisi $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$.

$$|r| = (x^2 + y^2 + z^2)^{\frac{1}{2}}$$

$$f(r) = f(x^2 + y^2 + z^2)^{\frac{1}{2}}$$

$$\nabla f(r) = \hat{i}\frac{\partial f(r)}{\partial x} + \hat{j}\frac{\partial f(r)}{\partial y} + \hat{k}\frac{\partial f(r)}{\partial z}$$

$$\frac{\partial f(r)}{\partial x} = \frac{df(r)}{dr}\cdot\frac{\partial r}{\partial x} \quad , \text{ dimana } \quad \frac{\partial(r)}{\partial x} = \frac{\partial(x^2+y^2+z^2)^{\frac{1}{2}}}{\partial x} = \frac{x}{r}$$

jadi: $\nabla f(r) = \left[\hat{i}\left(\frac{x}{r}\right) + \hat{j}\left(\frac{y}{r}\right) + \hat{k}\left(\frac{z}{r}\right)\right]\cdot\frac{df(r)}{dr}$ $\quad \frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}}(2x)$

$$= \frac{\vec{r}}{r}\frac{df(r)}{dr} = \hat{r}\frac{df(r)}{dr}.$$

$$\nabla f(r) = \hat{r}\frac{df(r)}{dr}$$

Dg. rumus diatas kita dapat menghitung:

$\nabla f(r)$, untuk $f(r) = r^{n-1}$.

$$\nabla(r^{n-1}) = \hat{r}\frac{d(r^{n-1})}{dr} = \hat{r}(n-1)r^{n-2} = (n-1)r^{n-2}\hat{r}$$

2. $\nabla\cdot\vec{r}f(r) = ?$

$$\nabla\cdot(\vec{r}f(r)) = \vec{r}\cdot(\nabla f(r)) + f(r)(\nabla\cdot\vec{r}).$$

$$= \vec{r}\cdot\hat{r}\frac{df(r)}{dr} + 3f(r).$$

$$= \frac{r^2}{r}\frac{df(r)}{dr} + 3f(r).$$

$$= r\frac{df(r)}{dr} + 3f(r)$$

Bila $f(r) = r^{n-1}$, maka $\nabla\cdot\vec{r}r^{n-1} = (n+2)r^{n-1}$.

3. $\nabla \times \vec{r} f(r) = ?$

$$\nabla \times \vec{r} f(r) = f(r)(\nabla \times \vec{r}) - \vec{r} \times (\nabla f(r)).$$

$$\nabla \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0.$$

$$\nabla f(r) = \hat{r} \frac{df(r)}{dr} \quad ; \quad \vec{r} \times \hat{r} \frac{df(r)}{dr} = 0 \, ; \quad \vec{r} \text{ dan } \hat{r} \text{ searah.}$$

$$\nabla \times \vec{r} f(r) = 0$$

1.2. Gradien, Divergensi, rotasi/curl

1.2.1. Gradien

Anggap medan skalar $\phi(x, y, z)$ sebagai fungsi skalar pada setiap titik ruang (x, y, z) dalam koordinat Cartes.
Sebagai fungsi skalar ia harus mempunyai nilai sama pada titik ruang dan tidak bergantung pada rotasi sistem koordinat

$$\phi'(x_1', y_1', x_3') = \phi(x_1, x_2, x_3)$$

$$\frac{\partial \phi'(x_1', x_2', x_3')}{\partial x_i'} = \frac{\partial \phi(x_1, x_2, x_3)}{\partial x_i}$$

$$= \sum_j \frac{\partial \phi}{\partial x_j} \frac{\partial x_j}{\partial x_i}$$

$$= \sum_j a_{ij} \frac{\partial \phi}{\partial x_j}$$

Jadi gradien itu merupakan suatu vektor dengan komponen $\frac{\partial \phi}{\partial x_j}$ yang disebut GRADIEN ϕ, dalam koordinat Cartesius ditulis

$$\nabla \phi = \hat{i} \frac{\partial \phi}{\partial x} + \hat{j} \frac{\partial \phi}{\partial y} + \hat{k} \frac{\partial \phi}{\partial z}$$

Bila $d\vec{r} = \hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz$.

$$\nabla \phi \cdot d\vec{r} = \left(\hat{i} \frac{\partial \phi}{\partial x} + \hat{j} \frac{\partial \phi}{\partial y} + \hat{k} \frac{\partial \phi}{\partial z}\right) \cdot (\hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz).$$

$$= \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy + \frac{\partial \phi}{\partial z} dz$$

$$\nabla \phi \cdot d\vec{r} = d\phi \quad \rightarrow$$

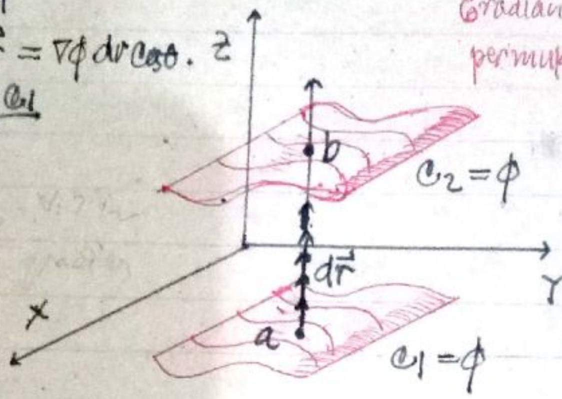
$$\Rightarrow \nabla\phi = \frac{d\phi}{d\vec{r}}$$

Kesimpulannya
Gradien ($\nabla\phi$) merupakan perubahan fungsi skalar ϕ terhadap perubahan posisi $d\vec{r}$. Sedang $\nabla\phi$ (gradien) merupakan suatu vektor, yang arahnya sejajar $d\vec{r}$ (arahnya ke lebih besar)

misal: suatu permukaan $\phi = C_1$. berubah ke arah $\phi = C_2$

$$d\phi = C_2 - C_1$$
$$= \nabla\phi \cdot d\vec{r} = \nabla\phi \, dr \cos\theta .$$
$$\nabla\phi = \frac{C_2 - C_1}{d\vec{r}}$$

Gradian selalu tegak lurus permukaan



bentuk integralnya gradien $\nabla\phi \cdot d\vec{r}$ dari titik a ke b adalah:

$$\int_a^b \nabla\phi \cdot d\vec{r} = \phi(b) - \phi(a).$$
$$= C_2 - C_1$$

arah gradien berlawan dg arah medan $\vec{E} = -\nabla V$

2.1.2. Divergensi

Bila $\vec{A} = \hat{i}A_x + \hat{j}A_y + \hat{k}A_z$, maka divergensi $\vec{A}$ dpt ditulis

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

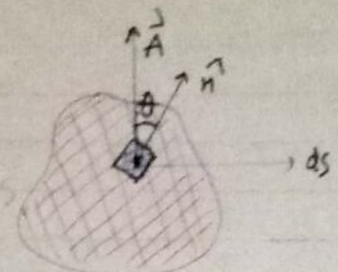
Pengertian divergensi dalam fisika, tidak dari

$$\lim_{\Delta V \to 0} \oint \frac{\vec{A} \cdot d\vec{s}}{\Delta V}$$

menurut teorema Gauss

$$\int \nabla \cdot \vec{A} \, d\tau = \oint_s \vec{A} \cdot d\vec{s}$$

$\vec{A}$ vektor sembarang yang melalui permukaan s tertutup.
Dari $\oint_s \vec{A} \cdot d\vec{s}$, maka dapat kita katakan $\vec{A}$ merupakan fluks yang melalui permukaan s tertutup.

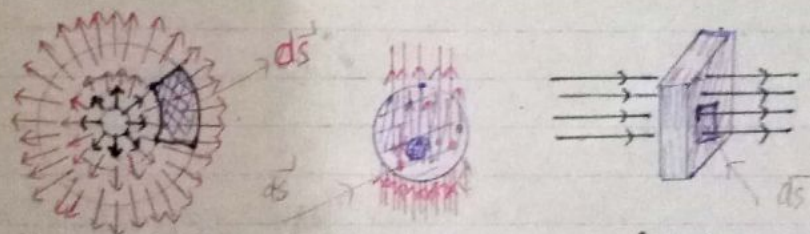


$$\oint_S \vec{A}\cdot\hat{n}\,ds = \int \nabla\cdot\vec{A}\,d\tau .$$

Jadi divergensi itu tidak ~~dari~~ lain dari perubahan garis gaya garis gaya yang melalui permukaan s tertutup yg. dirumuskan

$$\nabla\cdot\vec{A} = \lim_{\Delta V\to 0} \frac{\oint \vec{A}\cdot d\vec{s}}{\Delta V}$$

untuk mengecek ada atau tidaknya suatu divergensi itu, dilihat dari jumlah garis gaya yg. melalui permukaan satu dengan jlm garis gaya yang melalui permukaan kedua (ada perubahan garis gaya yang melalui permukaan tertutup)



$\oint_S \vec{A}\cdot d\vec{s} \neq 0$ $\qquad$ $\oint \vec{A}\,d\vec{s} \neq 0$. $\qquad$ $\oint \vec{A}\cdot d\vec{s} = 0$ (bukan divergensi).

Kalau tidak nol, lantas berapa harga $\oint \vec{A}\cdot d\vec{s}$?

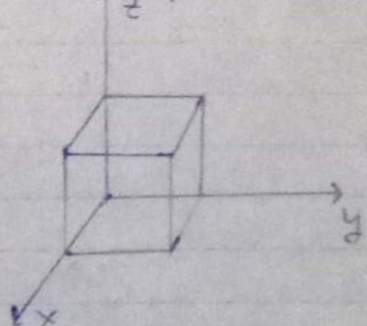
Contoh:

Vektor posisi $\vec{v}$ diberikan oleh:

$$\vec{v} = y^2\hat{i} + (2xy + z^2)\hat{j} + 2yz\hat{k}$$

Berapa nilai divergensinya.

solusi:



$$\nabla\cdot\vec{v} = 2y + 2x.$$
$$= 2(x+y).$$
$$\int_v \nabla\cdot\vec{v}\,d\tau = \int_v 2(x+y)\,d\tau.$$
$$= 2\int_v (x+y)\,d\tau.$$
$$= 2\iiint (x+y)\,dx\,dy\,dz.$$
$$= 2\left[\int (x+y)\,dx \int (x+y)\,dy \int dz\right]$$
$$= 2.$$

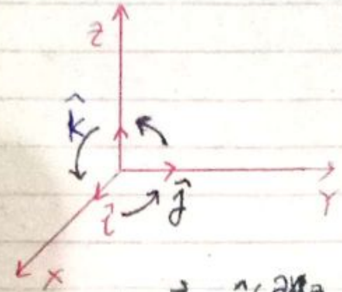
1.2.3. Rotasi / Curl

Bila vektor $\vec{V} = \hat{i}V_x + \hat{j}V_y + \hat{k}V_z$

maka rotasi dari $\vec{V}$ adalah:

$$\nabla \times \vec{V} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times \left(\hat{i}V_x + \hat{j}V_y + \hat{k}V_z\right).$$

dengan menggunakan aturan perkalian silang seperti



$$\hat{i} \times \hat{j} = \hat{k} \;;\; \hat{j} \times \hat{i} = -\hat{k}$$
$$\hat{j} \times \hat{k} = \hat{i} \;;\; \hat{k} \times \hat{j} = -\hat{i}$$
$$\hat{k} \times \hat{i} = \hat{j} \;;\; \hat{i} \times \hat{k} = -\hat{j}$$

$$\nabla \times \vec{V} = \hat{i}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) + \hat{j}\left(\frac{\partial V_x}{\partial z} - \frac{\partial V}{\partial x}\right) + \hat{k}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right).$$

atau dg. metoda Geometri

$$\nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix} = \hat{i}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) + \hat{j}\left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right) + \hat{k}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right).$$

Dalam Fisika pengertian rotasi dituliskan sbg

$$\nabla \times \vec{V} = \lim_{\Delta s \to 0} \frac{\Delta \tau}{\Delta s} = \frac{d\tau}{ds}$$

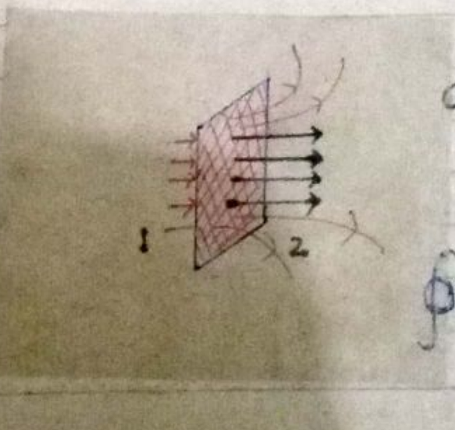
menurut teorema stokes

$$\int \nabla \times \vec{V}\, d\vec{s} = \oint_C \vec{V} \cdot d\vec{\ell}$$

Dengan demikian rotasi itu ada jika:

$$\oint \vec{V} \cdot d\vec{\ell} \neq 0 \text{ , atau}$$
$$\nabla \times \vec{V} \neq 0$$

$$\oint \vec{V} \cdot d\vec{\ell} \neq 0$$



$$d\ell_1 \neq d\ell_2$$

$$\oint \vec{V} \cdot d\vec{\ell} \neq 0.$$

1.3 Teorema Gauss dan Stokes

Teorema Gauss

$$\boxed{\oint_S \vec{A}\, d\vec{s} = \int \nabla\cdot\vec{A}\, d\tau}$$

Bila $\vec{A}(x,y,z) = A(x,y,z)\vec{a}$

$\vec{a}$ suatu vektor konstan yang arahnya sembarang $U(x,y,z)$

sehingga:

$$\vec{a}\cdot\oint_S A\, d\vec{s} = \int \nabla\cdot\vec{a}A\, d\tau$$

$$= \vec{a}\cdot\int_V \nabla A\, d\tau$$

$$\vec{a}\cdot\oint A\, d\vec{s} - \vec{a}\cdot\int \nabla A\, d\tau = 0$$

Bila $\vec{A} = \vec{a}\times\vec{P}$, $\vec{a}$ vektor konstan

maka bentuk teorema Gauss menjadi

$$\oint_S d\vec{s}\times\vec{P} = \int \nabla\times\vec{P}\, d\tau$$

menurut teorema Stokes perumusannya:

$$\oint_C \vec{A}\, dx = \int \nabla\times\vec{A}\cdot d\vec{s}\,, \quad \text{atau}$$

$$\boxed{\oint_C \vec{A}\cdot d\vec{l} = \int \nabla\times\vec{A}\cdot d\vec{s}}$$

$$\oint_C \vec{A}\cdot\hat{n}\, d\vec{l} = \int \nabla\times\vec{A}\cdot\hat{n}\, ds$$

Bila $\vec{A} = \vec{b}\phi$, maka:

$$\oint \vec{b}\phi\cdot d\vec{l} = \int \nabla\times(\vec{b}\phi)\cdot d\vec{s}$$

$$\vec{b}\cdot\oint\phi\, d\vec{l} = \int\left[\phi(\nabla\times\vec{b}) - \vec{b}\times(\nabla\phi)\right]d\vec{s}$$

$$= \int\left[\overbrace{\phi(\nabla\times\vec{b})}^{=0} + \cancel{(\nabla\phi)\times\vec{b}}\right]d\vec{s} - \int\vec{b}\times(\nabla\phi)\cdot d\vec{s}$$

$$= \int \cancel{\nabla\phi\times\vec{b}\cdot d\vec{s}} = \int d\vec{s}(\nabla\phi)\times\vec{b}$$

$$= \vec{b}\int d\vec{s}\times\nabla\phi$$

$$\vec{b}\left[\oint\phi\, d\vec{l} - \int d\vec{s}\times\nabla\phi\right] = 0.$$

$$\oint\phi\, d\vec{l} = \int d\vec{s}\times\nabla\phi.$$

Bila $\vec{A} = \vec{b} \times \vec{P}$, maka :

$$\oint_C (\vec{b} \times \vec{P}) \cdot d\vec{l} = \int \nabla \times (\vec{b} \times \vec{P}) \cdot d\vec{s}$$

$$\vec{b} \cdot \oint_C \vec{P} \times d\vec{P} = \int \nabla \cdot [(\vec{b} \times \vec{P}) \times d\vec{s}$$

$$= \int d\vec{s} \, [\nabla \times (\vec{b} \times \vec{P})]$$

$$= \int (\vec{b} \times \vec{P})(d\vec{s} \times \nabla) .$$

$$= \int \vec{b} \, (\vec{P} \times (d\vec{s} \times \nabla) .$$

$$= -\vec{b} \int (d\vec{s} \times \nabla) \times \vec{P}$$

$$\vec{b} \oint_C \vec{P} \times d\vec{l} = -\vec{b} \int (d\vec{s} \times \nabla) \times \vec{P} .$$

$$-\vec{b} \oint_C d\vec{l} \times \vec{P} = -\vec{b} \int (d\vec{s} \times \nabla) \times \vec{P}$$

$$\vec{b} \left[\oint_C d\vec{l} \times \vec{P} - \int (d\vec{s} \times \nabla) \times \vec{P} \right] = 0 .$$

$$\boxed{\oint_C d\vec{l} \times \vec{P} = \int (d\vec{s} \times \nabla) \times \vec{P} .}$$

1.3. Kordinat cartesius, kordinat bola, koordinat silinder.

1.3.1. Koordinat cartesius (x, y, z)

$d\vec{r} = \hat{i} dx + \hat{j} dy + \hat{k} dz$

$d\tau = dx\, dy\, dz$

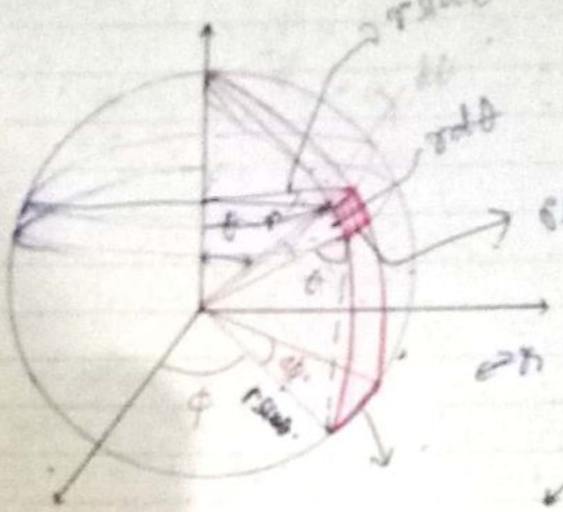
- gradien : $\nabla t = \hat{i} \frac{\partial t}{\partial x} + \hat{j} \frac{\partial t}{\partial y} + \hat{k} \frac{\partial t}{\partial z}$

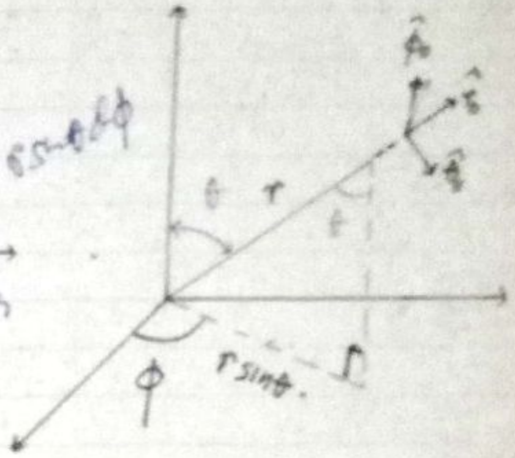
- Divergensi : $\nabla \cdot \vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$

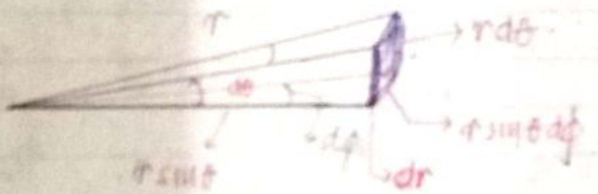
- Rotasi : $\nabla \times \vec{V} = \hat{i} \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z} \right) + \hat{j} \left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x} \right) + \hat{k} \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \right) .$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

1.3.2. Koordinat bola (r, θ, ϕ)







$d\vec{a} = p \times l$

$= r\sin\theta\, d\phi \cdot r\, d\theta$

$= r^2 \sin\theta\, d\theta\, d\phi$

$d\tau = p \times l \times t$

$= r\sin\theta\, d\phi \cdot r\, d\theta \cdot dr$

$= r^2 \sin\theta\, d\theta\, dr\, d\phi$

$d\vec{l} = \hat{r}\, dr + \hat{\theta}\, r\, d\theta + \hat{\phi}\, r\sin\theta\, d\phi$

- Gradiennya: $\nabla t = \hat{r}\frac{\partial t}{\partial r} + \hat{\theta}\left(\frac{1}{r}\frac{\partial t}{\partial \theta}\right) + \hat{\phi}\left(\frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\right)$

- Divergensinya: $\nabla \cdot \vec{V} = \frac{1}{r^2}\left(\frac{\partial}{\partial r} r^2 V_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(V_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial V_\phi}{\partial \phi}$

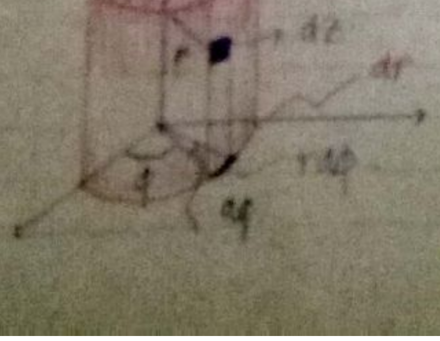
1.3.3. Koordinat silinder (r, ϕ, z)

$d\vec{l} = \hat{r}\, dr + \hat{\phi}\, r\, d\phi + \hat{z}\, dz$

$d\tau = r\, dr\, d\phi\, dz$

Gradiennya: $\nabla t = \hat{r}\left(\frac{\partial t}{\partial r}\right) + \hat{\phi}\left(\frac{1}{r}\frac{\partial t}{\partial \phi}\right) + \hat{z}\left(\frac{\partial t}{\partial z}\right)$

Divergensinya: $\nabla \cdot \vec{V} = \frac{1}{r}\frac{\partial}{\partial r}(r V_r) + \frac{1}{r}\frac{\partial V_\phi}{\partial \phi} + \frac{\partial V_z}{\partial z}$



1. $f(r) = ar^n$

$\nabla f(r) = ?$

solusi : $\nabla f(r) = \hat{r}_0 \dfrac{df(r)}{dr}$

$= \hat{r}_0 \dfrac{d(ar^n)}{dr}$

$= \hat{r}_0 \, n a r^{n-1}$

$= a n r^{n-1} . \hat{r}_0$

2. Bila $\vec{r} = ix + \hat{j}y + \hat{k}z$

$\nabla . \vec{r} = ?$

solusi : $\nabla . \vec{r} = \left[\dfrac{\partial}{\partial x} i + \dfrac{\partial}{\partial y}\hat{j} + \dfrac{\partial}{\partial z}\hat{k}\right]\left[ix + \hat{j}y + \hat{k}z\right]$

$\nabla . \vec{r} = 3$

3. Bila $f(r) = r^{n-1}$, hitung $\nabla . \vec{r} r^{n-1}$

solusi :

$\nabla . (\vec{r} r^{n-1}) = \vec{r} . \nabla r^{n-1} + r^{n-1}(\nabla . \vec{r})$

$= \vec{r} . \dfrac{\partial (r^{n-1})}{\partial r} \hat{r}_0 + 3r^{n-1}$

$= \vec{r}\hat{r}_0 (n-1) . r^{n-2} + 3r^{n-1}$; $\hat{r}_0 = \dfrac{\vec{r}}{r} \Rightarrow \vec{r} = \hat{r}_0 . r$

$= (\hat{r}_0 . r) . \hat{r}_0 (n-1) r^{n-2} + 3r^{n-1}$

$= (n-1) . r^{n-1} + 3r^{n-1}$.

$= (n-1+3) . r^{n-1}$

$\therefore \nabla . \vec{r} r^{n-1} = (n+2) . r^{n-1}$

4. $\nabla \times \vec{r} f(r) = f(r) \nabla \times \vec{r} + \nabla f(r) \times \vec{r}$

$\nabla \times \vec{r} = 0$; $\nabla f(r) \times r = \hat{r}_0 \dfrac{df(r)}{dr} \times r = 0$.

$= 0$

5. Hitung Laplace dari fungsi

a). $T = x^2 + 2xy + 3z + 4$.

b). $T = \sin x \sin y \sin z$

c). $T = e^{-5x} \sin 4y \cos 3z$

d). $v = x^2 i + 3xz^2 \hat{j} - 2xz\hat{k}$

$\nabla^2 V =$

solusi : a). $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \nabla^2 T$

$= 2 + 0 + 0 = \nabla^2 T.$

$\nabla^2 T = 2.$

b) Dengan cara yang sama

$\nabla^2 T = -3 \sin x \sin y \sin z$

c). $\nabla^2 T = 0$

d). $\nabla^2 V = 2i + 6x\hat{j}.$

Problem.

1. Tentukan vektor satuan yang tegak lurus pada permukaan $x^2 + y^2 + z^2 = 3$, di titik $(1,2,1)$
2. Diketahui fungsi skalar $s(x,y,z) = (x^2+y^2+z^2)^{-3/2}$
   a). Hitung ∇s di titik $(1,2,3)$.
   b). Hitung harga gradien s di titik $(1,2,3)$.

3. Bila $\vec{r}$ suatu vektor jarak dari titik asal O ke sebarang titik ruang (x,y,z).
   a). Buktikan bahwa $(\vec{U}\cdot\nabla)\vec{r} = \vec{U}$, bila sebarang vektor
   b). Buktikan bahwa $\nabla(\vec{A}\cdot\vec{r}) = \vec{A}$, bila vektor konstan.

4. Tunjukkan bahwa:
   a). $\frac{1}{3}\oint \vec{r}\cdot d\vec{s} = V$; dimana V = Volume permukaan tertutup.
   b). $\vec{B} = \nabla\times\vec{A}$, Tunjukan bahwa $\oint \vec{B}\, d\vec{s} = 0$, untuk setiap permukaan tertutup.

5. Suatu partikel bergerak dalam lintasan lingkaran
   $\vec{r} = \hat{i}\, r \cos\omega t + \hat{j}\, r \sin\omega t$
   a). Hitung $\vec{r} \times \dot{\vec{r}}$
   b). Tunjukkan bahwa $\ddot{\vec{r}} \times \omega^2 \vec{r} = 0$